# Paper Review

Hasan Tutkun Şen

March 10, 2011

**Title:** Constrained Cartesian Motion Control for Teleoperated Surgical Robots

**Authors:** Janez Funda, Russell H. Taylor, Benjamin Eldridge, Stephan Gomory, Kreg G. Gruben

**Summary:** In this paper, the optimal motion control of teleoperated surgical robots confined in a limited workspace is discussed. The problem is focused on determining which degrees of freedom at hand optimally. In this method the desired motion is defined as separate tasks in different coordinate frames subject to additional linear constraints. So the control problem proposed is minimizing some objective function using quadratic optimization techniques subject to some constraints.

In the motivation of the problem, laparoscopic surgery is given as an example due to its limited workspace. In this kind of surgeries, known as minimally invasive surgery in robotics world, the surgeon performs the surgery by inserting some tools into the abdomen part of the body through a single hole. This procedure is the best example of RCM mode of operation.

In analyzing the method discussed, the paper gives an example of RCM procedure in forming the optimization function and forming the constraining matrices. In this example the problem is: given the desired Cartesian displacement of the gaze frame: "$\Delta^g x_d = [x, y, z, 0,0,0]^T$, what is the appropriate robot motions which will move the gaze center to the new location while enforcing a number of constraints. Below is how this mathematical approach works:

1) Specifying Task Frame Objective Functions and Constraints:

Let us first Giving a tolerance, $\epsilon_g$, to hit the target location such that:

$$\left\| \begin{bmatrix} \Delta^g x[1] \\ \Delta^g x[2] \end{bmatrix} \quad \begin{bmatrix} \Delta^g x_d[1] \\ \Delta^g x_d[2] \end{bmatrix} \right\| \leq \epsilon_g \qquad (1)$$

where $\Delta^g x_d$ and $\Delta^g x$ are the desired and actual gaze displacements.

(1) can be approximated as a family of linear equations of the form:

$$[\cos(\theta_k), \sin(\theta_k), 0,0,0,0]^T (\Delta^g x - \Delta^g x_d) \leq \epsilon_g \quad (2)$$

and this can be rewritten in the form:

$$H_g \Delta^g x \geq h_g \qquad (3)$$

in which dim{$H_g$}= n x 6 and dim{$h_g$ }= nx1.

If one also wants to minimize the rotational error about the z-axis which is the viewing axis of the surgeon, then we need to minimize $\|\Delta^g x[6] - \Delta^g x_d[6]\|$ subject to above constraints. In doing this a weighting diagonal matrix is formed, such that the coefficient of the matrix corresponding to each actuator will penalize the actuator motion proportional to these coefficients. So in doing this we have another function to be optimized such that:

$$\left\|W_g(\Delta^g x - \Delta^g x_d)\right\| \qquad (4)$$

where $W_g$ is a diagonal matrix specifying relative importance of each actuator.

In the paper presented two more constraints, $H_e$, $H_j$ representing the end effector constraint and joint limit constraint are defined. Moreover, two more diagonal weighting matrices, $W_e$, $W_j$ representing minimization of the end effector movement and minimization of the total joint motion, in forming the objection function matrix are defined.

2) <u>Putting All Together :</u>

If all defined matrices are combined in a single equation the objection function and the constraint inequality equations become:

$$\left\|\begin{bmatrix} W_g & & \\ & W_e & \\ & & W_j \end{bmatrix}\left(\begin{bmatrix} J(q)^g \\ J(q)^e \\ I \end{bmatrix}\Delta q - \begin{bmatrix} (\Delta^g x_d \\ 0 \\ 0 \end{bmatrix}\right)\right\| \qquad (5)$$

$$\begin{bmatrix} H_g & & \\ & H_e & \\ & & H_j \end{bmatrix}\begin{bmatrix} J(q)^g \\ J(q)^e \\ I \end{bmatrix}\Delta q \geq \begin{bmatrix} h_g \\ h_e \\ h_j \end{bmatrix} \qquad (6)$$

These set of equations are solved numerically for the set of actuator displacement values.

3) <u>Assignment of Optimization Weights:</u> This part is important in assigning weighting coefficients for each actuator for a specific purpose of an optimization process. In this part the weighting factors are divided into two parts such that:

$$w_f[i] = u_f[i]v_f[i] \qquad (7)$$

where $u_f[i]$ stands for relative importance of minimizing the objection function for a particular DOF and $v_f[i]$ provides a scaling factor to adjust dimensionality and it takes care of the differences between the translational and rotational errors.

In the paper, it gives results for a sample experiment in which the above mathematic expressions are replaced with a real RCM operation robot having redundant DOF and it is seen that the results are pretty satisfactory except for the pivot-gaze trajectories in which the DOF being used is not enough for that particular goal.

In the following part of this paper it explains their software tools in implementing such algorithm, however; since this part is unrelated with my project I skipped this part in this paper review.

### Critique:

- This technique is useful for surgical systems for which the robot motion is slow compared to its internal loop so that one can approximate the incremental joint motion as joint velocities in that specified period of time.
- The paper also gives an example of RCM process which is very informative in showing how it works, besides it also shows the cases in which the algorithm fails and this is very useful for the users to avoid such cases in his/her programs.
- The algorithm needs a good initial guess for the initial position of the robot. However, its importance is overlooked in the paper.
- User must be careful in task-deficient operations as in the case of RCM mode operation since the required position may not be achievable with such constraints.

**Relevance:** The system to be developed has 7+4=11 degrees of freedom. So the system is already kinematically redundant such that, for 3 translational and 3 rotational unknowns (x, y, z, $R_x$, $R_y$, $R_z$) there are 11 variables to be determined.

The LARSnake robot motion can be classified as:

1) Coarse Motion
2) Fine Motion

In coarse motion, the snake will not move so the system will turn out to be 7 degrees of freedom LARS. So in implementing the constrained optimization algorithm, in developing the objection function matrix the coefficients of the snake actuators should be penalized more such that they should be larger.

In fine motion, it is aimed that x, y, z axis actuators will not move and the system will be an 8 dof system. In this motion RCM mode operation can also be developed as given as an example in the paper.

In the optimization problem, $||A\Delta q - b||$ is the objection function under the constraints $C\Delta q - d \geq 0$. As noted above the first critical part of the implementation process is the forming the objection "A and b" matrices. The first part of the "A" matrix will be made of the combined Jacobian of the system under concern. Below this, the weights of each actuator will reside diagonally. Depending on the type of the motion described above the penalization constants will differ. Likewise, the first part of the "b" column vector will be 6x1 the desired position and orientation of the end effector. Below it there will be zero.

Now that the objection function is formed, the next job is the formation of the constraint matrices "C" and "d". In snake side of the system, the constraints play a very important role because the capabilities of the snake robot are limited. It can bend around -45, +45 degrees from its equilibrium point so certain constrains are needed in order not converge to a value out of this range. In snake currently it is planned to be implemented:

1) Snake Joint Positive Limit Constraint
2) Snake Joint Negative Limit Constraint
3) Snake Joint Positive Rate Limit Constraint
4) Snake Joint Negative Rate Limit Constraint
5) Backbone Rate Constraint

Besides, in RCM mode certain constraints at the LARS side of the robot should be put. In doing that, the objection Jacobian should be changed such that the Jacobian should be recalculated at the RCM point. Again in this motion Cartesian axis actuators of the LARS side should be penalized more in order the keep that RCM point at a fixed location in space.