Seminar Presentation Systems Identification (RCM)

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Project Background

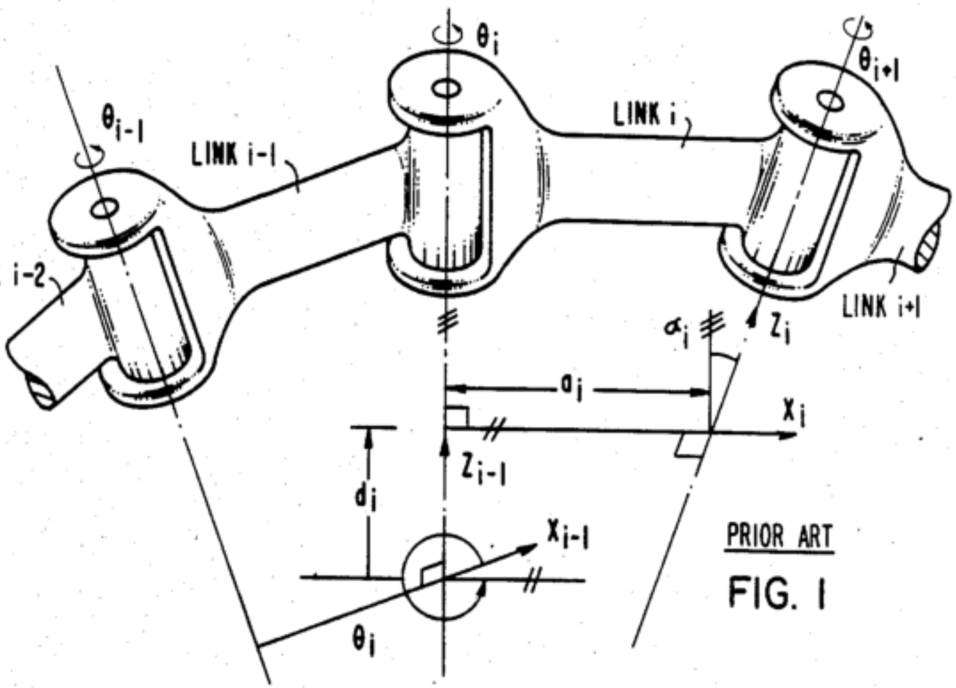
- There is some error in Revolving Needle Driver (RND) robot
- Actual kinematic description does not match idealized version
- Reality may be complicated
- Must determine a more accurate model based on behavior
- Must do Systems Identification
  - White-box
  - Grey-box
  - Black-box

Paper selection and why?

- *Nonlinear black-box modeling in system identification - a unified overview*
- Jonas Sjöberg et al.
- Great starting point for those trying to identify parameters in unknown systems
- Broadly applicable
- Written for the user

Necessary background

- Systems ID can't be 'formalized, automated'
- Methods can be novel



PRIOR ART
FIG. 1

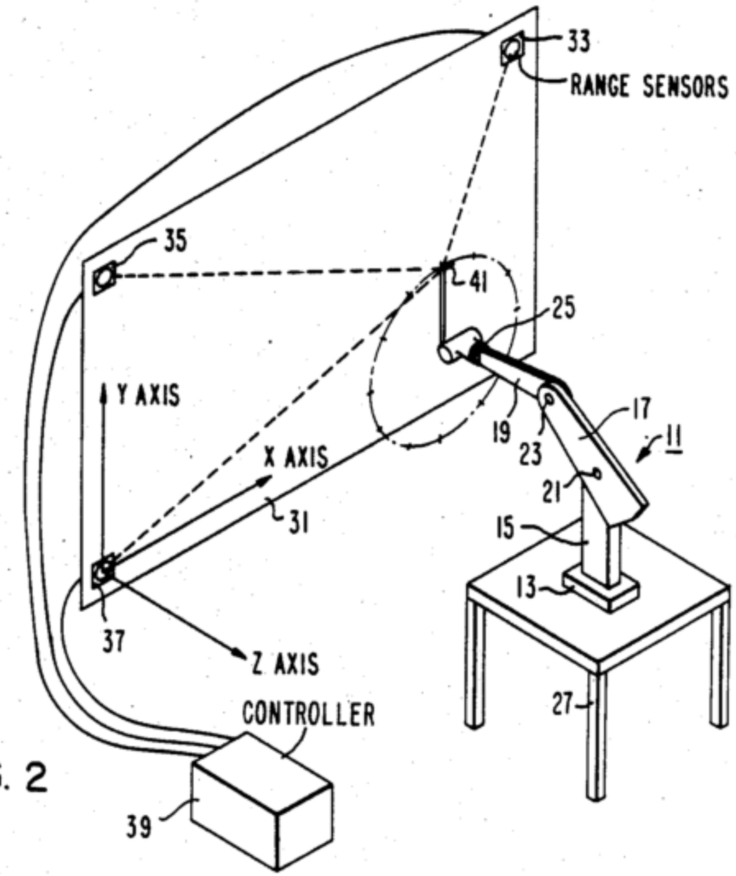


FIG. 2

Background continued

- Systems ID finds a relationship between input and output
- Must choose basis functions and parameters

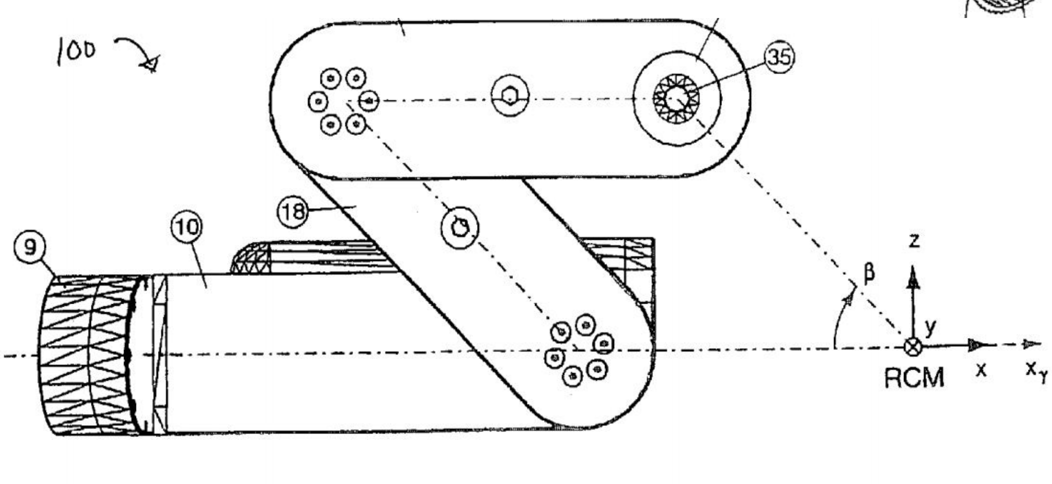
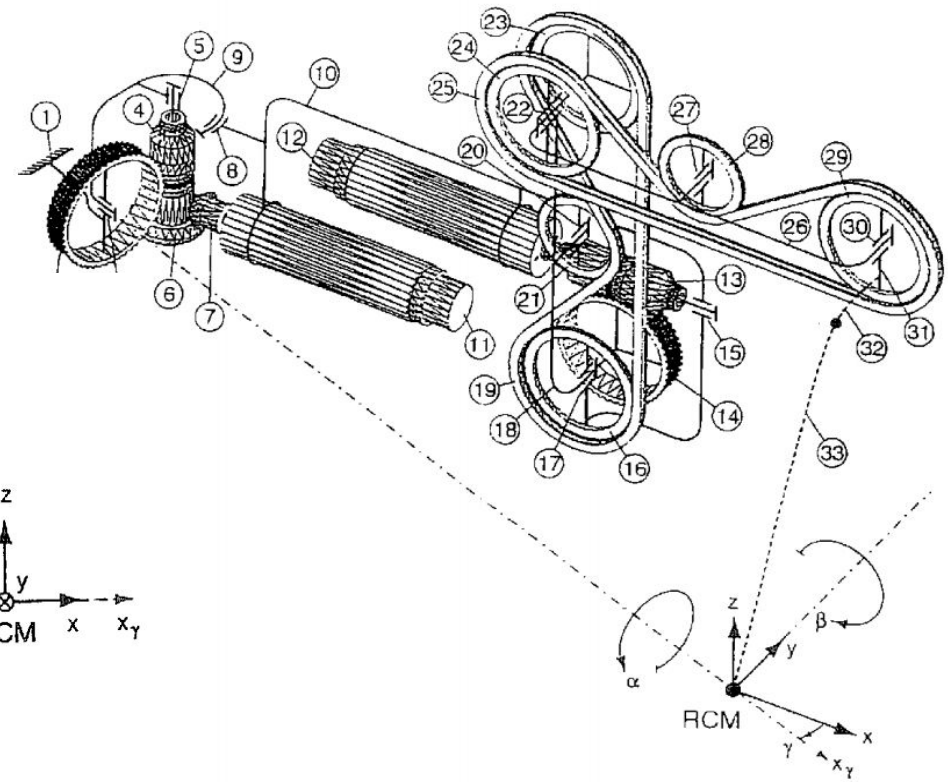


FIG. 1C



What the authors did

- Summarize the field of systems ID with a common approach
- Explain major choices systems identifiers will face
- Provide examples which help answer structural issues (of function choice)

General Approach

Inputs and Outputs

$$u^t = [u(1) \quad u(2) \quad \ldots \quad u(t)],$$

$$y^t = [y(1) \quad y(2) \quad \ldots \quad y(t)].$$

Looking for relationship

$$y(t) = g(u^{t-1}, y^{t-1}) + v(t).$$

Approach continued

Relationship can be decomposed into <u>parameters</u> and function of past I/O

$$g(u^{t-1}, y^{t-1}, \theta) = g(\varphi(t), \theta),$$

where

$$\varphi(t) = \varphi(u^{t-1}, y^{t-1}).$$

Function of past I/O pairs is <u>regression vector</u>

Approach continued

The mapping

$$g(\varphi, \theta),$$

which for any given θ goes from $\mathbb{R}^d$ to $\mathbb{R}^p$.

Can be written as a sum (family) of basis functions

$$g(\varphi, \theta) = \sum \alpha_k g_k(\varphi).$$

Approach continued

Basis functions can be constructed from single 'mother basis function'

$$g_k(\varphi) = \kappa(\varphi, \beta_k, \gamma_k) \quad `= \kappa(\beta_k(\varphi - \gamma_k))'.$$

With scaling (directional) and offset terms determining region of support

Ex: k(x) = cos(x) : Basis functions = Fourier

Basis function construction

Radial Construction

$$g_k(\varphi) = g_k(\varphi, \beta_k, \gamma_k) = \kappa(\|\varphi - \gamma_k\|_{\beta_k}),$$

Support diminishes by scale factor with distance from offset

The most homogeneous choice

Basis function construction (cntd)

Ridge Construction

$$g_k(\varphi) = g_k(\varphi, \beta_k, \gamma_k)$$
$$= \kappa(\beta_k^{\mathrm{T}}\varphi + \gamma_k), \quad \varphi \in \mathbb{R}^d.$$

Unbounded support in subspace along 'ridge' of scaling (direction) function

- These basis functions can yield recognizable structures

Basis function construction (cntd)

Tensor Product of d (dimension) basis functions

$$g_1(\varphi_1) \cdots g_d(\varphi_d).$$

Can behave very differently in arbitrary directions

Computationally expensive for high-dimension case

Model Quality

True model

$$y(t) = g_0(\varphi(t)) + e(t)$$

Quality

$$\begin{aligned}\bar{V}(\theta) &= E \,\| y(t) - g(\varphi(t), \theta) \|^2 \\ &= \lambda + E \,\| g_0(\varphi(t)) - g(\varphi(t), \theta) \|^2\end{aligned}$$

Model Quality and Variance

Best fit (#parameters=m) minimizes variance

$$\theta_*(m) = \arg\min_\theta \bar{V}(\theta)$$

Quality of a specific model with N I/O pairs

$$E\bar{V}(\hat{\theta}_N) = V_*(m).$$

Bias and Variance

Decompose deviation from true model

$$
\begin{aligned}
V_*(m) &= E\bar{V}(\hat{\theta}_N) \\
&= \lambda + E\,\|g_0(\varphi(t)) - g(\varphi(t), \hat{\theta}_N)\|^2 \\
&\approx \underbrace{\lambda}_{\text{noise}} + \underbrace{E\,\|g_0(\varphi) - g(\varphi, \theta_*(m))\|^2}_{\text{bias}} \\
&\quad + \underbrace{E\,\|g(\varphi, \theta_*(m)) - g(\varphi, \hat{\theta}_N)\|^2}_{\text{variance}}.
\end{aligned}
$$

What will help with bias?

Within a known model family:

- Increasing number of parameters will give better overall model quality
- Increasing number of basis functions will reduce variance

$$\hat{\theta}_N \rightarrow \theta_*(m)$$

How will this affect variance in unknown model families with novel mother basis functions?

What will help with variance?

Variance is a function of # of parameters and # of regressor-output pairs (w/ error variance)

$$E\,\|g(\varphi(t),\hat{\theta}_N) - g(\varphi(t),\theta_*(m))\|^2 \approx \lambda\frac{m}{N}$$

Giving model quality succinctly

$$\begin{aligned} V_*(m) = E\bar{V}(\hat{\theta}_N) &= \lambda + \lambda\frac{m}{N} \\ &\quad + E\,\|g_0(\varphi) - g(\varphi,\theta_*(m))\|^2 \\ &= \bar{V}(\theta_*(m)) + \lambda\frac{m}{N}. \end{aligned}$$

Advice

- Look at data
- Try and find physically intuitive explanation
- Pick efficient basis functions
- Do not add parameters 'spuriously'
- Do not add basis functions unless you are confident about the mother basis function
- Remember tradeoff between bias and variance

Significance of key result

- No new 'results'
- Significant due to comprehensive nature of paper - all described from a common framework
- Has great benefit for researchers in an array of fields - user focused

My assessment

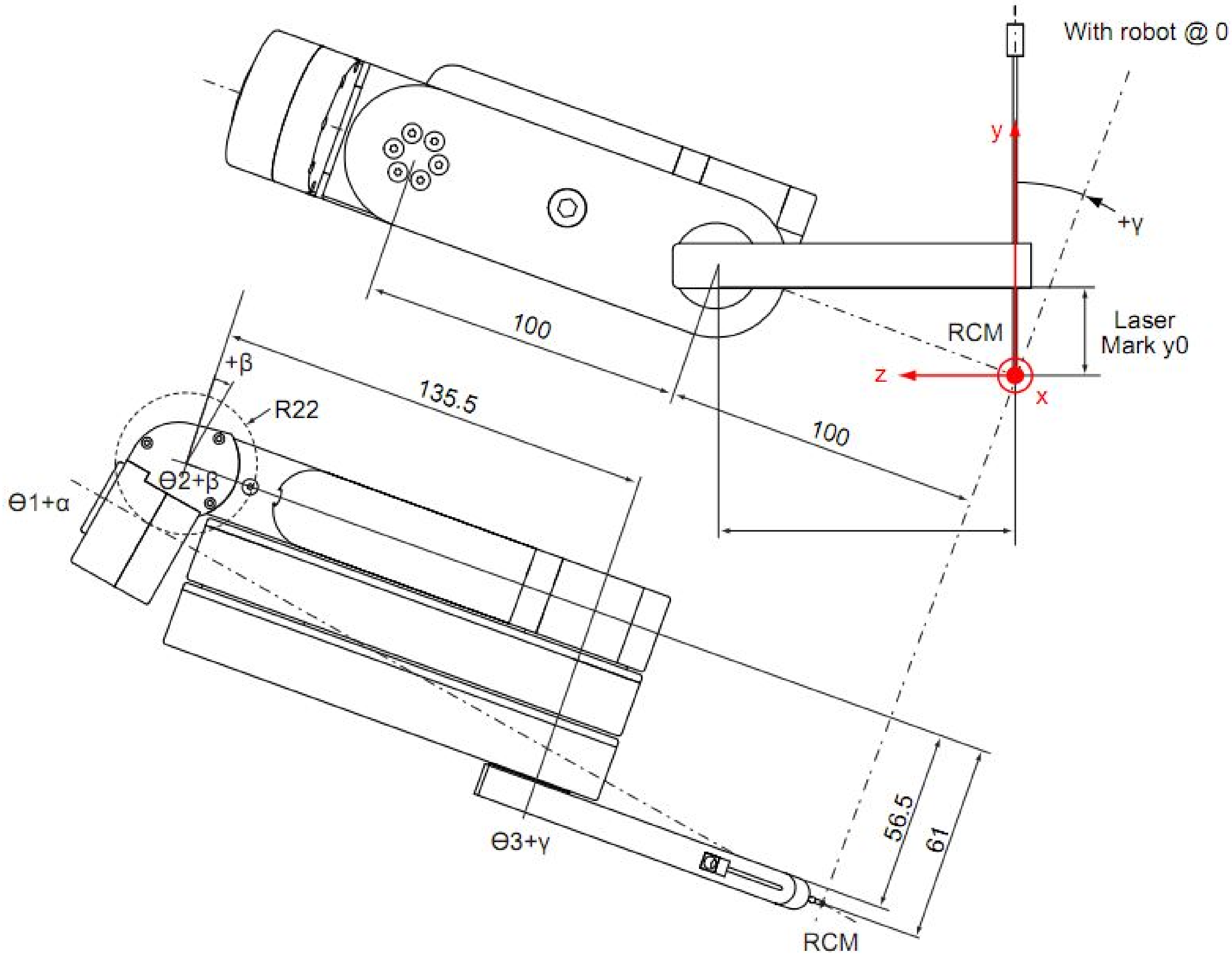
- Important as a blueprint for Systems ID
- Only most general abstractions reviewed are applicable to RCM link parameter estimation
- Good
  - Broad
  - Written from users perspective
- Bad
  - Not all applicable
  - Desire to be broad leaves out many specific mathematical processes

Applicability

Suppose needle tip position in base coordinates is a function of 3 parameters: Ɵ1, Ɵ3+ϒ (offset), L

- Rotation about Z $$Rb1 = \begin{pmatrix} \text{Cos}[\theta 1] & -\text{Sin}[\theta 1] & 0 \\ \text{Sin}[\theta 1] & \text{Cos}[\theta 1] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
- Rotation about X $$R2t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\theta 3 + \gamma] & -\text{Sin}[\theta 3 + \gamma] \\ 0 & \text{Sin}[\theta 3 + \gamma] & \text{Cos}[\theta 3 + \gamma] \end{pmatrix}$$
- Idealized model: [xyz]=Rb1*R2t*[0;-L;0]

'Reality'



With robot @ 0
y
+γ
100
Laser
Mark y0
RCM
z
x
+β
135.5
R22
100
Θ2+β
Θ1+α
Θ3+γ
56.5
61
RCM

Accounting for additional parameter

- Rotation about Y

$$R12 = \begin{pmatrix} \mathrm{Cos}[\beta] & 0 & \mathrm{Sin}[\beta] \\ 0 & 1 & 0 \\ -\mathrm{Sin}[\beta] & 0 & \mathrm{Cos}[\beta] \end{pmatrix}$$

- Resulting in a new model: [xyz] =Rb1*R12*R2t*[0;-L;0]

Next steps

- Future research in Systems ID should focus on expanding catalogue of basis functions
- This will move more problems from black-box to grey-box, white-box
- Future reading for our project should focus on grey-box systems identification

Conclusions

- Great paper!
- Very useful to many people
- Needs some interpretation to apply
- Desire to be broad excludes specific path
    - Kind of like variance vs bias

so,

- Questions?