

Problem 1

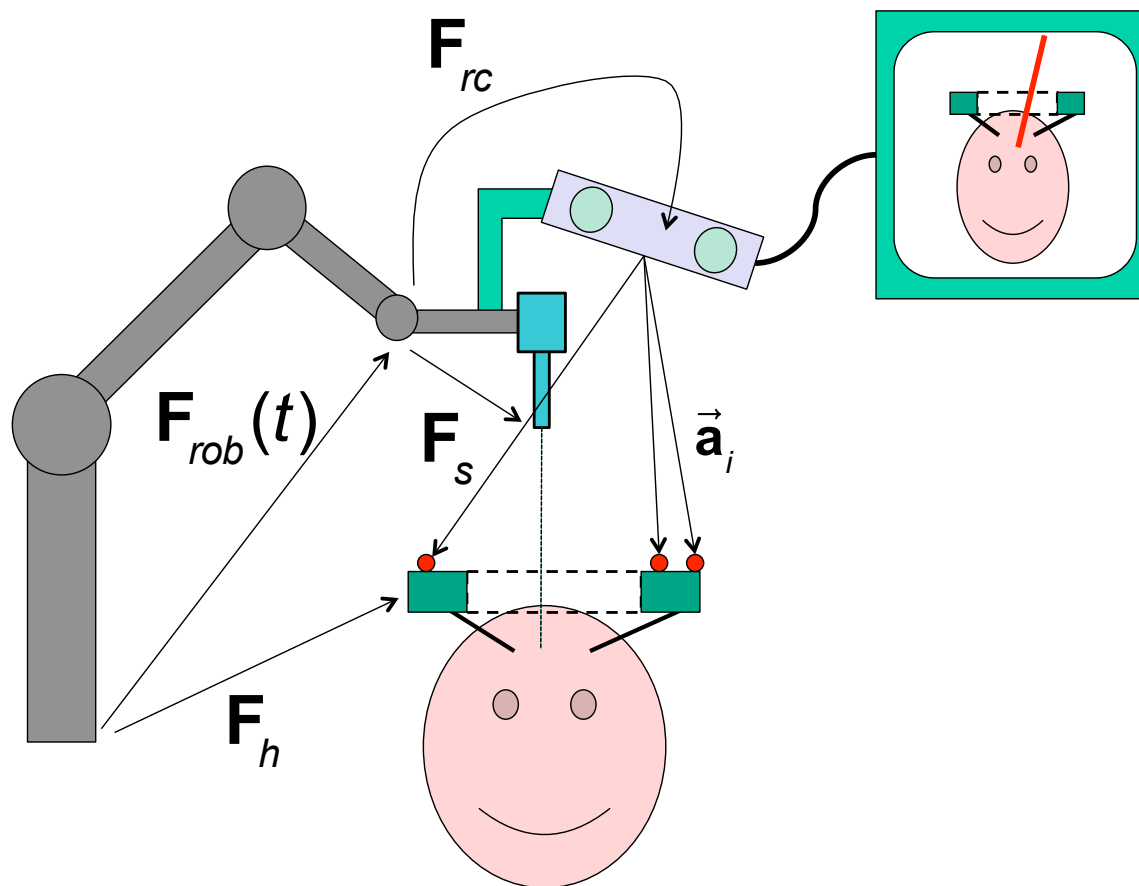


Fig. 1: Robot sensing setup

Consider the somewhat artificial situation shown in Fig. 1, although it is inspired by recent work at T.U. Munich by Nassir Navab and his students. Here we have a surgical robot that can be positioned to high accuracy at positions $\mathbf{F}_{rob}(t)$. A SPECT imaging sensor is attached at known pose \mathbf{F}_s relative to the robot's end effector. This sensor senses information along a highly collimated path along the z-axis of the sensor.

Also, a stereo vision system is attached at unknown pose \mathbf{F}_{rc} relative to the robot's end effector. A head-holding device is placed at unknown, but fixed pose \mathbf{F}_h relative to the robot's base. A patient is affixed to the holding device, to which are attached a number of reflective markers. The vision system is capable of capturing stereo images of the surgical scene and projecting them onto a stereo TV set. Given the parameters $\mathbf{L}_{line} = [\vec{\mathbf{p}}_{line}, \vec{\mathbf{n}}_{line}]$ of a line in space consisting of points $\vec{\mathbf{p}}_{line} + \lambda \vec{\mathbf{n}}_{line}$ relative to the camera coordinate system, where $\vec{\mathbf{p}}_{line}$ is a point on the line, $\vec{\mathbf{n}}_{line}$ is the direction of the line, and $|\vec{\mathbf{n}}_{line}| = 1$, the system is capable of superimposing computer graphics showing the corresponding position of the line on the stereo video monitor. Also, the system has image processing software capable of finding the markers in the video scene and stereo triangulation software for computing their positions $\vec{\mathbf{a}}_i(t)$ at time t .

- A. Describe a procedure for determining \mathbf{F}_{rc} . Include a complete workflow, specification of what measurements are to be taken, and sufficient algorithmic and mathematical detail so that one of ordinary skill could implement it. Use only the apparatus described in the problem statement. **Hint:** You will probably need to move the robot.
- B. Once \mathbf{F}_{rc} has been determined, explain how to compute the parameters in camera coordinates of a line $\mathbf{L}_s = [\vec{\mathbf{p}}_s, \vec{\mathbf{n}}_s]$ passing through the origin of the sensor coordinate system and running parallel to its z-axis
- C. Suppose that the calibrated value of \mathbf{F}_s is inaccurate, so that the true value is $\mathbf{F}_s^* = \mathbf{F}_s \Delta \mathbf{F}_s$ where $\Delta \mathbf{F}_s \approx [\mathbf{I} + sk(\alpha), \vec{\boldsymbol{\varepsilon}}]$. How will this affect your estimate of \mathbf{L}_s ?

Problem 2 – Programming

Suppose that you know that all the markers are visible to the stereo camera system whenever the robot is at a pose $\mathbf{F}_{rob}(t) = [\mathbf{R}_{rob}(t), \vec{\mathbf{p}}_{rob}(t)]$ where $\|\vec{\mathbf{z}} \times \mathbf{R}_{rob}(t) \vec{\mathbf{z}}\| \leq 0.5$ (i.e., the robot's end effector is tilted at most 30 degrees from vertical) and $[300, 300, 400] \leq \vec{\mathbf{p}}_{rob}(t) \leq [700, 700, 1000]$. Write a MATLAB

program to determine \mathbf{F}_{rc} . In addition to the MATLAB programs available to you for Homework 1, you have the following MATLAB class and methods available:

```
H = HW2Scenario(dP)
    % Sets up scenario.
    % dP is the amount of random "noise" associated with finding one marker via
    the stereo camera
aMarkers = H.PositionRobotAndFindMarkers(Frob)
    % Frob = desired pose of robot end effector wrt base
    % aMarkers = vct3Array of marker positions wrt the stereo camera
    % aMarkers.NumEl() = Number of markers
[aE,pE] = H.FrcErrorMag(FrcGuess)
    % FrcGuess = Estimated value for Frc
    % aE = error in degrees of FrcGuess.R compared to true value
    % pE = error in mm of FrcGuess.P compared to true value
```

Hint: You should place the robot in some known pose and observe a set of marker positions WRT the camera using the above function. Use these values to compute a nominal set of marker positions relative to some coordinate system fixed relative to the head holder. If $A = \{\vec{\mathbf{a}}_1, \dots, \vec{\mathbf{a}}_N\}$ are the observed values, it is OK to use $M = A - \text{mean}(A)$ or even just A . Then define a series of positions $\mathbf{F}_{rob}(t)$ and use the function

$A(t) = \text{PositionRobotandFindMarkers}(\mathbf{F}_{rob}(t))$ to find move to those positions

and measure marker positions $A(t)$. From these, you can use $\mathbf{F}_{ch}(t) = \text{FindBestRigidTransformation}(M, A(t))$ to find a series of transformations for the camera relative to the head-holder frame. Then solve an $AX=XB$ problem to find \mathbf{F}_{rc} and \mathbf{F}_h using the values $\{\dots[\mathbf{F}_{rob}(t), \mathbf{F}_{ch}(t)]\dots\}$.

For your programming assignment, you should include a runnable file HW2Script that looks something like this:

```
disp('No noise');
HW2 = HW2Scenario(0);
Frc = HW2CalibFrc(HW2,...) % add extra parameters for search pattern
Err = HW2.FrcErrorMag(Frc)

disp('Small noise');
HW2 = HW2Scenario(0.5);
Frc = HW2CalibFrc(HW2,...) % add extra parameters for search pattern
Err = HW2.FrcErrorMag(Frc)

disp('More noise');
HW2 = HW2Scenario(1.0);
Frc = HW2CalibFrc(HW2,...) % add extra parameters for search pattern
```

Err = HW2.FrcErrorMag(Frc)

Problem 3 (optional)

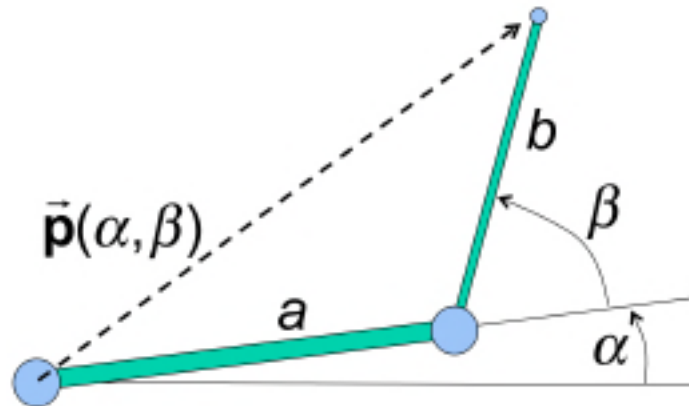


Fig. 2: 2 link manipulator

Consider the simple 2 link manipulator shown in Fig. 2, above. The links of this manipulator have lengths a and b , and the angles of the two links are measured as α and β , as shown. Unfortunately, the encoders of the manipulator are not accurate, so that the actual angles are $\alpha^* = \alpha + \Delta\alpha_0 + f(\alpha)$ and $\beta^* = \beta + \Delta\beta_0 + g(\beta)$. All we know about these error parameters are that they are fairly small and that

$$|\Delta\alpha_0| \leq \delta \quad |f(\alpha)| \leq \nu \quad |f'(\alpha)| \leq \gamma$$

$$|\Delta\beta_0| \leq \delta \quad |g(\alpha)| \leq \nu \quad |g'(\alpha)| \leq \gamma$$

In answering the questions below, you should express your answers in terms of the symbols, but you can assume the following order-of-magnitude values: $a \approx 500, b \approx 400, \delta \approx 0.01, \nu \approx 0.01, \gamma \approx 0.01$

- A. Give a formula for the nominal position $\vec{\mathbf{p}}(\alpha, \beta)$ of the end link.
- B. Assuming small errors, give a formula estimating the actual position $\vec{\mathbf{p}}(\alpha, \beta)^*$ of the end link that is linear in $\Delta\alpha_0, \Delta\beta_0, f(\alpha), g(\beta)$.
- C. Suppose that $\alpha = 0$ and $\beta = \pi/2$ estimate bounds on the error in $\vec{\mathbf{p}}(\alpha, \beta)$
- D. Suppose that the end of the manipulator has been placed accurately on a point known to be at position $[a, b]$ relative to the manipulator. The manipulator is now moved to a new point $[a + u, b + v]$ nearby and values for α and β are read. Estimate bounds on the error in calculating $\vec{\mathbf{p}}(\alpha, \beta)$, assuming that $|u| \leq b/10$ and $|v| \leq a/10$ and that you have used the available information.
- E. Now suppose that instead of precisely positioning the manipulator at $[a, b]$, the manipulator has been precisely placed at 4 points $[a \pm b/10, b \pm a/10]$. Estimate the maximum error for placing the manipulator within this rectangle, assuming that you have used the available information to perform some sort of calibration.