Stereo Matching Using Belief Propagation
Paper Reading for CIS II Course Project

Xiang Xiang

Mentors: Dr. Dan Mirota, Dr. Greg Hager and Dr. Russ Taylor

Computer Science, JHU

March 14, 2013
Outline

- Background
- Paper Review: Approach
- Discussion
Matching

Matching, correspondence, alignment, registration, ...

- String matching: theoretic CS, brute force, KMP.
- Color matching: color space, perception theory

- **Point (cloud) matching**: image registration, ICP, kd-tree

- **Feature (point) matching**: NNDR, kd-tree BBF, Scott and Longuet-Higgins

- Shape matching: distance transform, chamfer matching, active contour, level set, shape context

- Surface matching: 3D shape matching, rendering

- **Stereo matching**: block matching, DP, MRF

- Temporal matching: HMM, DWT
Assumption: parallel optic axis

\[ d = x - x' \]

\[ Z = f \frac{T}{d} \]
Loosen assumption using rectification
Stereo algorithms generally perform:
1. matching cost computation;
2. cost (support) aggregation;
3. disparity computation and optimization;
4. disparity refinement.

Joint probability of the MRF:

\[ P(D, C, G) = \prod_{p \in \mathcal{P}} \Phi(d_p, c_p) \prod_{(p, q) \in \mathcal{N}} \Psi(d_p, d_q, g_{pq}) \]

Disparity optimization: M.L.E by (Loopy) Belief Propagation
Max Likelihood in Belief Propagation (BP)  
\[ E(f) = E_{data}(f) + E_{smooth}(f) \]
\[ = \sum_{p \in \mathcal{P}} U(d_p, c_p) + \sum_{(p,q) \in \mathcal{N}} V(d_p, d_q, g_{pq}) \]
\[ = \sum_{p \in \mathcal{P}} -\log \Phi(d_p, c_p) + \sum_{(p,q) \in \mathcal{N}} -\log \Psi(d_p, d_q, g_{pq}) \]

\[ P(\mathcal{D}, \mathcal{C}, \mathcal{G}) = \prod_{p \in \mathcal{P}} \Phi(d_p, c_p) \prod_{(p,q) \in \mathcal{N}} \Psi(d_p, d_q, g_{pq}) \]

Taking negative log-likelihood:
\[ -\log P(\mathcal{D}, \mathcal{C}, \mathcal{G}) = \sum_{p \in \mathcal{P}} -\log \Phi(d_p, c_p) + \sum_{(p,q) \in \mathcal{N}} -\log \Psi(d_p, d_q, g_{pq}) \]
Paper Review: Approach

**Stereo Matching Using Belief Propagation**

Jian Sun  
Nan-Ning Zheng  
Heung-Yeung Shum

Min Energy $\Rightarrow$ Max Likelihood $\Rightarrow$ Max a Posterior
Bayesian Model

\(D: \text{ Disparity field}\)

\(L: \text{ Line process field}\)

\(O: \text{ Occlusion field}\)

\[
P(D, L, O | I) = \frac{P(I | D, L, O) P(D, L, O)}{P(I)}
\]

Likelihood (independence assumption)

\[
P(I | D, O, L) = P(I | D, O)
\]

Prior (independence assumption)

\[
P(D, O, L) = P(D, L) P(O)
\]

Markov property

\[
P(D, L, O) \propto \prod_{s} \prod_{t \in N(s)} \exp(-\varphi_c(d_s, d_t, l_{s,t})) \prod_{s} \exp(-\eta_c(o_s))
\]

\(l_{s,t}, l_{s,t} \) is the line variable between \(d_s\) and \(d_t\)

\[
\varphi_c(d_s, d_t, l_{s,t}) = \varphi(d_s, d_t)(1 - l_{s,t}) + \gamma(l_{s,t})
\]

Combine all

\[
(P(D, O, L | I) \propto \prod_{s \notin O} \exp(-F(s, d_s, I)) \prod_{s} \exp(-\eta_c(o_s)) \prod_{s} \prod_{t \in N(s)} \exp(-\varphi_c(d_s, d_t)(1 - l_{s,t}) + \gamma(l_{s,t})))
\]

\[
F(s, d_s, I) \text{ is the matching cost function of pixel } s \text{ with disparity } d_s \text{ given observation } I
\]

\[
F(s, d_s, I) = \min\{\overline{d}(s, s', I)/\sigma_f, \overline{d}(s', s, I)/\sigma_f\}
\]
MAP \approx \text{MLE}

\[ (P(D, O, L|I) \propto \prod_{s \in O} \exp(-F(s, d_s, I)) \prod_s \exp(-\eta_c(o_s)) \prod_{s, t \in N(s)} \exp(-\varphi(d_s, d_t)(1 - l_{s,t}) + \gamma(l_{s,t}))) \]

\[ P(D, C, G) = \prod_{p \in P} \Psi(d_p, c_p) \prod_{(p, q) \in N} \Psi(d_p, d_q, g_{pq}) \]

M.A.P.

\[ \max_{D, L, O} P(D, L, O|I) = \]

\[ \max_D \left\{ \max_O \prod_s \exp(-(F(s, d_s, I)(1 - o_s) + \eta_c(o_s)o_s)) \right\} \]

\[ \max_L \prod_{s, t \in N(s)} \exp(-\varphi(d_s, d_t)(1 - l_{s,t}) + \gamma(l_{s,t})) \]

Relaxation

\[ \max_O \prod_s \exp(-(F(s, d_s, I)(1 - o_s^a) + \eta_c(o_s^a)o_s^a)) \]

\[ = \exp(-\min_O \sum_s (F(s, d_s, I)(1 - o_s^a) + \eta_c(o_s^a)o_s^a)) \]

Robust estimator

\[ \rho_d(d_s) = \min_{o_s^a} (F(s, d_s, I)(1 - o_s^a) + \eta_c(o_s^a)o_s^a) \]

\[ \rho_p(d_s, d_t) = \min_{l_{s,t}^a} (\varphi(d_s, d_t)(1 - l_{s,t}^a) + \gamma(l_{s,t}^a)) \]

\[ \max_D \left\{ \prod_s \exp(-\rho_d(d_s)) \prod_s \prod_{t \in N(s)} \exp(-\rho_p(d_s, d_t)) \right\} \]
Approximate MAP by BP

Simplify notation

\[
\prod_s \exp(-\rho_d(d_s)) \prod_{t \in N(s)} \prod_s \exp(-\rho_p(d_s, d_t))
\]

\[
\prod_{p \in P} \Phi(d_p, c_p) \prod_{(p,q) \in N} \Psi(d_p, d_q, g_{pq})
\]

\[
\prod_s \psi_s(x_s, y_s) \prod_{s \in N(s)} \prod_{t \in N(s)} \psi_{st}(x_s, x_t)
\]

\[
\psi_{st}(x_s, x_t) = m_{st}(x_s, x_t) = m_{st}(x_t)
\]

\[
\psi_s(x_s, y_s) = m_s(x_s, y_s) = m_s(x_s)
\]

Loopy BP by Max-Product

1. Initialize all messages \(m_{st}(x_t)\) as uniform distributions and messages \(m_s(x_s) = \psi_s(x_s, y_s)\).

2. Update messages \(m_{st}(x_t)\) iteratively for \(i = 1:T\)

\[
m_{st}^{i+1}(x_t) \leftarrow \kappa \max_{x_s} \psi_{st}(x_s, x_t) m_s^i(x_s) \prod_{x_k \in N(x_s) \setminus x_t} m_{ks}^i(x_s).
\]

3. Compute beliefs

\[
b_s(x_s) \leftarrow \kappa m_s(x_s) \prod_{x_k \in N(x_s)} m_{ks}(x_s)
\]

\[
x_s^{MAP} = \arg \max_{x_k} b_s(x_k).
\]

Power of Bayesian model: incorporating multi-priors

Multi-view extension: Modifying matching cost \(F\)
Results

(a) Ground truth. (b) Image segmentation result. (c) Textureless regions. (d) Max-product result without segmentation. (e) Discontinuity (white) and occlusion (black) regions. (f) Max-product result with segmentation.
Reference


Thank you! Comments!