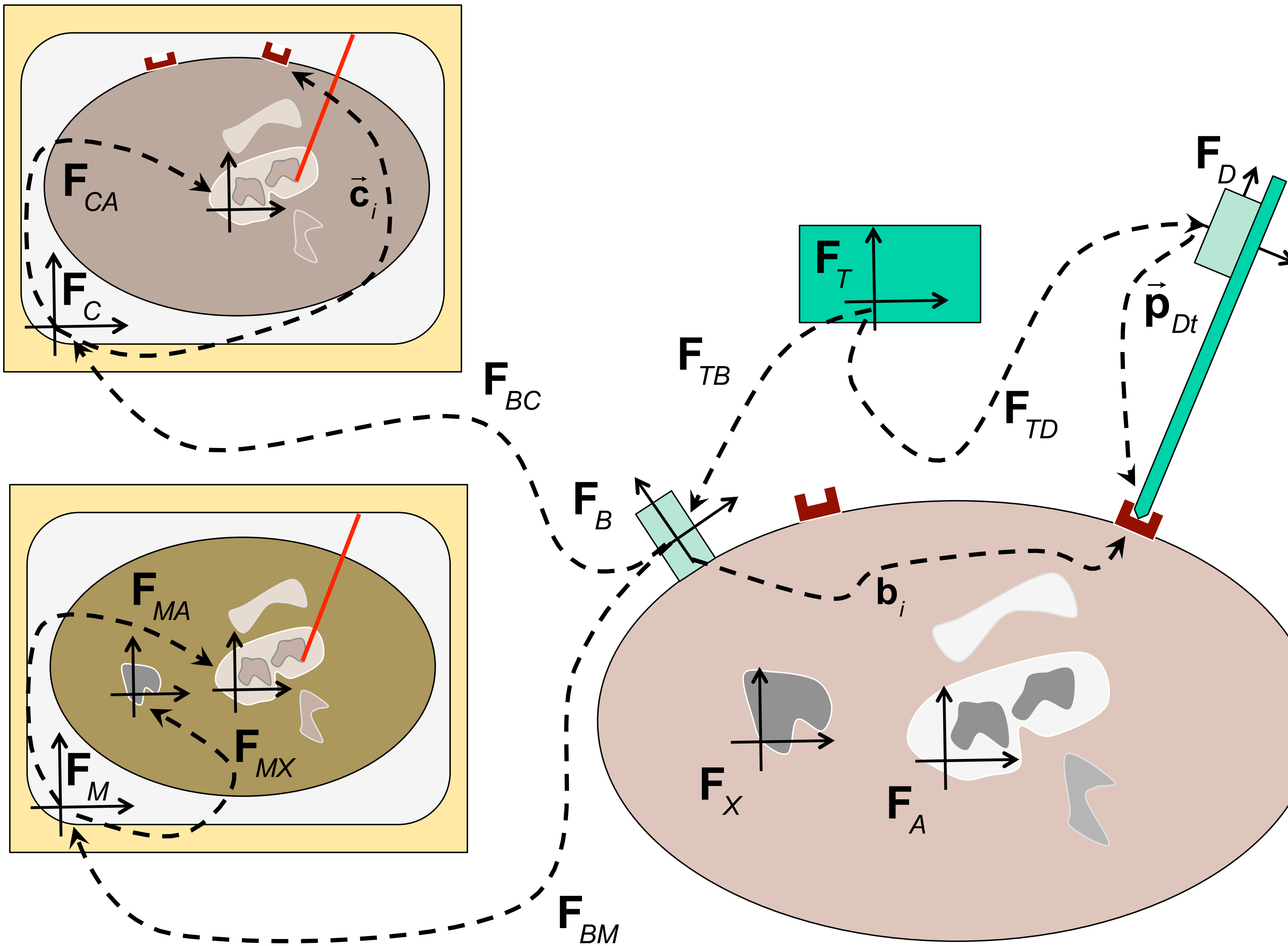
Homework 1 Scenario



Consider the stereotactic navigation scenario illustrated in the figure above. Here, we have a stereotactic tracking system whose coordinate system is represented by $\mathbf{F}_T$, capable of tracking the pose $\mathbf{F}_{TB}$ of a tracker body $\mathbf{F}_B$ attached to the patient and the pose $\mathbf{F}_{TD}$ of another tracker body $\mathbf{F}_D$ attached to a pointer tool. The position of the tip of the pointer tool may be calibrated to be at a position $\vec{\mathbf{p}}_{Dt}$ relative to $\mathbf{F}_D$ (we will do this as part of the assignment).

CT and MRI images of the patient are available. The anatomic structure $\mathbf{F}_X$ of greatest interest (which you may think of as a tumor or other malformation) is visible in the MRI image at pose $\mathbf{F}_{MX}$ in MRI coordinates but not in the CT image. However, another anatomic structure $\mathbf{F}_A$ is visible at pose $\mathbf{F}_{MA}$ in MRI coordinates and $\mathbf{F}_{CA}$ in CT coordinates.

After the MRI image was acquired, but before the CT image was acquired, small fiducial objects were pasted to the patient's skin. These markers are visible at locations $\vec{\mathbf{c}}_i$ in CT coordinates. During a registration step, the tracked pointer is placed on each of the small fiducials, and the corresponding position $\vec{\mathbf{b}}_i$ relative to $\mathbf{F}_B$ is computed (see questions, below). The corresponding values of $\vec{\mathbf{b}}_i$ and $\vec{\mathbf{c}}_i$ are used to compute the registration transformation $\mathbf{F}_{BC}$ between CT and patient tracker body coordinates, so that $\mathbf{F}_{BC}\vec{\mathbf{c}}_i \approx \vec{\mathbf{b}}_i$. At some point, we will want to know the registration $\mathbf{F}_{BM}$ between MRI and tracker body coordinates, but this has not yet been computed.

Question 1

A. Assuming that the value $\vec{\mathbf{p}}_{Dt}$ is known, and given values for $\mathbf{F}_{TB}$ and $\mathbf{F}_{TD}$ when the pointer tip is touching fiducial i, give a formula for computing $\vec{\mathbf{b}}_i$. Give the answer first in terms of the "$\mathbf{F}_{pq}$" variables and then in terms of the corresponding **"$\mathbf{R}_{pq}$"** and "$\vec{\mathbf{p}}_{pq}$" variables.

B. Now, suppose that $\vec{\mathbf{p}}_{Dt}$ is not known, outline a procedure for determining both $\vec{\mathbf{p}}_{Dt}$ and the $\vec{\mathbf{b}}_i$. Explain the workflow, explain what values will be measured, and give the computational algorithm. (Hint: this will involve a pivot calibration).

C. Assuming that $\mathbf{F}_{BC}$ has been computed by some registration process, give a formula for computing $\mathbf{F}_{BM}$, given the other information available. Give the answer first in terms of the "$\mathbf{F}_{pq}$" variables and then in terms of the corresponding **"$\mathbf{R}_{pq}$"** and "$\vec{\mathbf{p}}_{pq}$" variables.

D. Given values for $\mathbf{F}_{TB}$ and $\mathbf{F}_{TD}$, give a formula for computing the position $\vec{\mathbf{p}}_{Mt}$ in MRI coordinates corresponding to the current position $\vec{\mathbf{p}}_{tip}$ of the pointer tip. Give the answer first in terms of the "$\mathbf{F}_{pq}$" variables and then in terms of the corresponding **"$\mathbf{R}_{pq}$"** and "$\vec{\mathbf{p}}_{pq}$" variables.

E. Suppose that the surgeon is now operating on the patient and has identified an anatomic feature located at position $\vec{\mathbf{p}}_{Xf}$ relative to a local coordinate system $\mathbf{F}_{X}$ associated with the tumor. Given values for $\mathbf{F}_{TB}$ and $\mathbf{F}_{TD}$, give a formula for computing the distance between the current position $\vec{\mathbf{p}}_{tip}$ of the pointer tip and the anatomic feature. Give the answer first in terms of the "$\mathbf{F}_{pq}$" variables and then in terms of the corresponding **"$\mathbf{R}_{pq}$"** and "$\vec{\mathbf{p}}_{pq}$" variables.

Question 2

Here we will implement simple MATLAB functions to do some of these computations. You will be provided with a MATLAB package implementing basic Cartesian data types and computations (described elsewhere), together with a special MATLAB class "HW1Scenario", which will implement the following methods

HW1 = HW1Scenario(dR,dP,jR,jP)

% dR, dP = amount of random error assumed for tracker system for each returned measurement

% jR, jP = amount of random jogging of tracker base relative to patient

[FB,FD] = HW1.SampleAtMarker(m,rx,ry,rz)

% m = patient marker number

% rx,ry,rz = angles of rotation (in degrees) about x,y,z axes of the pointer probe relative to the tracking system when the pointer tip is placed onto patient marker m

[FCA,cMarkers] = HW1.SegmentCT()

% FCA = $\mathbf{F}_{CA}$; cMarkers = vct3Array of the $\vec{\mathbf{c}}_i$

[FMA,FMX] = HW1.SegmentMRI()

% FMA = $\mathbf{F}_{MA}$; FMX = $\mathbf{F}_{MX}$

A. Implement your answer to Question 1B using the MATLAB functions provided. You should produce a program with the general format

```
[pDt,b] = PerformPivotCalibration(HW1,m)
    %  m = patient marker number
    %  call the function HW1.SampleAtMarker(m,xD,yD,zD) multiple
       times to simulate taking data for pivot calibration
```

B. Implement a MATLAB function to compute $\mathbf{F}_{BC}$ and $\mathbf{F}_{BM}$ from the results of question 2A and a call to HW1.SegmentCT(). For this purpose you can also use the provided MATLAB function

```
F=FindBestRigidTransformation(A,B)
    % A,B = vct3Arrays of  3D points
    % F = Transformation such that F*A(i)=B(i)
```

Note that you will be implementing this function in a later assignment

C. Implement a MATLAB function to compute FBX = the pose of anatomic structure X relative to patient attached marker B.

Your homework should include a script that looks something like this

```
disp('No Error');
HW1 = HW1Scenario(0,0,0,0);
[b,dPt] = HW1PivotCal(HW1,1); disp([b.el';dPt.el']);
[fbc,fbm,fba,fbx]=HW1Register(HW1)

disp('Jogging Only is next');
HW1 = HW1Scenario(0,0,3,5);
[b,dPt] = HW1PivotCal(HW1,1); disp([b.el';dPt.el']);
[fbc,fbm,fba,fbx]=HW1Register(HW1)

disp('Tiny Noise is next');
[b,dPt] = HW1PivotCal(HW1,1); disp([b.el';dPt.el']);
[fbc,fbm,fba,fbx]=HW1Register(HW1)

disp('Larger Noise is next');
[b,dPt] = HW1PivotCal(HW1,1); disp([b.el';dPt.el']);
[fbc,fbm,fba,fbx]=HW1Register(HW1)
```