Homework Assignment 6 – 600.455/655 Fall 2021

Instructions and Score Sheet (hand in with answers)

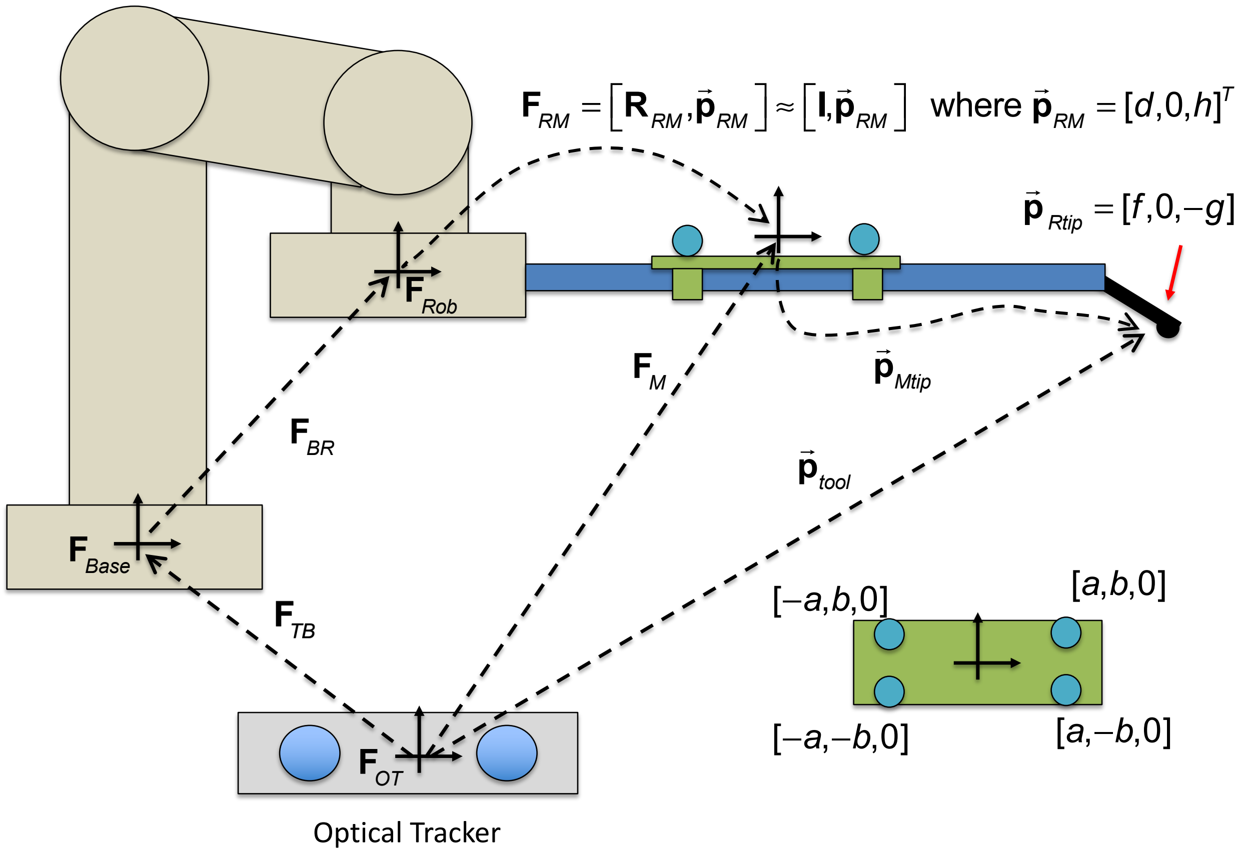
Name	Name
Email	Email
Other contact information (optional)	Other contact information (optional)
I have followed the rules in completing this assignment (signature)	I have followed the rules in completing this assignment (signature)

Question	Points		Question	Points		Total
1A	5		2A	5		
1B	10		2B	10		
1C	10		2C	10		
1D	10		2D	20		
1E	15		2E	15		
			2F	15		
			2G	20		
Subtotal	50		Subtotal	95		145

Note: There are 145 points on the assignment, but the most that will count toward your course grade is 100. Note that you do not have to attempt all the questions to get a good score. The intent is to give you some options for what questions to attempt.

1. Remember that this is a graded homework assignment. It is the functional equivalent of a take-home exam.
2. You are to work **alone or with your partner** and are **not to discuss the problems with anyone** other than the TAs or the instructor. **(NOTE:** You are strongly encouraged to work with a partner).
3. It is otherwise open book, notes, and web. But you should cite any references you consult.
4. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
5. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web. See the course organizational materials.
6. Sign and hand in the score sheet as the first sheet of your assignment.
7. You will submit the assignment in PDF form to Gradescope, as discussed in class.

Question 1



Consider the situation shown above. Here, we have a tool attached to a robot's end-effector. The coordinate system of the end-effector is $\mathbf{F}_{Rob}$, and the tip of the tool is located at $\vec{\mathbf{p}}_{Rtip} = [f,0,-g]$ relative to $\mathbf{F}_{Rob}$. An optical tracking marker body has been attached to the shaft of the tool at $\mathbf{F}_{RM} = [\mathbf{R}_{RM}, \vec{\mathbf{p}}_{RM}]$ relative to the end-effector, where $\vec{\mathbf{p}}_{RM} = [d,0,h]^T$. The optical marker body has four optically tracked spheres located at $\vec{\mathbf{p}}_{Mk} = [\pm a, \pm b, 0]$ relative to the coordinate system of the marker body.

The optical tracker can locate optical spheres to an accuracy of δ. I.e., if the tracker measures the position of a sphere as $\vec{\mathbf{p}}_k$ relative to the tracker, then we know that $\left\| \vec{\mathbf{p}}^* - \vec{\mathbf{p}} \right\| \le \delta$, where $\vec{\mathbf{p}}^*$ is the actual position of the sphere.

A. Give an expression for the nominal position $\vec{\mathbf{p}}_{Mtip}$ of the tool tip relative to the marker body. I.e. what is $\vec{\mathbf{p}}_{Mtip}$ such that $\mathbf{F}_{RM}\vec{\mathbf{p}}_{Mtip} = \vec{\mathbf{p}}_{Rtip}$. (**Note:** Due to a typo, this was originally $\mathbf{F}_{RM}\vec{\mathbf{p}}_{Mtip} = \vec{\mathbf{p}}_{tip}$. I.e., $\vec{\mathbf{p}}_{tip} = \vec{\mathbf{p}}_{Rtip}$.)

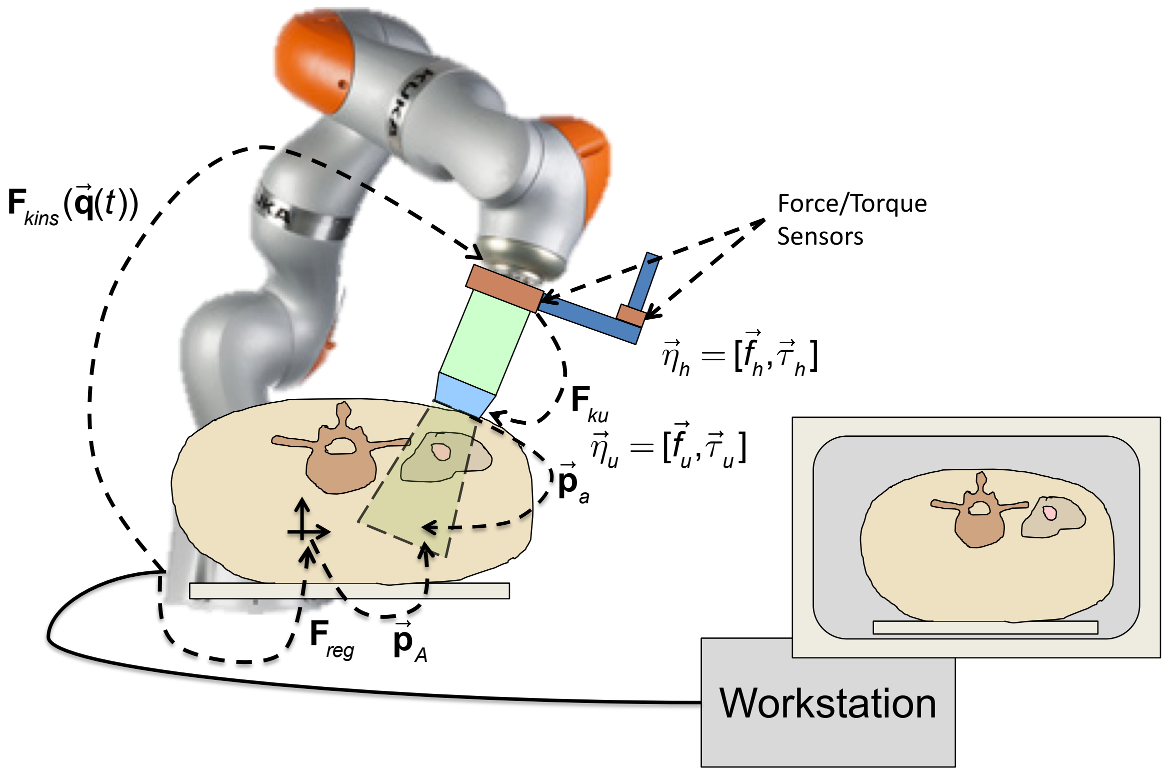
B. Give an expression estimating how accurately the optical tracking system can locate the pose $\mathbf{F}_M$ of the marker body relative to the optical tracking system. I.e., if $\mathbf{F}_M^*$ is the actual pose, then give expressions limiting the magnitude of $\vec{\alpha}_M$ and $\vec{\varepsilon}_M$ where $\mathbf{F}_M^* = \Delta\mathbf{F}_M\mathbf{F}_M$ and $\Delta\mathbf{F}_M \approx [\mathbf{I} + sk(\vec{\alpha}_M), \vec{\varepsilon}_m]$. **HINT:** Think about the geometry of the problem. **(NOTE:** You can give either element-wise limits on the components of $\vec{\alpha}_M$ and $\vec{\varepsilon}_M$ or an approximate upper bound on the overall magnitudes $\|\vec{\alpha}_M\|$ and $\|\vec{\varepsilon}_M\|$).

C. Suppose, for the moment, that $\mathbf{R}_{RM} = \mathbf{I}$. For $\mathbf{F}_M^* = \Delta\mathbf{F}_M\mathbf{F}_M$ and $\Delta\mathbf{F}_M \approx [\mathbf{I} + sk(\vec{\alpha}_M), \vec{\varepsilon}_m]$, give an expression for the error $\Delta\vec{\mathbf{p}}_{tool} = \vec{\varepsilon}_{tool}$ in measuring the position $\vec{\mathbf{p}}_{tool}$ of the tool tip relative to the optical tracking system in terms of $\vec{\alpha}_M$, $\vec{\varepsilon}_M$, and $\vec{\mathbf{p}}_{Mtip}$. Express your answer in normalized linear form.

D. Combine your answers to Questions 1B and 1C to estimate a worst-case error for the magnitude of $\vec{\varepsilon}_{tool}$

E. Suppose that the optical marker may have been shifted and twisted slightly on the shaft of the tool. Describe a calibration procedure for estimating the value of $\mathbf{F}_{RM}$. You may assume that the pose $\mathbf{F}_{TB}$ of the robot base coordinate system $\mathbf{F}_{Base}$ relative to the optical tracker is fixed, though unknown. Also, you should use the notation $\mathbf{F}_{BR,t}$ to represent the pose of the robot's end-effector relative to its base at time *t* and $\mathbf{F}_{M,t}$ to represent the measured pose of the tracker marker body relative to the optical tracker. You may assume that the robot and optical tracking system are sufficiently accurate so that $\mathbf{F}_{BR,t}$ and $\mathbf{F}_{M,t}$ have negligible errors. **Hint:** this can be reduced to an AX=XB problem. Give sufficient details of the workflow and formulation so that it is clear how you will solve the problem, but you do not need to recite all the details of the AX=XB solver algorithm, provided that you give enough of the formulation so that it is clear how this is an AX=XB problem.

Question 2



Note that, in general, we will adopt the notation $\vec{\xi} = [\vec{\alpha}^T, \vec{\varepsilon}^T]^T$ to indicate a set of small orientation and position variables. We will use $\Delta\mathbf{F}(\vec{\xi}) \approx [\mathbf{I} + sk(\vec{\alpha}), \vec{\varepsilon}]$ to indicate the corresponding pose change.

Consider the robotically-assisted ultrasound system shown above. This system has a workstation, an ultrasound system, and a robot and two force/torque (F/T) sensors. One of these sensors is attached to a handle, which, in turn is attached to the tooling attachment plate of the robot. When the human user exerts forces or torques on this handle, the F/T sensor senses these values and the workstation computes a corresponding F/T vector $\vec{\eta}_h = [\vec{\tau}_h, \vec{f}_h]$ resolved in the coordinate system of the robot's tooling attachment plate, where $\vec{\tau}$ represents torque and $\vec{\mathbf{f}}$ represents force. The robot has another F/T sensor that is also attached to the tooling plate and to an ultrasound probe sensor. When forces or torques are applied to the ultrasound probe (e.g., when the probe is pressed against the patient) this sensor measures them and the workstation computes an F/T vector $\vec{\eta}_u = [\vec{\tau}_u, \vec{f}_u]$ resolved in the coordinate associated with the ultrasound sensor probe (i.e., where the ultrasound sensor contacts the patient's anatomy).

The workstation is able to read the joint values $\vec{\mathbf{q}}(t)$ of the robot and has a function $\mathbf{F}_{kins}(\vec{\mathbf{q}})$ that computes the pose of the tooling plate relative to the base coordinate system of the robot. It can output a new set of position goals at every sample interval. The workstation also has a function

$$\mathbf{J}_{kins}(\vec{\mathbf{q}}) = \begin{bmatrix} \mathbf{J}_{\alpha}(\vec{\mathbf{q}}) \\ \mathbf{J}_{\varepsilon}(\vec{\mathbf{q}}) \end{bmatrix}$$

such that for small changes $\Delta\vec{\mathbf{q}}$, the corresponding pose of the robot's tooling plate pose is given by $\mathbf{F}_{kins}(\vec{\mathbf{q}} + \Delta\vec{\mathbf{q}}) = \mathbf{F}_{kins}(\vec{\mathbf{q}})\Delta\mathbf{F}_{kins}(\vec{\mathbf{q}}, \Delta\vec{\mathbf{q}})$, where $\Delta\mathbf{F}_{kins} \approx \Delta\mathbf{F}_k(\vec{\xi}_k) = [\mathbf{I} + sk(\vec{\alpha}_k), \vec{\varepsilon}_k]$ and

$$\vec{\xi}_k = \begin{bmatrix} \vec{\alpha}_k \\ \vec{\varepsilon}_k \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{\alpha}(\vec{\mathbf{q}}) \\ \mathbf{J}_{\varepsilon}(\vec{\mathbf{q}}) \end{bmatrix} \Delta\vec{\mathbf{q}}$$

The joints of the robot have limited range. I.e., $\vec{\mathbf{q}}^{lower} \le \vec{\mathbf{q}} \le \vec{\mathbf{q}}^{upper}$ and also have limited velocity $\dot{\vec{\mathbf{q}}}^{lower} \le \dot{\vec{\mathbf{q}}} \le \dot{\vec{\mathbf{q}}}^{upper}$. (You can also assume $\dot{\vec{\mathbf{q}}}^{lower} \le \vec{\mathbf{0}} \le \dot{\vec{\mathbf{q}}}^{upper}$)

The workstation has a "mid-level" motion control loop that runs at a sample interval time ΔT (i.e., every ΔT seconds), and a low-level controller that controls joint velocities, but stops motion if it does not receive a new command within a short interval after the time it expects to see one. Typical values for ΔT are usually somewhere between 2 and 10 ms. For the purposes of this assignment, you can assume that the computer workstation is fast enough to finish whatever it needs to do in the assigned time. To simplify the problem, we will also assume that the mid-level controller runs at regular ΔT intervals and that the overall control loop logic is as follows:

Step 1. Read robot state ($\vec{\mathbf{q}}, \dot{\vec{\mathbf{q}}}, etc.$) and sensor values.
Step 2. Perform safety checks and stop motion if needed.
Step 3. Output $\Delta\vec{\mathbf{q}} / \Delta T$ values computed on previous clock tick (done now so commands to low level controller come at predictable times).
Step 4. Interpret any commands from task level controller.
Step 5. Compute $\mathbf{F}_{kins}(\vec{\mathbf{q}})$ and $\mathbf{J}_{kins}(\vec{\mathbf{q}})$.

Step 6. Based on the current commanded behavior, robot state & kinematics, sensor values, and registered anatomic model, compute the desired incremental joint motion $\Delta\vec{\mathbf{q}}$ for the next clock tick. Typically, this will involve formulating and solving an optimization problem.

Step 7. Go to sleep until the next clock tick.

The workstation also has a model of the patient's anatomy, which has been registered to the robot, so that a position $\vec{\mathbf{p}}_A$ in the patient coordinate system corresponds to $\mathbf{F}_{reg}\vec{\mathbf{p}}_A$ in the robot coordinate system. We will also assume that for every point $\vec{\mathbf{p}}_s$ on the surface of the patient model, we are able to compute a surface normal $\vec{\mathbf{n}}_s$ pointing outward from the surface.

Similarly, a calibration has been performed so that a point at location $\vec{\mathbf{p}}_a = [x_a, 0, z_a]$ in an ultrasound image corresponds to a point $\mathbf{F}_{ku}\vec{\mathbf{p}}_a$ relative to the tooling plate of the robot. The pose of the ultrasound probe relative to the base frame of the robot is thus $\mathbf{F}_{Bu} = \mathbf{F}_{kins}\mathbf{F}_{ku}$.

Note: We sometimes describe haptic interfaces in which the human pushes on the robot and the robot moves accordingly as "admittance-type" interfaces. Similarly, we refer to interfaces where the robot pushes back on the human in response to motion by the human as "impedance-type" interfaces. The "steady hand" robot virtual fixtures described in class were of the admittance type.

In the questions below, you can assume that the basic optimization problem solved in Step 6 of the controller has the general form of the following constrained least-squares problem, as discussed in class

$$\Delta\vec{\mathbf{q}} = \underset{\Delta\vec{\mathbf{q}}}{\arg\min} \left\| \mathbf{A}_{\vec{\xi}}\vec{\xi} - \vec{\mathbf{b}}_{\vec{\xi}} \right\|^2 + \left\| \mathbf{A}_{\Delta\vec{\mathbf{q}}}\Delta\vec{\mathbf{q}} - \vec{\mathbf{b}}_{\Delta\vec{\mathbf{q}}} \right\|^2$$

$$\vec{\xi} = \mathbf{J}_{kins}(\vec{\mathbf{q}})\Delta\vec{\mathbf{q}}$$

$$\mathbf{C}_{\vec{\xi}}\vec{\xi} \le \vec{\mathbf{d}}_{\vec{\xi}}$$

$$\mathbf{C}_{\Delta\vec{\mathbf{q}}}\Delta\vec{\mathbf{q}} \le \vec{\mathbf{d}}_{\Delta\vec{\mathbf{q}}}$$

where the **A**'s**, b**'s, **C**'s, and **d**'s may have many rows, corresponding to different considerations in the objective function and constraints.

A. Write an optimization problem for performing basic admittance control for the robot, subject to the constraint that the joints should never go outside of their allowable range or velocity. I.e., the commanded Cartesian velocity of the robot's end effector should be proportional to the forces and torques exerted on the control handle ($\vec{\xi}_{cmd} = \mathbf{K}\vec{\eta}_h$). **Hint:** This is very similar to what you might see in your lecture notes.

B. Now, extend your optimization problem to include a constraint on the velocity of the ultrasound probe relative to the base of the robot. **Hint:** Let $\mathbf{F}_u = \mathbf{F}_{kins}(\vec{\mathbf{q}}(t))\mathbf{F}_{Ku}$ and $\Delta\mathbf{F}_u\mathbf{F}_u = \mathbf{F}_{kins}(\vec{\mathbf{q}}(t)+\Delta\vec{\mathbf{q}})\mathbf{F}_{Ku}$. $\Delta\mathbf{F}_u \approx [\mathbf{I}+sk(\vec{\alpha}_u),\vec{\varepsilon}_u]$ and $\vec{\xi}_u = [\vec{\alpha}_u{}^T,\vec{\varepsilon}_u{}^T]^T$. Provide formulas for computing the components of $\vec{\varepsilon}_u$. The constraint will then be $\vec{\varepsilon}_u^{lower} \le \vec{\varepsilon}_u \le \vec{\varepsilon}_u^{upper}$. **NOTE:** This is simplified from the original question, which also asked for the components of $\vec{\alpha}_u$. **Hint:** You may still need to compute something like $sk(\vec{\alpha}_u)$ and insert its definition into your formula for $\vec{\varepsilon}_u$.

C. What would you add to the objective function of problem 2A to minimize the angle between the ultrasound probe's z-axis and the surface normal $\vec{\mathbf{n}}_s$ of the patient, when the probe is at position $\vec{\mathbf{p}}_s$ on the patient's surface.

D. In performing synthetic aperture imaging, it is often desirable that a 2D ultrasound probe moves so that the pixels in successive images taken as the probe moves across the patient's anatomy are all co-planar. What would you add to the optimization problem of Question 2C so that all the 2D images are co-planar and so that the probe remains as close as possible to the surface of the patient while moving across the surface? To reduce dependence on the answer to Question 2b, you may assume that the probe is currently at a pose $\mathbf{F}_u = [\mathbf{R}_u,\vec{\mathbf{p}}_u]$. Also, you may express your answer in terms of $\vec{\alpha}_u$ and $\vec{\varepsilon}_u$ where $\mathbf{F}_{kins}\Delta\mathbf{F}_{kins}\mathbf{F}_{ku} = \Delta\mathbf{F}_u\mathbf{F}_u$. **Hint:** Here, you will be adding things to the constraints and to the objective function.

E. Suppose that there is an anatomical feature at position $\vec{\mathbf{p}}_a$ in the ultrasound image corresponding to an anatomical feature at $\vec{\mathbf{p}}_A$ in the CT image. Express constraints that will keep this feature within a distance ρ of $\vec{\mathbf{p}}_a$ in the ultrasound image. **Note:** Remember that as the ultrasound probe moves, the feature will appear to move in the ultrasound image, so it won't always be at $\vec{\mathbf{p}}_a$. You may find it convenient to define a formula for $\vec{\mathbf{p}}_{im}(\vec{\mathbf{q}}+\Delta\vec{\mathbf{q}})$, the point in the image where $\vec{\mathbf{p}}_A$ appears as the ultrasound probe moves To reduce dependence on the answer to Question 2b, you may assume that the probe is currently at a pose $\mathbf{F}_u = [\mathbf{R}_u, \vec{\mathbf{p}}_u]$. Also, you may express your answer in terms of $\vec{\alpha}_u$ and $\vec{\varepsilon}_u$ where $\mathbf{F}_{kins} \Delta \mathbf{F}_{kins} \mathbf{F}_{ku} = \Delta \mathbf{F}_u \mathbf{F}_u$. **Hint:** First express this with a quadratic constraint, then produce a set of linear constraints that approximate this result.

F. How would you modify your answer to Question 2C to ensure that the angle θ between the probe's z-axis and $\vec{\mathbf{n}}_s$ never exceeds an angle $\theta^{\max}$. Also how would you modify the objective function so that there is no penalty for small angles $\theta < \theta^{thresh}$ but you penalize any angles in the range $\theta^{thresh} < \theta < \theta^{\max}$. To reduce dependence on the answer to Question 2b, you may assume that the probe is currently at a pose $\mathbf{F}_u = [\mathbf{R}_u, \vec{\mathbf{p}}_u]$. Also, you may express your answer in terms of $\vec{\alpha}_u$ and $\vec{\varepsilon}_u$ where $\mathbf{F}_{kins} \Delta \mathbf{F}_{kins} \mathbf{F}_{ku} = \Delta \mathbf{F}_u \mathbf{F}_u$. **Hint:** refer to the lecture notes for similar problems.

G. Suppose that an anatomic feature with CT coordinates $\vec{\mathbf{p}}_A$ is visible in the ultrasound image at coordinates $\vec{\mathbf{p}}_a$ and the probe position is at position $\vec{\mathbf{p}}_s$ on the patient. The surgeon has released the guiding handle, so that the handle force/torque sensor reading is $\vec{\eta}_h = \vec{\mathbf{0}}$. Suppose that there is some small patient motion (possibly deformable) and that we want the probe to comply to sensed forces and torques between the ultrasound probe and the surface of the patient to maintain a desired ultrasound force/torque reading $\vec{\eta}_u^{des}$. The patient is slightly compliant so that a small motion $\vec{\xi}_u$ of the ultrasound probe produces a change in force/torque sensor reading $\Delta\vec{\eta}_u = \mathbf{K}_s \vec{\xi}_u$ when the probe is at position $\vec{\mathbf{p}}_s$. At the same time, you wish to minimize the excursion of the feature in the ultrasound image and strictly limit the forces on the patient so that $\vec{\mathbf{f}}_u \le \vec{\mathbf{f}}_u^{\max}$. Formulate an optimization problem that will implement this behavior. To reduce dependence on the answer to Question 2b, you may assume that the probe is currently at a pose $\mathbf{F}_u = [\mathbf{R}_u, \vec{\mathbf{p}}_u]$. Also, you may express your answer in terms of $\vec{\alpha}_u$ and $\vec{\varepsilon}_u$ where $\mathbf{F}_{kins} \Delta \mathbf{F}_{kins} \mathbf{F}_{ku} = \Delta \mathbf{F}_u \mathbf{F}_u$. You may also assume (somewhat unrealistically) that the only force exerted by the probe on the patient is in the direction of $\vec{\mathbf{n}}_s$ and that the force exerted by the probe at $\vec{\mathbf{p}}_s$ is $\vec{\mathbf{0}}$. You may also want to limit the torques on the patient (I.e., $\left|\vec{\eta}_u\right| \le \vec{\eta}^{\max}$)