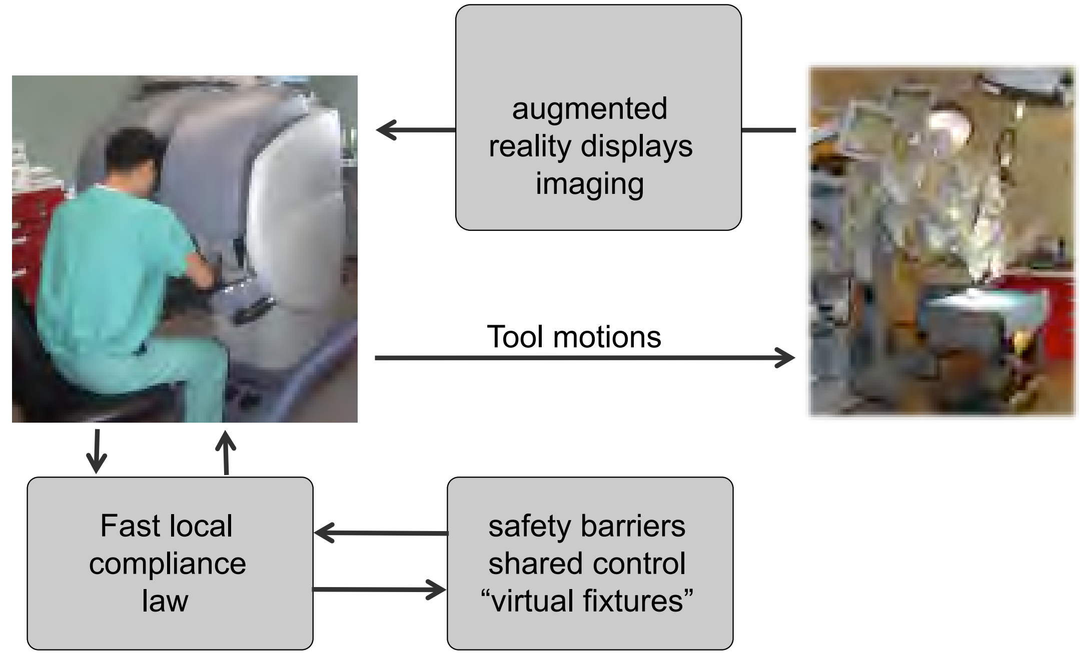
Problem 1: Haptic Virtual Fixtures

Consider the telesurgical system shown in the figure at right. In this system, the “patient side” robot’s tools always follow the positions of the corresponding hand controller manipulators held by the surgeon.



A very fast local (~1 kHz) compliance loop is able to observe the positions of the hand controllers, compute forces and torques to be exerted by the hand controllers on the surgeon’s hands, and output these forces and torques. A somewhat slower (~50 Hz) loop is also able to observe the robot’s state and other information and modify the parameters used by the fast compliance loop.

The control parameters are as follows:

$\mathbf{F} = \left[\mathbf{R}, \vec{\mathbf{p}}\right] =$ current pose; $\dot{\mathbf{p}} =$ current velocity

$\mathbf{F}_c = [\mathbf{R}_c, \vec{\mathbf{p}}_c] =$ position compliance frame

$\vec{\mathbf{k}}^{(+)}, \vec{\mathbf{k}}^{(-)} =$ position stiffness factors

$\vec{\mathbf{b}}^{(+)}, \vec{\mathbf{b}}^{(-)} =$ damping factors

$\vec{\mathbf{g}}^{(+)}, \vec{\mathbf{g}}^{(-)} =$ force bias terms

$\mathbf{R}_o =$ orientation compliance frame

$\vec{\mathbf{k}}_o^{(+)}, \vec{\mathbf{k}}_o^{(-)} =$ orientation stiffness factors

$\vec{\tau}^{(+)}, \vec{\tau}^{(-)} =$ torque bias terms

$t =$ time remaining on timeout counter

The fast control loop works as follows:

if (t>0) then

begin

$t = t - 1$

$\vec{\mathbf{q}} = \mathbf{F}_C^{-1}\vec{\mathbf{p}} = \mathbf{R}_C^{-1}\left(\vec{\mathbf{p}} - \vec{\mathbf{p}}_c\right)$

$\vec{\mathbf{v}} = \mathbf{R}_C^{-1}\dot{\mathbf{p}}$

$\vec{\mathbf{h}} = \vec{\mathbf{0}}; \vec{\psi} = \vec{\mathbf{0}}$

for $i \in \{x, y, z\}$ do

$\left\{\text{if } \vec{\mathbf{q}}_i \le 0 \text{ then } \vec{\mathbf{h}}_i = \vec{\mathbf{g}}_i^{(-)} + \vec{\mathbf{k}}_i^{(-)}\vec{\mathbf{q}}_i + \vec{\mathbf{b}}_i^{(-)}\vec{\mathbf{v}}_i \text{ else } \vec{\mathbf{h}}_i = \vec{\mathbf{g}}_i^{(+)} + \vec{\mathbf{k}}_i^{(+)}\vec{\mathbf{q}}_i + \vec{\mathbf{b}}_i^{(+)}\vec{\mathbf{v}}_i\right\};$

$\vec{\mathbf{f}} = \mathbf{R}_C\vec{\mathbf{h}}$; add $\vec{\mathbf{f}}$ to the forces exerted on the master

$\vec{\theta} =$ Rodrigues vector corresponding to $\Delta\mathbf{R} = \mathbf{R}_o^{-1}\mathbf{R}$

for $i \in \{x, y, z\}$ do

$\left\{\text{if } \vec{\theta}_i \le 0 \text{ then } \vec{\psi}_i = \vec{\tau}_i^{(-)} + \vec{\mathbf{k}}_i^{(-)}\vec{\theta}_i \text{ else } \vec{\psi}_i = \vec{\tau}_i^{(+)} + \vec{\mathbf{k}}_i^{(+)}\vec{\theta}_i\right\};$

add $\mathbf{R}_o\vec{\psi}$ to the torques exerted on the master

end

A MATLAB class is available to define these parameters, together with a method to compute the forces and torques.

$$\text{function } V\text{=HW4VirtualFixture}(\mathbf{F}_c, \vec{\mathbf{k}}^{(+)}, \vec{\mathbf{k}}^{(-)}, \vec{\mathbf{b}}^{(+)}, \mathbf{b}^{(-)}, \vec{\mathbf{g}}^{(+)}, \vec{\mathbf{g}}^{(-)} \ \ldots$$
$$\mathbf{R}_o, \vec{\mathbf{k}}_o^{(+)}, \vec{\mathbf{k}}_o^{(-)}, \tau^{(+)}, \tau^{(-)})$$

⋮

end

$$\text{function } [\vec{f}, \vec{\tau}] = \text{EvaluateHapticForceTorque}(V, \mathbf{F}_h, \vec{\mathbf{v}})$$

% $V =$ virtual fixture parameter block

% $\mathbf{F}_\mathrm{h} = [\mathbf{R}_h, \vec{\mathbf{p}}_h] =$ current hand pose

% $\vec{\mathbf{v}} =$ current velocity

⋮

end

A. Suppose that $\mathbf{F}_c$ is a point on an anatomic surface close to $\mathbf{F}_h$, with $\mathbf{F}_c \bullet \vec{\mathbf{z}}$ as the outward facing normal for the surface. Write a MATLAB function to define a virtual fixture that will produce a bias force of magnitude f_{bias} toward the surface if $\vec{\mathbf{p}}_h$ is above the surface, and produce a spring force with spring constant κ resisting penetration into the surface, but otherwise not constraining the motion. And

B. Use the function of Question 1.A to define a virtual fixture with $\mathbf{F}_c = [\mathbf{I},[100,200,300]]$, $f_{bias} = 10$ and $\kappa = 100$. Evaluate the resulting virtual fixture for $\mathbf{R}_h = RotMx.xyzD(10,30,45)$ and $\vec{\mathbf{p}}_h = [105,195,310]$, $\vec{\mathbf{p}}_h = [103,201,302]$, $\vec{\mathbf{p}}_h = [95,205,299]$, and $\vec{\mathbf{p}}_h = [101,199,298]$.

C. Starting from your answer to Question 1A, now write a virtual fixture that also generates a spring force pulling $\vec{\mathbf{p}}_h$ toward a line parallel to $\mathbf{R}_c\vec{\mathbf{x}}$ and passing through $\vec{\mathbf{p}}_c$ with spring constant κ_{line}. If $\vec{\mathbf{p}}_h$ is below the surface, then the restoring force in the direction of the surface should be proportional to κ_{surf}.

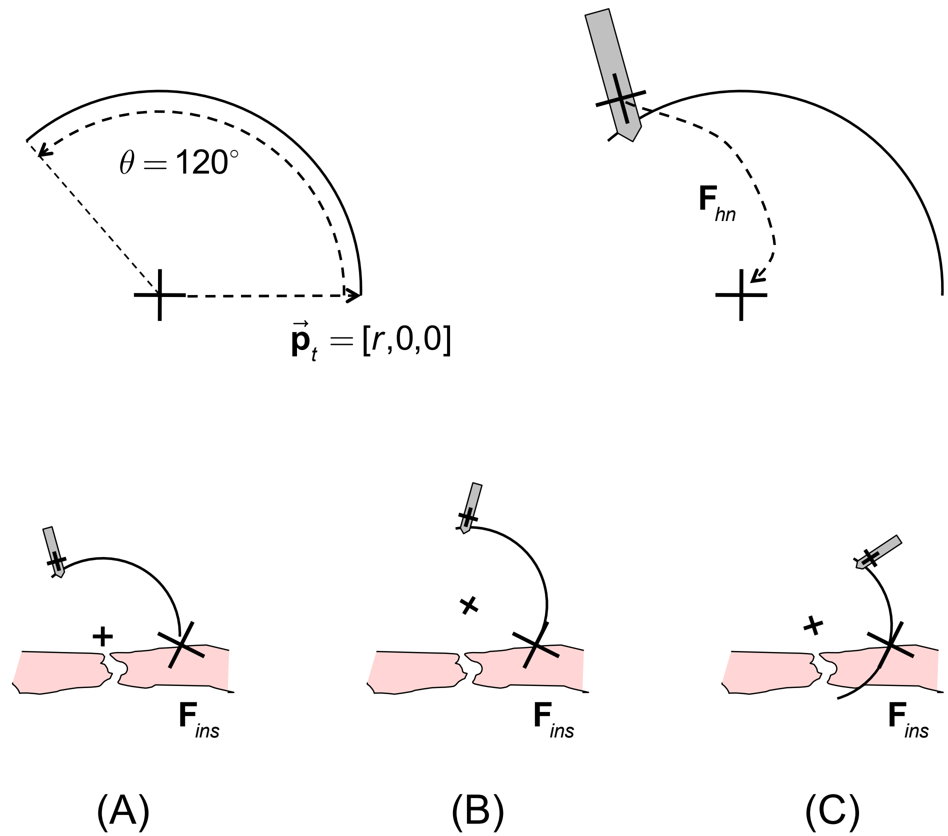
D. Use the function of Question 1.C to define a virtual fixture with $\mathbf{F}_c = [\mathbf{I},[100,200,300]]$, $f_{bias} = 5$, $\kappa_{line} = 25$ and $\kappa_{surf} = 100$. Evaluate the resulting virtual fixture for $\mathbf{R}_h = RotMx.xyzD(10,30,45)$ and $\vec{\mathbf{p}}_h = [105,195,310]$, $\vec{\mathbf{p}}_h = [103,201,302]$, $\vec{\mathbf{p}}_h = [95,205,299]$, and $\vec{\mathbf{p}}_h = [101,199,298]$.

E. Starting from your answer to Question 1A, now write a virtual fixture that also generates a spring torque aligning $\mathbf{F}_h\vec{\mathbf{z}}$ to $\mathbf{F}_c\vec{\mathbf{z}}$ with torque spring constant γ_{align}. The motion should not constrain motion about $\mathbf{F}_c\vec{\mathbf{z}}$.

F. Use the function of Question 1.C to define a virtual fixture with $\mathbf{F}_c = [\mathbf{I},[100,200,300]]$, $f_{bias} = 5$, $\kappa_{line} = 25$ and $\kappa_{surf} = 100$. Evaluate the resulting virtual fixture for $\mathbf{R}_h = RotMx.xyzD(1,3,5)$ and $\vec{\mathbf{p}}_h = [105,195,310]$, $\vec{\mathbf{p}}_h = [103,201,302]$, $\vec{\mathbf{p}}_h = [95,205,299]$, and $\vec{\mathbf{p}}_h = [101,199,298]$.

Problem 2: Virtual Fixtures for Suture Passing

Consider the curved surgical needle illustrated in the figure above. The sharp end of the needle is at point $\vec{\mathbf{p}}_t = [0,0,r]$ relative to the coordinate system of the needle. The needle is bent counterclockwise in a circular arc about an axis parallel to the $\vec{\mathbf{z}}$ axis of the needle coordinate system. The needle is held by the robot in a pose $\mathbf{F}_{hn}$, so that the coordinate system of the needle is given by $\mathbf{F}_n = \mathbf{F}_h \mathbf{F}_{hn}$.



The task is to insert this needle into tissue at $\mathbf{F}_{ins} = [\mathbf{R}_{ins}, \vec{\mathbf{p}}_{ins}]$ relative to robot coordinates. There are three phases: A) Move the needle so that its tip is positioned at $\vec{\mathbf{p}}_{ins}$ while maintaining orientation; B) align the needle so that $\mathbf{R}_n \vec{\mathbf{z}}$ is parallel to $\mathbf{R}_{ins} \vec{\mathbf{z}}$ while keeping the needle tip at $\vec{\mathbf{p}}_{ins}$; and C) advance the needle into the tissue so that $\mathbf{R}_n\vec{\mathbf{z}}$ remains parallel to $\mathbf{R}_{ins}\vec{\mathbf{z}}$ while the needle continues to pass through $\vec{\mathbf{p}}_{ins}$.

A. Define the parameters for a virtual fixture to assist in performing step A of the above procedure.

B. Define the parameters for a virtual fixture to assist in performing step B of the above procedure..

C. Define the parameters for a virtual fixture to assist in performing step C of the above procedure.

D. (Optional) Implement MATLAB functions for 2.A, 2.B, 2.C and demonstrate them by writing a suitable test program.

Hint: Your answers may depend on hand pose and will need to be continually updated.