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Seminar On
Point Set Registration: Coherent Point Drift

1 Selection

Point Set Registration: Coherent Point Drift Andriy Myronenko and Xubo Song present in this paper an algorithm for fast robust point set registration even in the presence of outliers, noise, and missing points. This paper was chosen for two reasons. The first is that the algorithm is particularly applicable to the ongoing project since we plan to use a point set registration technique to help determine the pose of the fiducial in our image (and as can be seen in the figure below: out detected points contain noise and outliers). Secondly, it was chosen since many of the students learned Iterative Closest Points last semester in CIS I, therefore I thought this would be a good chance to extend that knowledge and learn a similar algorithm that is based on the probabilistic Gaussian Mixture Model technique. Before I begin, I will note, that while the paper introduces algorithms for rigid, affine, and non-rigid registration, I will only be presenting on the rigid and affine. This is done primarily for their applicability to the project, but also for the sake of time and completeness for presenting all three would be quite rushed in a twenty minute setting.

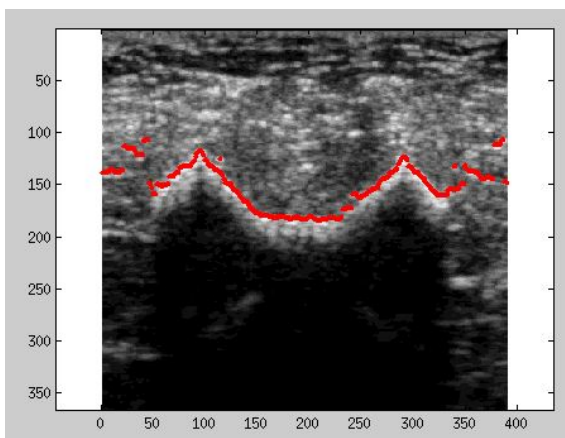

Figure 1: Detected Points

2 Overview

General Approach This paper approaches point set registration as a probability density estimation problem. This is done by setting the points in the moving set to be centroids of a Gaussian Mixture Model(GMM). Alignment is then attained my maximizing the likelihood for the data by iterating through Expectation-Maximization (EM). On each update, the points are forced to move "coherently" by applying the same transformation (rigid or affine) to each of the centroids rather than allowing each centroid to drift independently. It is important to note that the novelty in this specific paper does not come from the use of GMM for point set registration, which had been done before this paper, however, this paper prevents the first closed form complete (without ignoring terms in the objective function) solution for the M step in the rigid registration.

Affine Technique I will begin by explaining the calculation for the affine registration. The affine registration is a bit more straight forward than the rigid, which will be explained later on.

The goal of the algorithm is to attain a affine transformation matrix $\mathbf{B}$ and a translation vector $\mathbf{t}$ to align dataset $\mathbf{Y}_{MxD}$ that contains M points of dimension D to $\mathbf{X}_{NxD}$, such that $\tau(\mathbf{Y}) = \mathbf{Y}\mathbf{B}^T + \mathbf{1}\mathbf{t}^T$ where $\mathbf{1}$ is a column vector of 1's.

First we will begin by defining our objective functions which is simply maximizing likelihood, or instead minimizing the negative log-likelihood.

$$Q = -\sum_{n=1}^{N}\sum_{m=1}^{M}\mathbf{P}(m|\mathbf{x}_n)\log(\mathbf{P}^{new}(m)p^{new}(\mathbf{x}_n|m))$$

Where $\mathbf{P}$is the posterior probability distribution and the "new" variable refers to cost with the new parameter values. This equation can be rewritten as.

$$Q(\theta,\sigma^2) = \frac{1}{2\sigma^2}\sum_{m,n=1}^{M,N}\mathbf{P}(m|\mathbf{x}_n)\|\mathbf{x}_n - \tau(\mathbf{y}_m,\theta)\|^2 + \frac{N_p D}{2}\log(\sigma^2)$$

Where θ is the transformation parameters and N_p is the sum of the components of the posterior probabilities.
Now we can begin with the algorithm.
Initialization:

$$\mathbf{B} = \mathbf{I}, \mathbf{t} = \mathbf{0}, 0 \le \omega \le 1$$
$$\sigma^2 = \frac{1}{NMD}\sum_{n=1}^{N}\sum_{m=1}^{M}\|\mathbf{x}_n - \mathbf{y}_m\|^2$$

For clarification, each Gaussian in the Mixture Model is assumed to have the same variance and to equally be likely to generate data. Also an additional uniform distribution weighted by ω is also put into the model to help account for noise and outliers.

EM Optimization: Repeat Until Convergence
E: Compute the Posterior Probabilities: $\mathbf{P}_{MxN}$
The following determines the probability of each model $\mathbf{y}$, given the data point $\mathbf{x}$. The second term in the denominator provides for the uniform distribution.

$$p_{mn} = \frac{e^{-\frac{1}{2\sigma^2}\|\mathbf{x}_n - (\mathbf{B}\mathbf{y}_m + \mathbf{t})\|^2}}{\sum_{k=1}^{M} e^{-\frac{1}{2\sigma^2}\|\mathbf{x}_n - (\mathbf{B}\mathbf{y}_k + \mathbf{t})\|^2} + (2\pi\sigma^2)^{D/2}\frac{\omega}{1-\omega}\frac{M}{N}}$$

M: Update $\mathbf{B}$, $\mathbf{t}$, and σ^2 based by minimizing the objective function Q

$$Q(\mathbf{B}, \mathbf{t}, \sigma^2) = \frac{1}{2\sigma^2} \sum_{m,n=1}^{M,N} \mathbf{P}(m|\mathbf{x}_n) \|\mathbf{x}_n - (\mathbf{B}\mathbf{y}_m + \mathbf{t})\|^2 + \frac{N_p D}{2} \log(\sigma^2)$$

Where N_p is the sum of the components of $\mathbf{P}$. As we can see, we are essentially minimizing distance weighted by the posterior probabilities. By taking the partial derivatives for each variable, equating them to zero, and solving, the following solution is derived.

Obtain the posterior probability weighted means.

$$\mu_x = \frac{1}{N_p} \mathbf{X}^T \mathbf{P}^T \mathbf{1}, \mu_y = \frac{1}{N_p} \mathbf{Y}^T \mathbf{P} \mathbf{1}$$

Recenter the Data Points.

$$\hat{\mathbf{X}} = \mathbf{X} - \mathbf{1}\mu_x^T, \hat{\mathbf{Y}} = \mathbf{Y} - \mathbf{1}\mu_y^T$$

Update $\mathbf{B}$.

$$\mathbf{B} = (\hat{\mathbf{X}}^T \mathbf{P}^T \hat{\mathbf{Y}})(\hat{\mathbf{Y}}^T d(\mathbf{P}\mathbf{1})\hat{\mathbf{Y}})^{-1}$$

Where d() is the diagonaliztion of vector.

Update $\mathbf{t}$.

$$\mathbf{t} = \mu_x - \mathbf{B}\mu_y$$

And finally, update σ^2.

$$\sigma^2 = \frac{1}{N_p D}(tr(\hat{\mathbf{X}}^T d(\mathbf{P}^T\mathbf{1})\hat{\mathbf{X}}) - tr(\hat{\mathbf{X}}^T \mathbf{P}^T \hat{\mathbf{Y}} \mathbf{B}^t))$$

Rigid Technique The rigid registration follows a very similar algorithm, however instead of $\mathbf{B}$, we have s$\mathbf{R}$, where s is a scaling factor which is initialized to 1, and $\mathbf{R}$ is a rotation matrix. However, now we have to constrain our update so that $\mathbf{R}^T\mathbf{R} = \mathbf{I}$ and $\det(\mathbf{R}) = 1$.

The novel approach in this paper was presenting the closed form solution to this constrained optimization, which is done as follows using Singular Value Decomposition (SVD) during the M step.

$$\mathbf{A} = \hat{\mathbf{X}}^T \mathbf{P}^T \hat{\mathbf{Y}}$$
$$SVD(\mathbf{A}) = \mathbf{U}\mathbf{S}\mathbf{V}^T$$

$$\mathbf{C} = d([1, 1, ..., 1, det(\mathbf{U}\mathbf{V}^T)])$$

$$\mathbf{R} = \mathbf{U}\mathbf{C}\mathbf{V}^T$$

$$s = \frac{tr(\mathbf{A}^T\mathbf{R})}{tr(\hat{\mathbf{Y}}^T d(\mathbf{P}\mathbf{1})\hat{\mathbf{Y}})}$$

$$\mathbf{t} = \mu_x - s\mathbf{R}\mu_y$$

$$\sigma^2 = \frac{1}{N_p D}(tr(\hat{\mathbf{X}}^T d(\mathbf{P}^T\mathbf{1})\hat{\mathbf{X}}) - s(tr(\mathbf{A}^T\mathbf{R})))$$

3 Results and Discussion

This figure below shows pretty well, how accurate CPD can be, working with missing points, outliers, noise, and the combination of them.

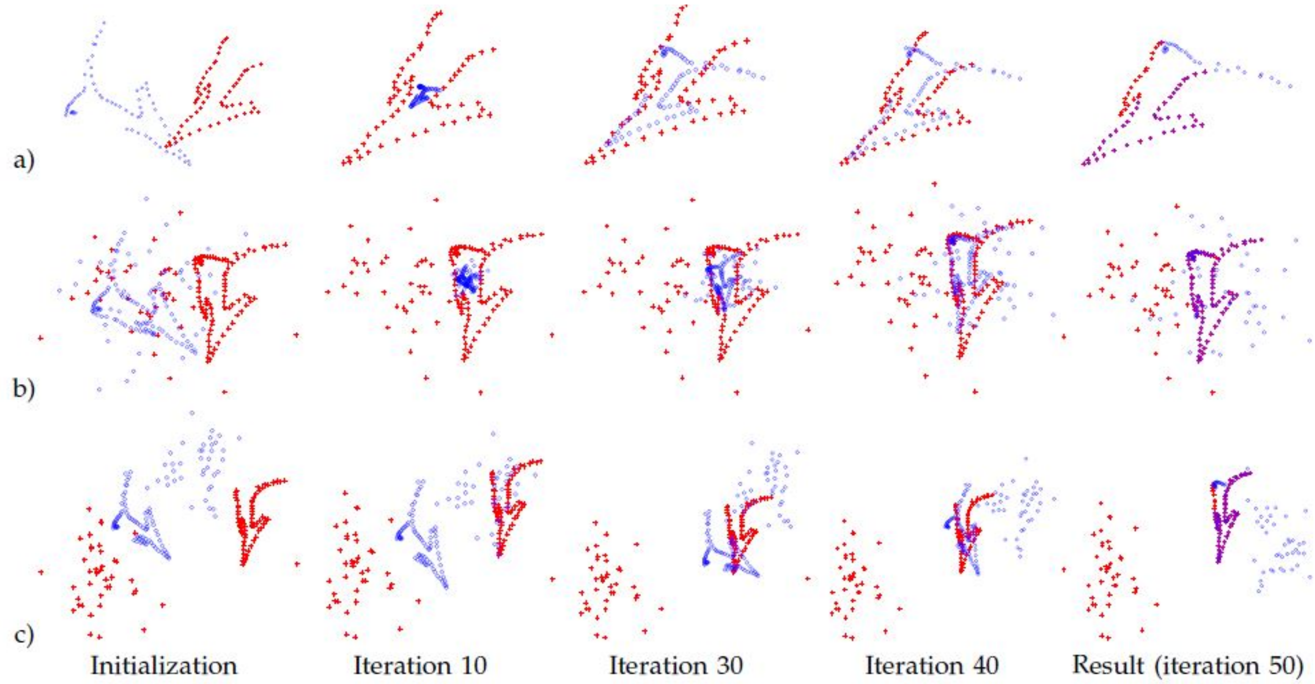

Figure 2: Taken from A. Myronenko and X.B. Song

As we can see in the below two figures as well, CPD works better than LM-ICP with both noise and outliers, however it works best if the outliers are in the moving point set, and the noise is in the stationary point set. This is useful information, however it would also be interesting if they took an exhaustive look at the combination, since it is often the case that one set will have both the outliers and the noise, as in the case for my project.

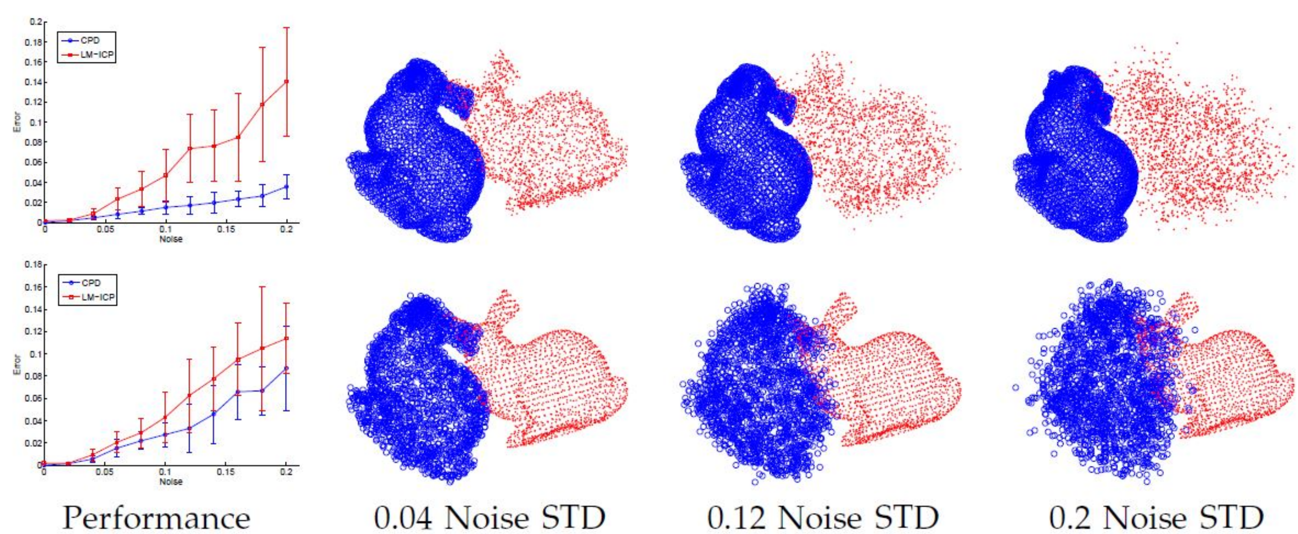

Figure 3: Noise: Taken from A. Myronenko and X.B. Song

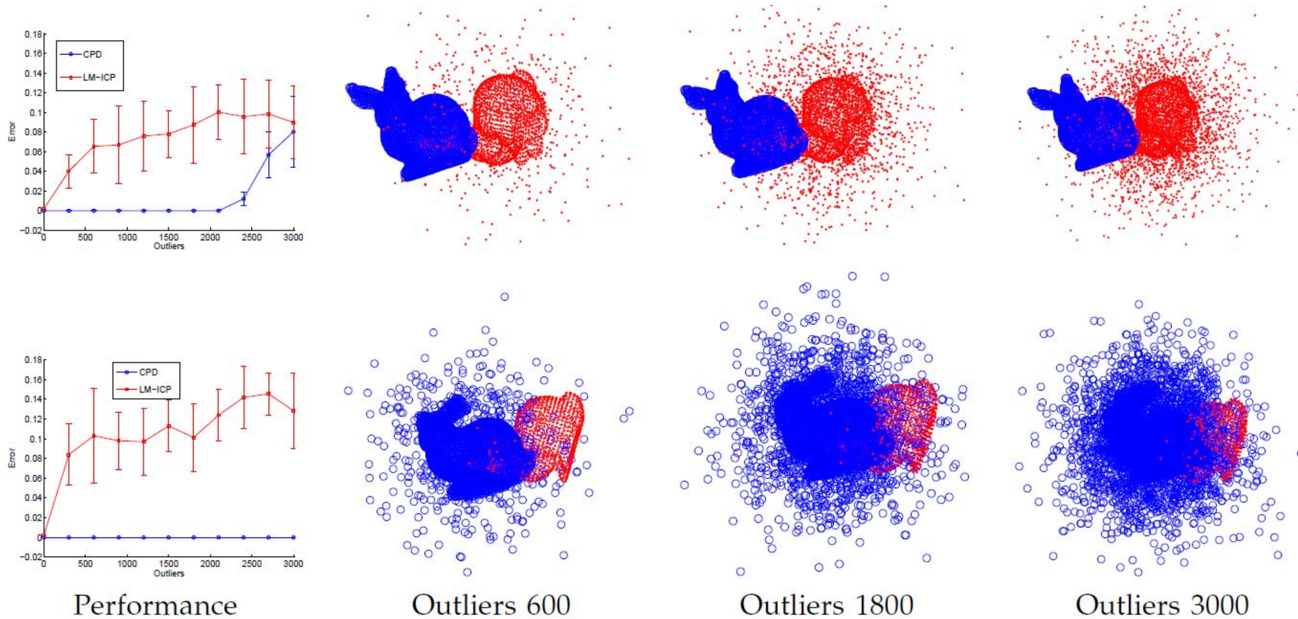

Figure 4: Outliers: Taken from A. Myronenko and X.B. Song

Furthermore, they never do an analysis of the value of ω and the effect it has in dealing with noise and outliers. It is the only parameter that is up to the user to determine, yet, the value is not analyzed in a rigorous fashion, leaving the reader uncertain as to when and to what degree it should be adjusted.

In reference to my project, the algorithm proves to be very useful, leading to accurate alignment as shown below.

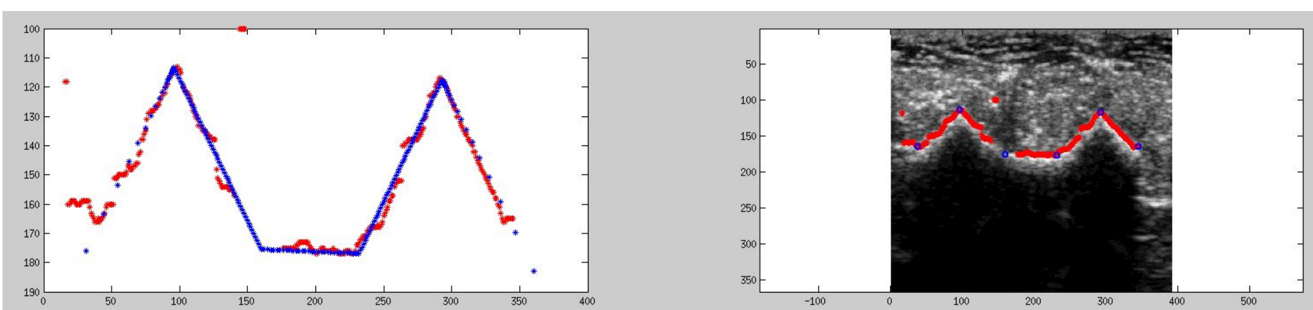

Figure 5: Alignment on Left, Final Detection on Right

4 References

A. Myronenko and X.B. Song, Point-Set Registration: Coherent Point Drift, IEEE Trans. Pattern Analysis and Machine Intelligence, vol. 32, no. 12, pp. 2262-2275, Dec. 2010.