Homework Assignment 4 – 600.455/655 Fall 2021

Instructions and Score Sheet (hand in with answers)

Name	Name
Email	Email
Other contact information (optional)	Other contact information (optional)
I have followed the rules in completing this assignment (signature)	I have followed the rules in completing this assignment (signature)

Question	Points		Question	Points		Total
1	10		5	20		
2	20		6	10		
3	20		7a	10		
4	20		7b	20		
			7c	10		
Subtotal	70		Subtotal	70		140

Note: There are 140 points on the assignment, but the most that will count toward your course grade is 100.

1. Remember that this is a graded homework assignment. It is the functional equivalent of a take-home exam.
2. You are to work **<u>alone or with your partner</u>** and are **<u>not to discuss the problems with anyone</u>** other than the TAs or the instructor. **(NOTE:** You are strongly encouraged to work with a partner).
3. It is otherwise open book, notes, and web. But you should cite any references you consult.
4. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
5. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web. See the course organizational materials.
6. Sign and hand in the score sheet as the first sheet of your assignment.
7. You will submit the assignment in PDF form to Gradescope, as discussed in class.

Scenario: Ultrasound-guided Intervention - Calibration

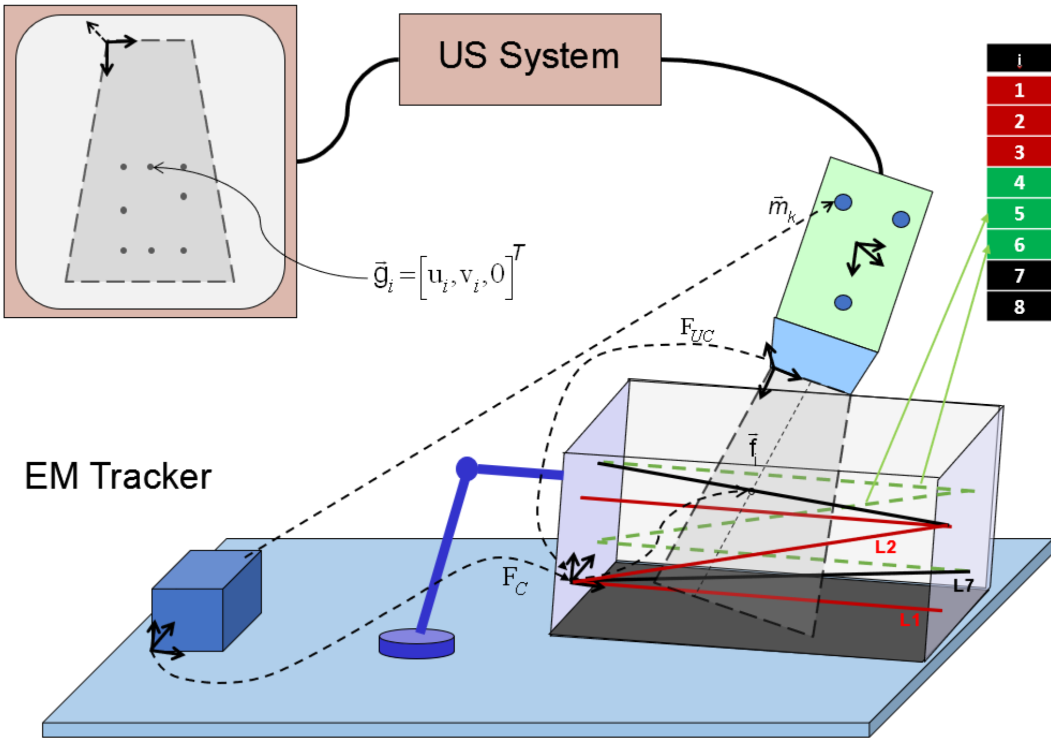

Table 1

i	Endpoints relative to object	
1	[0,0,0]	[0,0,c]
2	[0,0,0]	[0,b,c]
3	[0,b,0]	[0,b,c]
4	[a,0,0]	[a,0,c]
5	[a,0,0]	[a,b,c]
6	[a,b,0]	[a,b,c]
7	[0,0,0]	[a,0,c]
8	[a,b,0]	[0,b,c]

Figure 1 Overall Calibration Setup

Consider the ultrasound system shown in Figure 1**.** The system consists of an ultrasound B-mode imaging system equipped with a linear array, an EM tracker system, and a calibration object. There are three EM markers mounted on the ultrasound probe at unknown locations relative to the probe. The EM tracker system is, however, able to measure the 3D position $\vec{\mathbf{m}}_k$ of each marker *k*. The calibration object is held in a fixed, but unknown, pose $\mathbf{F}_C$ relative to the EM tracker. Inside the calibration object are eight wires or thin rods whose endpoints are as indicated in Figure 1 (note three rods (1,2,and 3) in RED forming Z pattern, three rods (4,5, and 6) in GREEN forming an opposite Z pattern, and two (7, and 8) BLACK rods connecting both Z patterns). The center of the image of each wire *i* in ultrasound image coordinates is given by $\vec{\mathbf{g}}_i = [u_i, v_i, 0]$. This position corresponds to an (unknown) position $\vec{\mathbf{f}}_i$ in calibration object coordinates where the ultrasound B-mode scan plane crosses wire *i*. The probe is positioned so that all eight wires are visible in the ultrasound image and that the image processing software is able to determine which wire is which, so that the $\vec{\mathbf{g}}_i$ are known unambiguously.

Questions

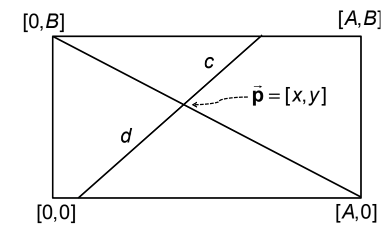

Figure 2

1. Consider the rectangle in Figure 2. A line cuts across the middle of the rectangle and intersects the $[0,B]-[A,0]$ diagonal at an unknown point $\vec{\mathbf{p}}=[x,y]$. The lengths of the two segments of this line are c and d, as shown. Give a formula for computing $\vec{\mathbf{p}}$.
2. Describe a method for computing the transformation F_{UC} between calibration object coordinates and ultrasound image coordinates. Give all pertinent formulas in sufficient detail so that it would be possible to implement your method. If there is a known algorithm from class, you can just use it, but you need to be clear what the inputs and outputs will be. **Hint:** you might want to consider how you might find the values of some of the $\vec{\mathbf{f}}_i$. You might wonder if Question 1 is somehow relevant to your answer.
3. Suppose that the image processing software is accurate to within an amount δ. I.e., a measured location $\vec{\mathbf{g}}_i$ will differ from its true value $\vec{\mathbf{g}}_i^*$ with a bound $\left|\vec{\mathbf{g}}_i^*-\vec{\mathbf{g}}_i\right|\le\delta$. This will introduce some error into your estimation of the transformation F_{UC}. If the actual value is F_{UC}^*, then we have $\mathrm{F}_{UC}^*=\mathrm{F}_{UC}\Delta\mathrm{F}_{UC}\approx\mathrm{F}_{UC}\bullet\left[\mathrm{I}+skew\left(\vec{\alpha}\right),\vec{\varepsilon}\right]$. Develop a set of constraints for estimating the limits on the components of $\vec{\alpha}$ and $\vec{\varepsilon}$. As in Homework Assignment #2, express your answer in terms of linearized constraints with no terms involving $skew(\vec{\alpha})$. Here, I am looking for a set of linear inequalities of the form

   $$\left|\mathbf{M}_k\vec{\alpha}+\vec{\varepsilon}\right|\le\left|\text{linearized expression involving }\delta,a,b,c\text{ and some of the }\vec{\mathbf{g}}_i\right|$$

   Hint: you might want to consider the effects of δ on how accurately you can find $\vec{\mathbf{f}}_k$.

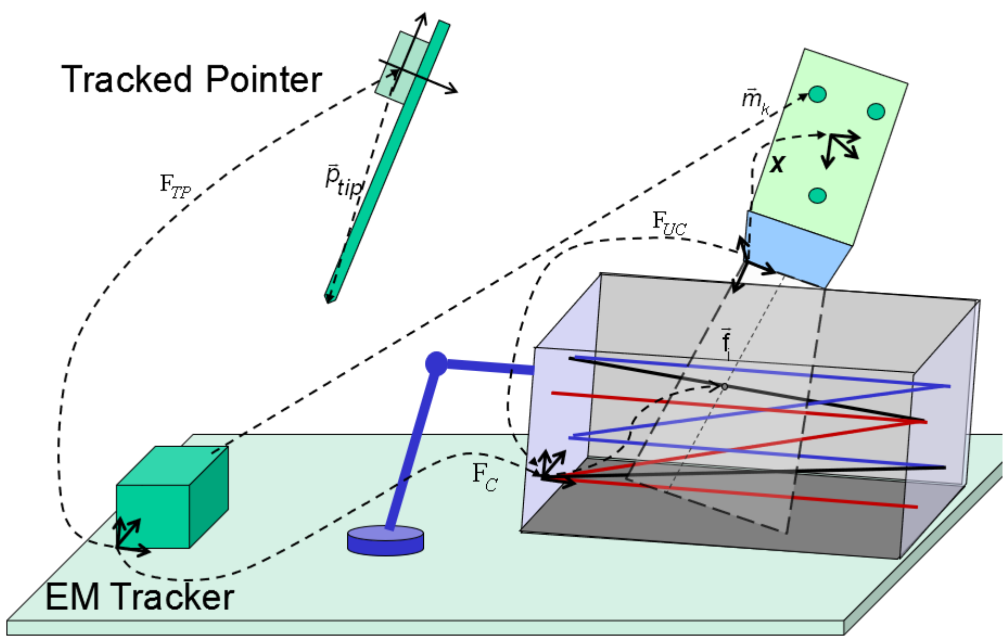

Figure 3 Tracked Pointer Calibration Approach

4. Suppose you have access to a tracked pointer as shown in Figure 3. Describe a method for determining the unknown transformation from the ultrasound probe rigid body to ultrasound image coordinates system (*X*). Give all pertinent formulas in sufficient detail so that it would be possible to implement your method.

5. Suppose you don't have access to the tracked pointer mentioned in Q4. Can you still calibrate for this unknown transformation? Give all pertinent formulas in sufficient detail so that it would be possible to implement your method. (Hint: try to formulate AX=XB setup as in Figure 4)

6. Discuss the pros and cons of both calibration approaches with a tracked pointer vs AX=XB. Include as many factors that may be pertinent to someone designing a system for actual use. Please justify your answers with specific reasons, including references to math, where appropriate.

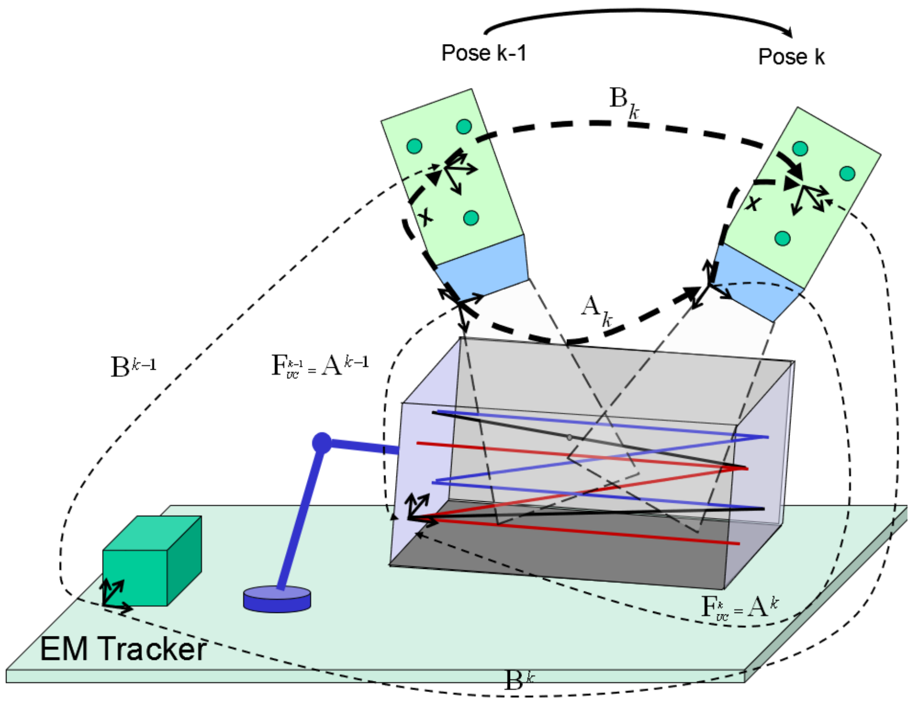

Figure 4 AX=XB Calibration Approach

7. As the ultrasound probe moves from one pose to another (Figure 4), the computed relative frame transformation of the ultrasound image coordinate motion A has a small error ΔA. Similarly, there is a small error ΔB in the relative EM tracking frame B of the ultrasound rigid-body during this motion. Starting from $A\Delta A \cdot X\Delta X = X\Delta X \cdot B\Delta B$, which we can write as $\mathrm{M} = \left[\mathrm{R}_M, \vec{\mathrm{P}}_M\right]$ and $\Delta\mathrm{M} = \left[\Delta\mathrm{R}_M, \Delta\vec{\mathrm{P}}_M\right]$ where M can be either A, B or X.

   a- Write expressions giving the error in the calibration matrix X. Give expressions for the errors ΔR_X and $\Delta\vec{p}_X$ in terms of the $\Delta R's$ and $\Delta\vec{p}'s$ of A's and B's.

   b- Give linearized approximations in "normalized" form for these error values.

   c- Now expand this formula from one motion to multiple motions A_K and B_K.