Project 5

EchoSure

Detecting Blood-Clots Post-Operatively In Blood Vessel Anastomoses

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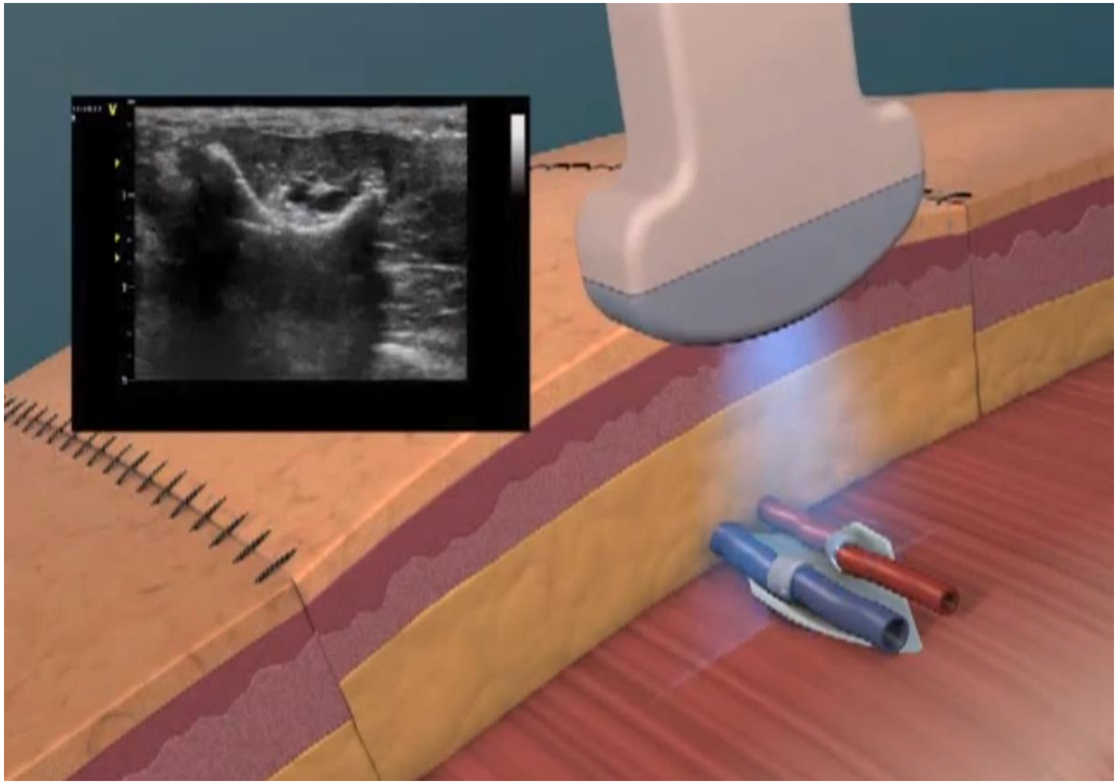
Recap of Project

Ultrasound Doppler Imaging for Tracking Changes in Blood Flow Velocity

Biodegradable Plastic Fiducial for Supplying Reliable Pose



Animation by David A. Rini

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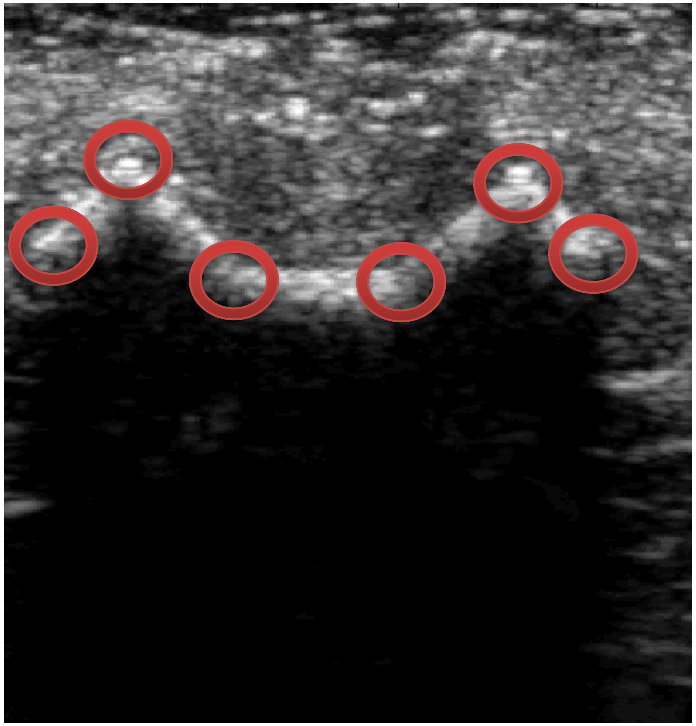
Goal



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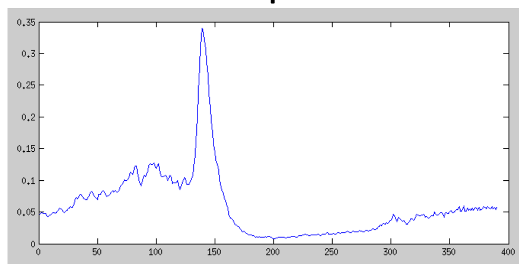
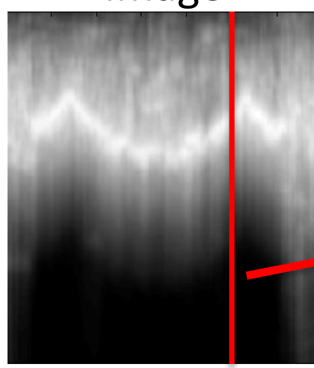
Template Matching By Columns

Original Image

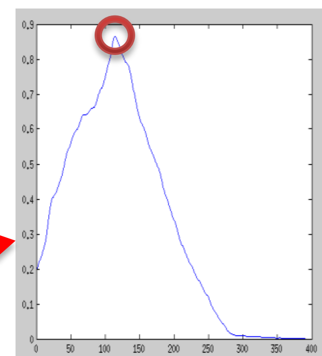
Template



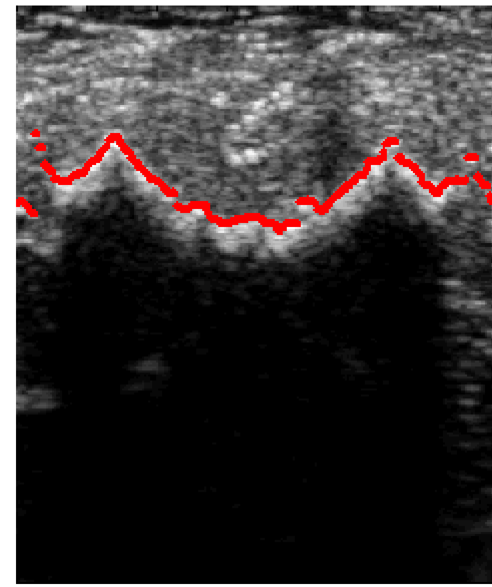
Correlation Image



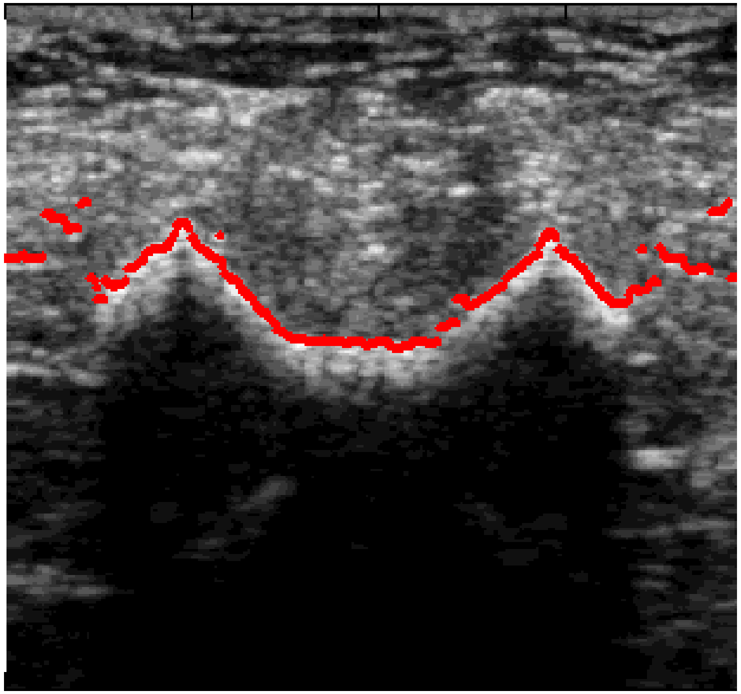
Selected Column

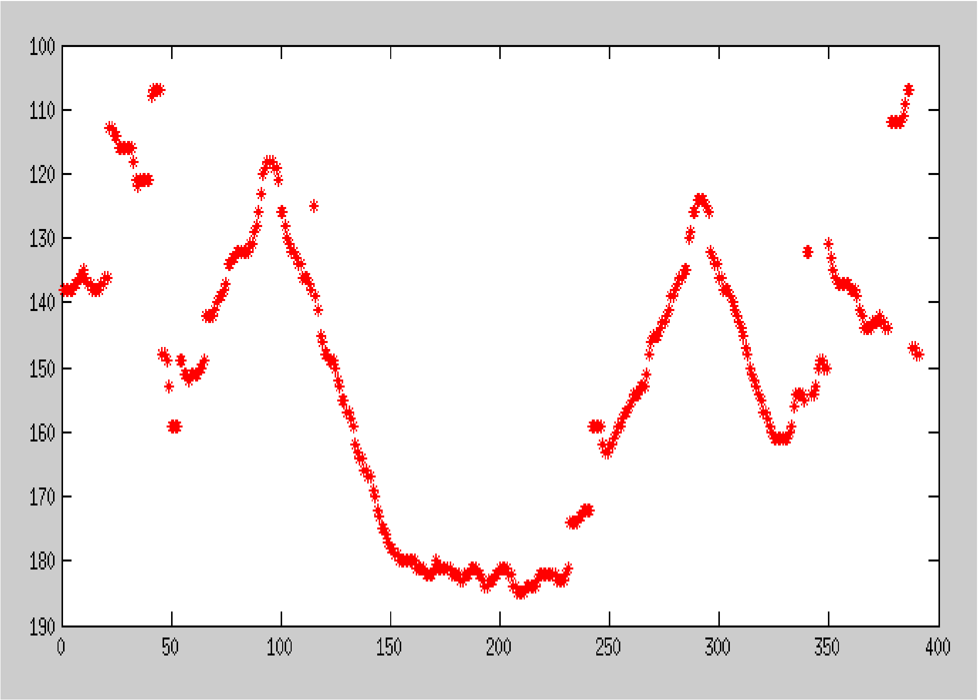


Peaks of Each Column

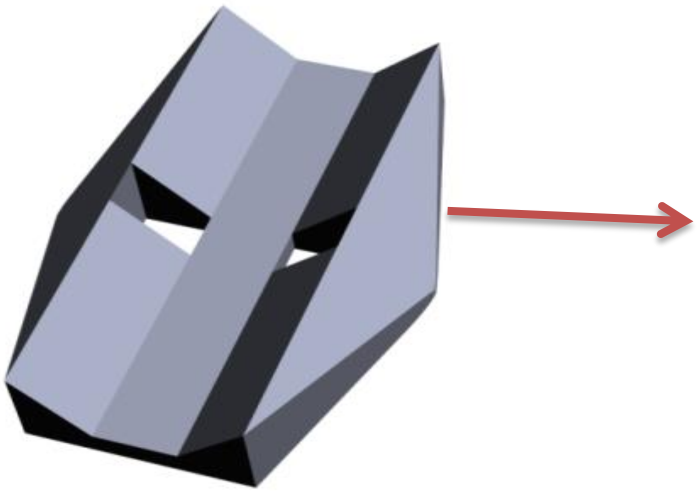


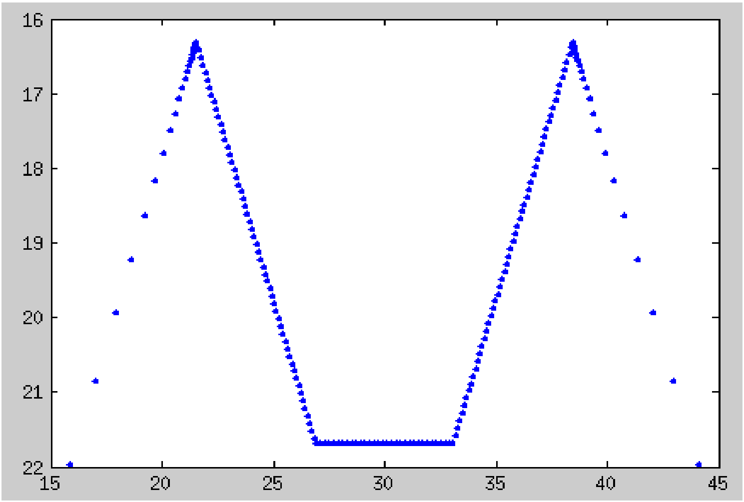
So We Have a Point Set



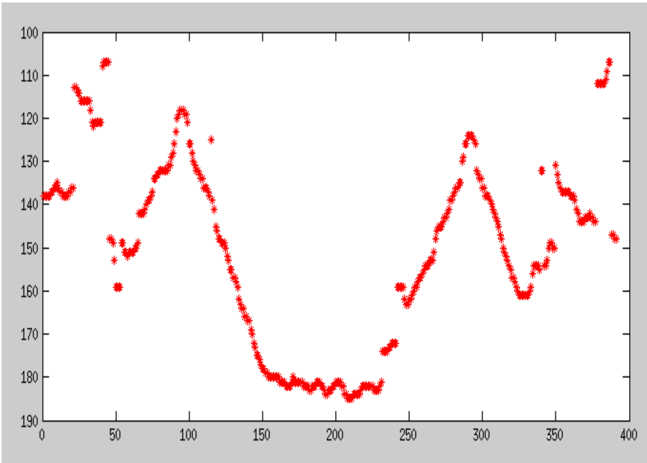


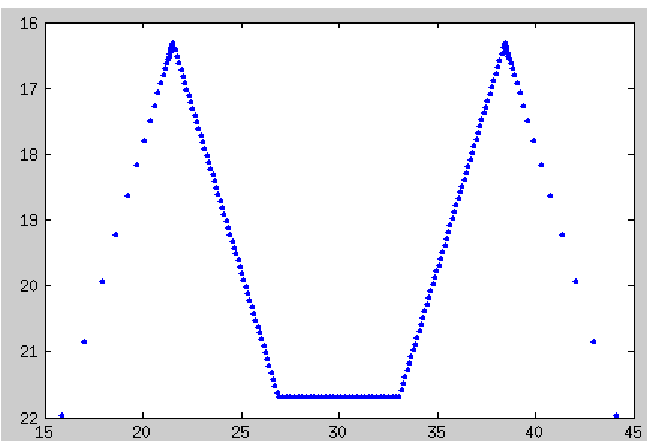
Attaining the Second Point Set





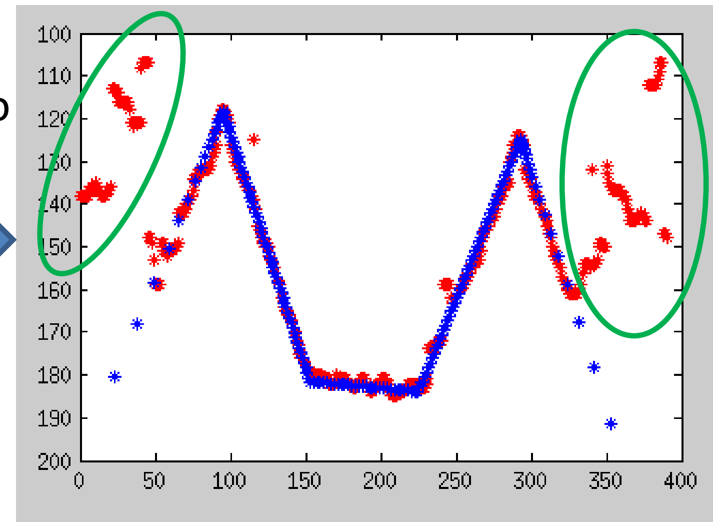
Affine Point Set Registration





Ideal Scenario

Problem: Outliers



The Paper

A. Myronenko and X.B. Song

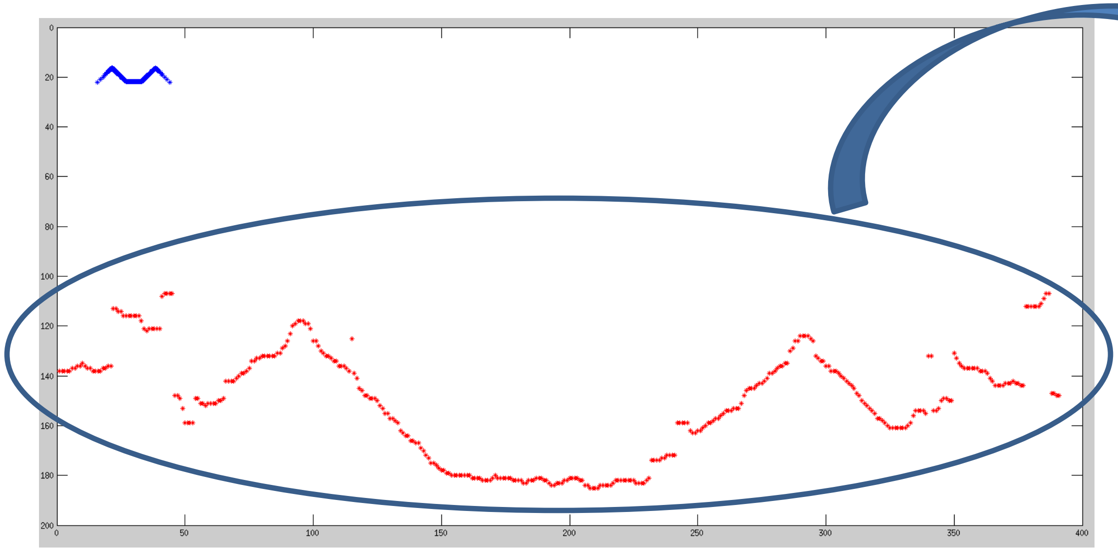
Point-Set Registration: Coherent Point Drift

IEEE Trans. Pattern Analysis and Machine Intelligence

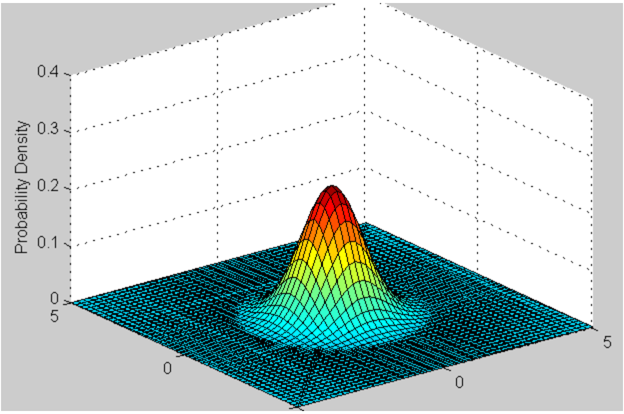
vol. 32, no. 12, pp. 2262-2275, Dec. 2010.

High Level Understanding

- The problem is set up as a probability density estimation problem using Gaussian Mixture Models



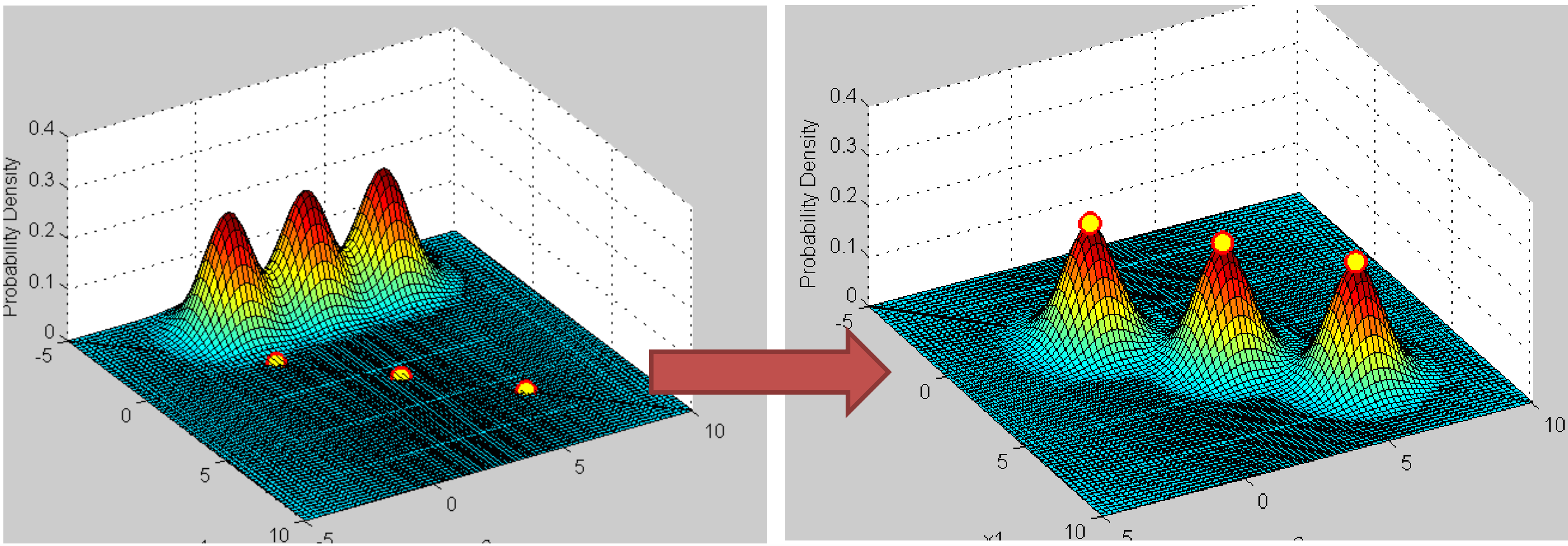
Each Point in The Moving Set is a Gaussian Centroid



A. Myronenko and X.B. Song, Point-Set Registration: Coherent Point Drift

High Level Understanding

- The problem is set up as a probability density estimation problem using Gaussian Mixture Models



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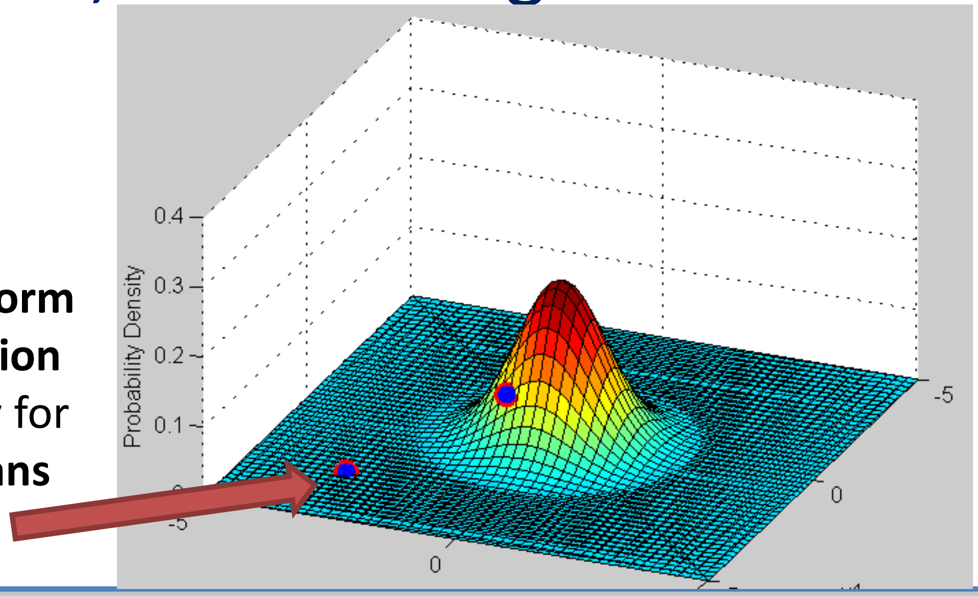
High Level Understanding

- In order to be **robust to outliers**, a **uniform distribution is added to the mixture model** so that points that do not fit well to any of the Gaussians, will be assigned to the outlier group.

Assigned to **Uniform Outlier Distribution** if low probability for **all other Gaussians** as well



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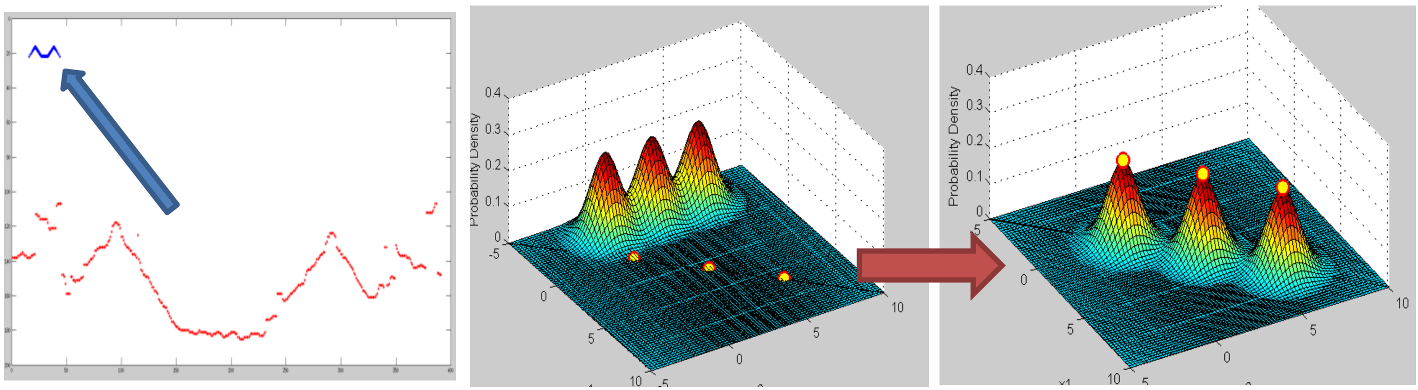
High Level Understanding

- The Gaussian Centroids are iteratively brought closer to the Data Points through **Maximizing Likelihood** using **Expectation-Maximization (EM)** as the optimizer



To restrict the points to moving “coherently”, the same transformation is applied to all of the points at each iteration

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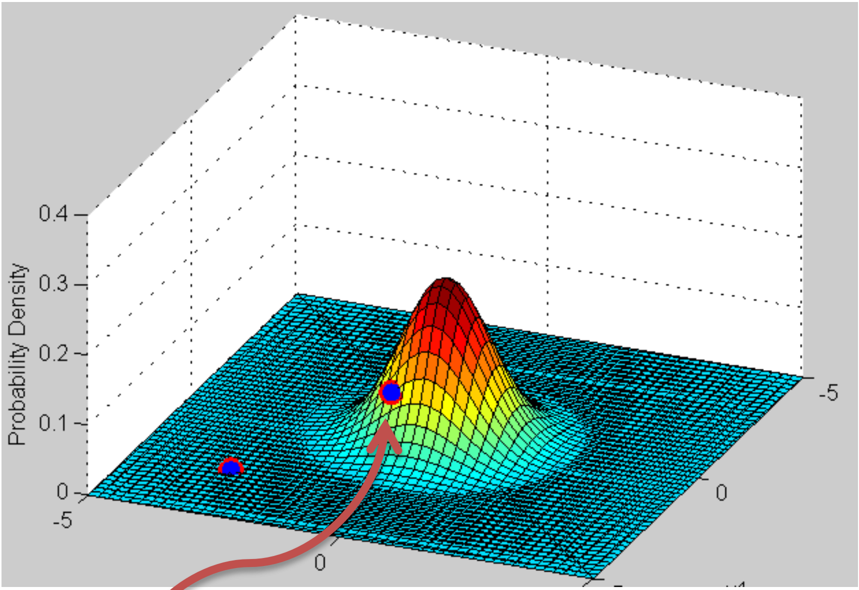
Expectation: Posterior Probabilities

Calculating P_{MxN}

D - Dimension of Data Set
X_{NxD} - Data Points (Stationary)
Y_{MxD} - Gaussian Centroids (Moving)
B – Affine Transformation Matrix
t – Translation Vector



$$p_{mn} = \frac{e^{-\frac{1}{2\sigma^2}\|\mathbf{x}_n-(\mathbf{B}\mathbf{y}_m+\mathbf{t})\|^2}}{\sum_{k=1}^{M} e^{-\frac{1}{2\sigma^2}\|\mathbf{x}_n-(\mathbf{B}\mathbf{y}_k+\mathbf{t})\|^2} + (2\pi\sigma^2)^{D/2}\frac{\omega}{1-\omega}\frac{M}{N}}$$

Partition Function

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Maximization: Objective Function

- Our goal is to **Maximize Likelihood**. This is Equivalent to **Minimizing Negative Log-Likelihood**

$$Q = -\sum_{n=1}^{N}\sum_{m=1}^{M+1} \mathbf{P}(m|\mathbf{x}_n)\log(\mathbf{P}^{new}(m)p^{new}(\mathbf{x}_n|m))$$

M Gaussian Centroids
1 Uniform Distribution
N Data Points

Distance Weighted By Posterior

Uniform Term

$$Q(\theta,\sigma^2) = \frac{1}{2\sigma^2}\sum_{m,n=1}^{M,N}\mathbf{P}(m|\mathbf{x}_n)\|\mathbf{x}_n - \tau(\mathbf{y}_m,\theta)\|^2 + \frac{N_p D}{2}\log(\sigma^2)$$

$$p_{mn} = \frac{e^{-\frac{1}{2\sigma^2}\|\mathbf{x}_n - (\mathbf{B}\mathbf{y}_m + \mathbf{t})\|^2}}{\sum_{k=1}^{M} e^{-\frac{1}{2\sigma^2}\|\mathbf{x}_n - (\mathbf{B}\mathbf{y}_k + \mathbf{t})\|^2} + (2\pi\sigma^2)^{D/2}\frac{\omega}{1-\omega}\frac{M}{N}}$$

D - Dimension of Data Set
X_{NxD} - Data Points
Y_{MxD} - Gaussian Centroids
B – Affine Transformation Matrix
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Expectation-Maximization

Expectation:

Update P

$$p_{mn} = \frac{e^{-\frac{1}{2\sigma^2}\|\mathbf{x}_n - (\mathbf{B}\mathbf{y}_m + \mathbf{t})\|^2}}{\sum_{k=1}^{M} e^{-\frac{1}{2\sigma^2}\|\mathbf{x}_n - (\mathbf{B}\mathbf{y}_k + \mathbf{t})\|^2} + (2\pi\sigma^2)^{D/2}\frac{\omega}{1-\omega}\frac{M}{N}}$$

Maximization:

Update B, t, and variance by taking partial derivatives of the objective function and setting them equal to zero

$$Q(\theta, \sigma^2) = \frac{1}{2\sigma^2}\sum_{m,n=1}^{M,N} \mathbf{P}(m|\mathbf{x}_n)\|\mathbf{x}_n - \tau(\mathbf{y}_m, \theta)\|^2 + \frac{N_p D}{2}\log(\sigma^2)$$

$$N_{\mathbf{P}} = \mathbf{1}^T\mathbf{P}\mathbf{1},\ \mu_{\mathbf{x}} = \frac{1}{N_{\mathbf{P}}}\mathbf{X}^T\mathbf{P}^T\mathbf{1},\ \mu_{\mathbf{y}} = \frac{1}{N_{\mathbf{P}}}\mathbf{Y}^T\mathbf{P}\mathbf{1},$$

$$\hat{\mathbf{X}} = \mathbf{X} - \mathbf{1}\mu_{\mathbf{x}}^T,\ \ \hat{\mathbf{Y}} = \mathbf{Y} - \mathbf{1}\mu_{\mathbf{y}}^T,$$

$$\mathbf{B} = (\hat{\mathbf{X}}^T\mathbf{P}^T\hat{\mathbf{Y}})(\hat{\mathbf{Y}}^T\,\mathrm{d}(\mathbf{P}\mathbf{1})\hat{\mathbf{Y}})^{-1},$$

$$\mathbf{t} = \mu_{\mathbf{x}} - \mathbf{B}\mu_{\mathbf{y}},$$

$$\sigma^2 = \frac{1}{N_{\mathbf{P}}D}(\mathrm{tr}(\hat{\mathbf{X}}^T\,\mathrm{d}(\mathbf{P}^T\mathbf{1})\hat{\mathbf{X}}) - \mathrm{tr}(\hat{\mathbf{X}}^T\mathbf{P}^T\hat{\mathbf{Y}}\mathbf{B}^T))$$

Taken from A. Myronenko and X.B. Song

D - Dimension of Data Set
X_{NxD} - Data Points
Y_{MxD} - Gaussian Centroids
B – Affine Transformation Matrix
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Note on Rigid Case

- M-Step More Complicated than Affine (Constrained Optimization)
  - Need to constrain B such that **$B^T B = I$ and det(B) = 1**
    - (make B a Rotation Matrix: R)

Authors find a complete closed form solution:

Lemma: Given a matrix A_{DxD}

The **optimal Rotation matrix (R_{DxD}) to maximize trace($A^T R$)** is:

$$\mathbf{R} = \mathbf{U}\mathbf{C}\mathbf{V}^T \qquad \text{Where: } SVD(\mathbf{A}) = \mathbf{U}\mathbf{S}\mathbf{V}^T$$

$$\mathbf{C} = d([1, 1, ..., 1, det(\mathbf{U}\mathbf{V}^T)])$$

Authors Rewrite Objective Function in form of trace:

$$Q = -c_1 \operatorname{tr}((\hat{\mathbf{X}}^T \mathbf{P}^T \hat{\mathbf{Y}})^T \mathbf{R}) + c_2 \qquad \mathbf{A} = \hat{\mathbf{X}}^T \mathbf{P}^T \hat{\mathbf{Y}}$$

Results

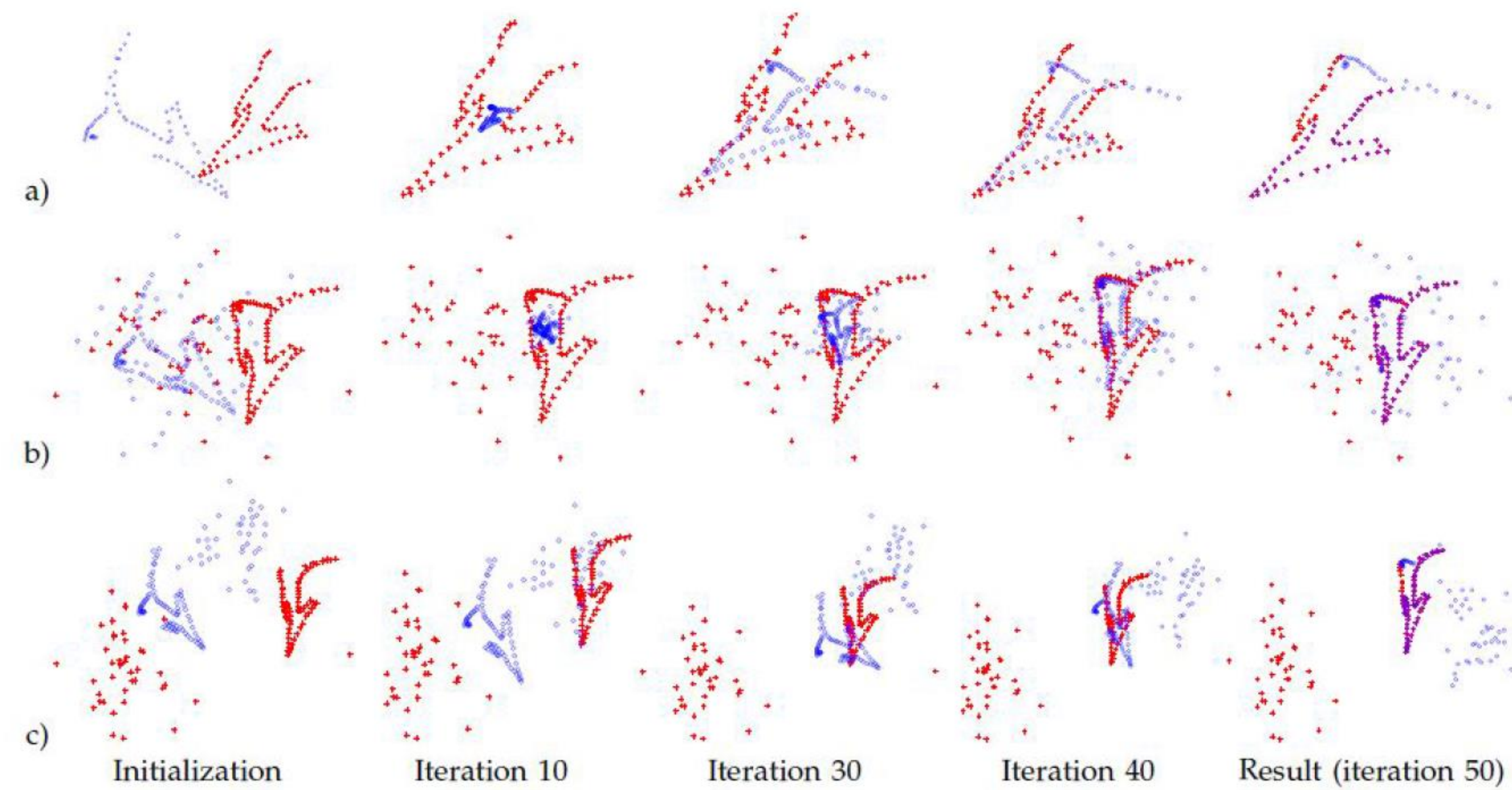

Figure 2: Taken from A. Myronenko and X.B. Song

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Results: Noise

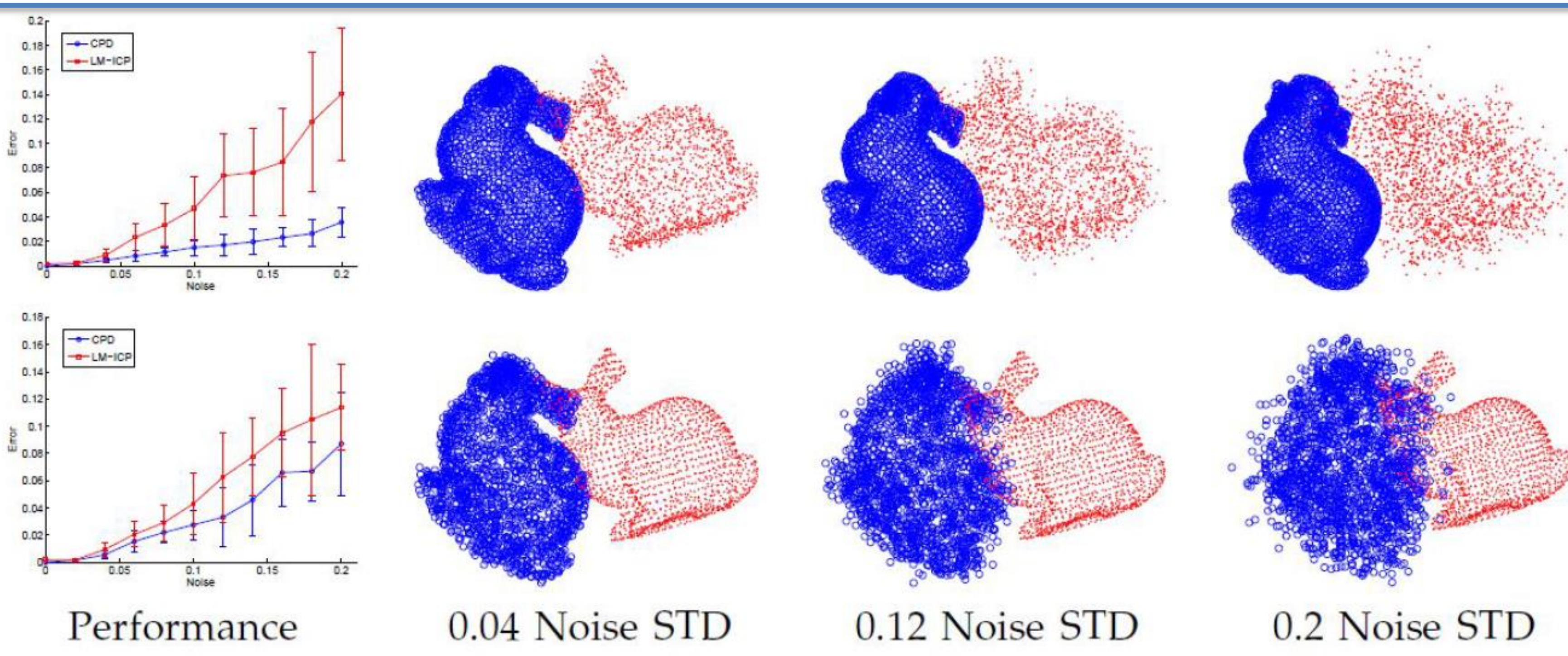

Figure 3: Noise: Taken from A. Myronenko and X.B. Song

Moving Point Set (Gaussian Centroids)

Stationary Point Set (Data Points)

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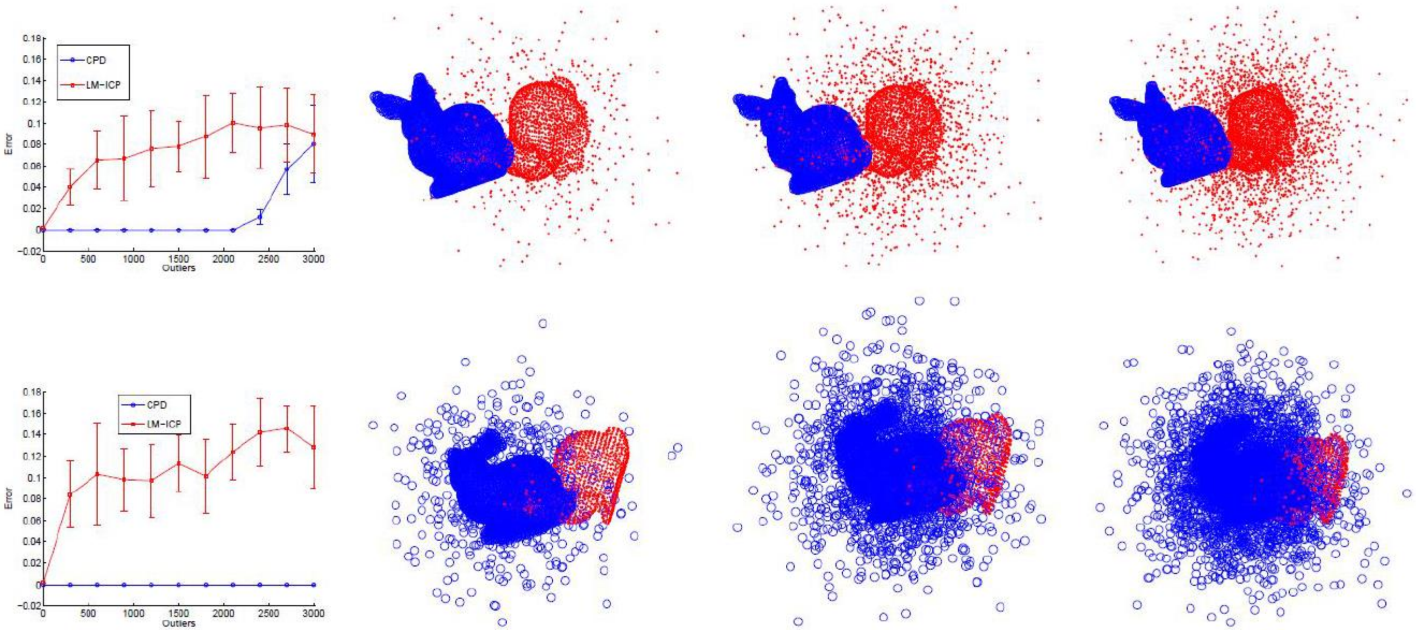
Results: Outliers



Performance | Outliers 600 | Outliers 1800 | Outliers 3000

Figure 4: Outliers: Taken from A. Myronenko and X.B. Song

Moving Point Set (Gaussian Centroids)

Stationary Point Set (Data Points)

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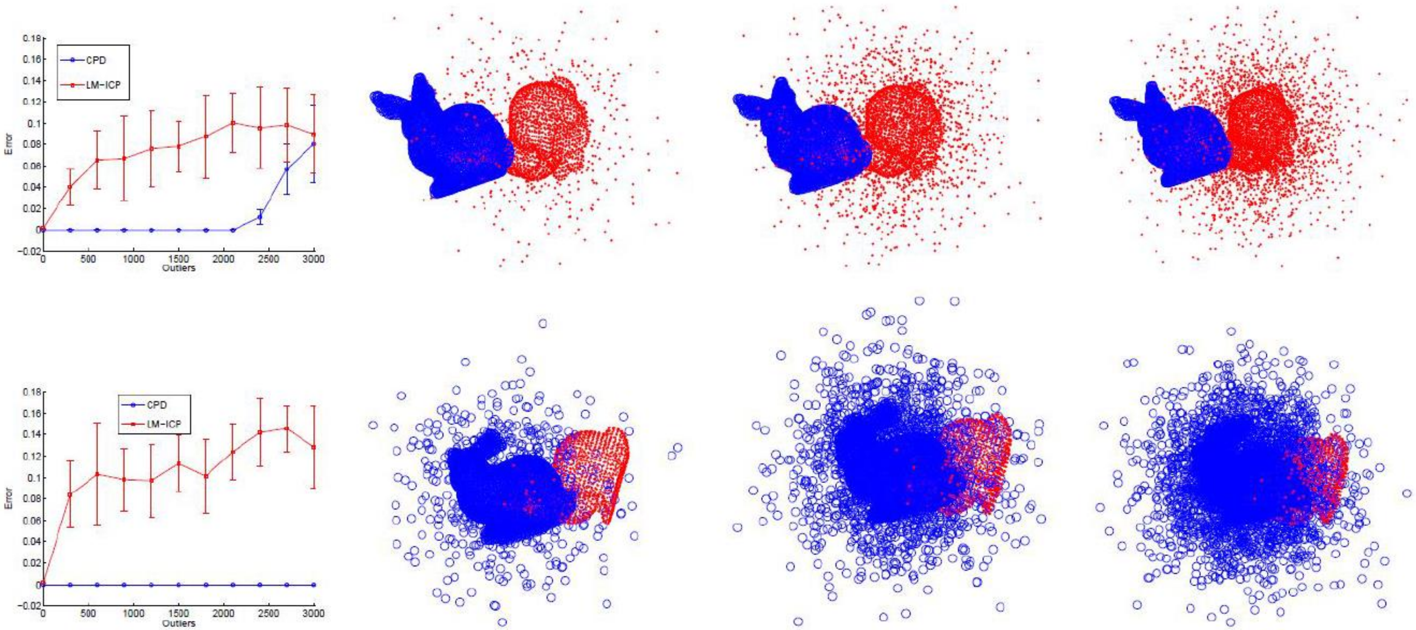
Results: Outliers



Performance | Outliers 600 | Outliers 1800 | Outliers 3000

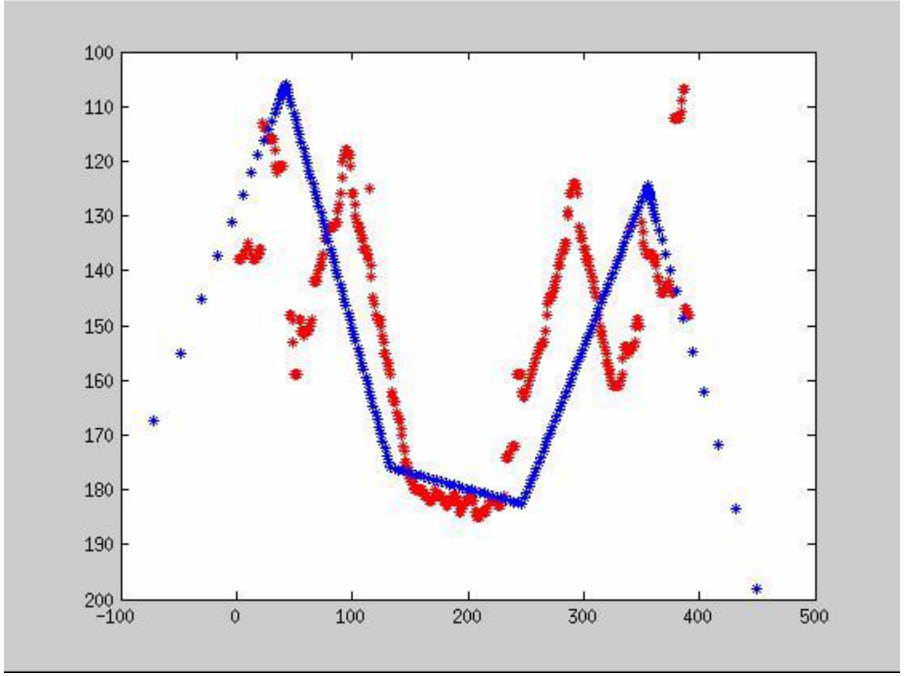
Figure 4: Outliers: Taken from A. Myronenko and X.B. Song

Moving Point Set (Gaussian Centroids)

Stationary Point Set (Data Points)

A. Myronenko and X.B. Song, Point-Set Registration: Coherent Point Drift

Results: From This Project



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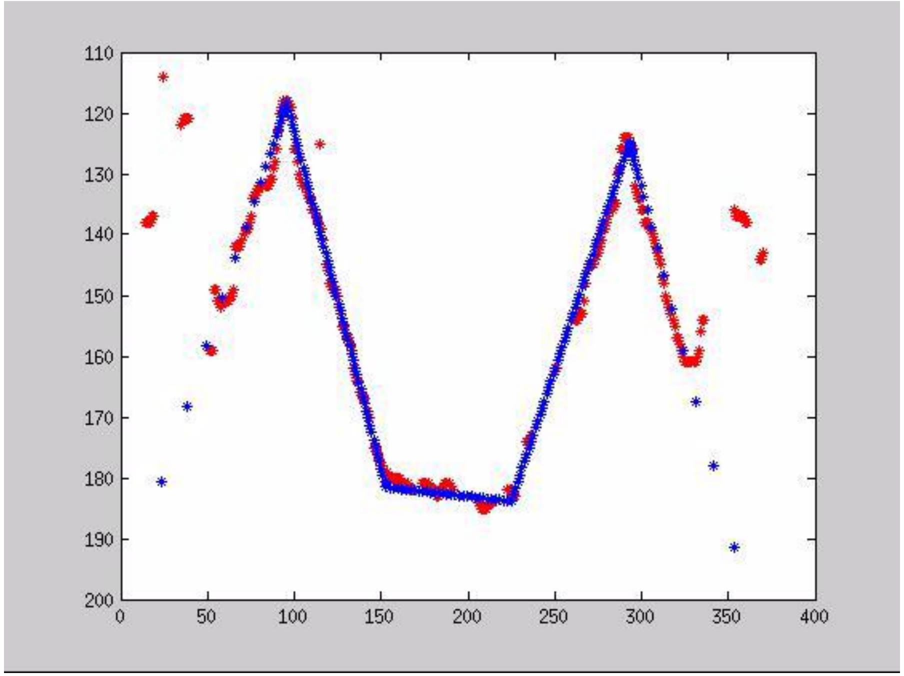
Results: From This Project

With Threshold Applied Based on the Cross -Correlation Level



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Discussion

- Positives
  - Is very robust compared to many of the alternatives
  - Nice Analysis of Outliers and Noise
  - Eloquent Closed Form Solution for M-Step
- Negatives
  - Outliers IID for testing (not always realistic)
  - No discussion on effect of w

Questions?

References

1. A. Myronenko and X.B. Song, Point-Set Registration: Coherent Point Drift, *IEEE Trans. Pattern Analysis and Machine Intelligence,* vol. 32, no. 12, pp. 2262-2275, Dec. 2010.

Appendix A

Rigid point set registration algorithm:

- Initialization: $\mathbf{R} = \mathbf{I}, \mathbf{t} = 0, s = 1, 0 \le w \le 1$
  $\sigma^2 = \frac{1}{DNM} \sum_{n=1}^{N} \sum_{m=1}^{M} \|\mathbf{x}_n - \mathbf{y}_m\|^2$
- EM optimization, repeat until convergence:
  - E-step: Compute $\mathbf{P}$,
    $$p_{mn} = \frac{\exp^{-\frac{1}{2\sigma^2}\|\mathbf{x}_n - (s\mathbf{R}\mathbf{y}_m + \mathbf{t})\|^2}}{\sum_{k=1}^{M} \exp^{-\frac{1}{2\sigma^2}\|\mathbf{x}_n - (s\mathbf{R}\mathbf{y}_k + \mathbf{t})\|^2} + (2\pi\sigma^2)^{D/2} \frac{w}{1-w} \frac{M}{N}}$$
  - M-step: Solve for $\mathbf{R}, s, \mathbf{t}, \sigma^2$:
    - $N_{\mathbf{P}} = \mathbf{1}^T\mathbf{P}\mathbf{1}, \mu_{\mathbf{x}} = \frac{1}{N_{\mathbf{P}}}\mathbf{X}^T\mathbf{P}^T\mathbf{1}, \mu_{\mathbf{y}} = \frac{1}{N_{\mathbf{P}}}\mathbf{Y}^T\mathbf{P}\mathbf{1}$,
    - $\hat{\mathbf{X}} = \mathbf{X} - \mathbf{1}\mu_{\mathbf{x}}^T, \;\; \hat{\mathbf{Y}} = \mathbf{Y} - \mathbf{1}\mu_{\mathbf{y}}^T$,
    - $\mathbf{A} = \hat{\mathbf{X}}^T\mathbf{P}^T\hat{\mathbf{Y}}$, compute SVD of $\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{S}\mathbf{V}^T$,
    - $\mathbf{R} = \mathbf{U}\mathbf{C}\mathbf{V}^T$, where $\mathbf{C} = \mathrm{d}(1, .., 1, \det(\mathbf{U}\mathbf{V}^T))$,
    - $s = \frac{\mathrm{tr}(\mathbf{A}^T\mathbf{R})}{\mathrm{tr}(\hat{\mathbf{Y}}^T \mathrm{d}(\mathbf{P}\mathbf{1})\hat{\mathbf{Y}})}$,
    - $\mathbf{t} = \mu_{\mathbf{x}} - s\mathbf{R}\mu_{\mathbf{y}}$,
    - $\sigma^2 = \frac{1}{N_{\mathbf{P}}D}(\mathrm{tr}(\hat{\mathbf{X}}^T \mathrm{d}(\mathbf{P}^T\mathbf{1})\hat{\mathbf{X}}) - s\,\mathrm{tr}(\mathbf{A}^T\mathbf{R}))$.
- The aligned point set is $\mathcal{T}(\mathbf{Y}) = s\mathbf{Y}\mathbf{R}^T + \mathbf{1}\mathbf{t}^T$,
- The probability of correspondence is given by $\mathbf{P}$.

Fig. 2. Rigid point set registration algorithm.

Affine point set registration algorithm:

- Initialization: $\mathbf{B} = \mathbf{I}, \mathbf{t} = 0, 0 \le w \le 1$
  $\sigma^2 = \frac{1}{DNM} \sum_{n=1}^{N} \sum_{m=1}^{M} \|\mathbf{x}_n - \mathbf{y}_m\|^2$
- EM optimization, repeat until convergence:
  - E-step: Compute $\mathbf{P}$,
    $$p_{mn} = \frac{\exp^{-\frac{1}{2\sigma^2}\|\mathbf{x}_n - (\mathbf{B}\mathbf{y}_m + \mathbf{t})\|^2}}{\sum_{k=1}^{M} \exp^{-\frac{1}{2\sigma^2}\|\mathbf{x}_n - (\mathbf{B}\mathbf{y}_k + \mathbf{t})\|^2} + (2\pi\sigma^2)^{D/2} \frac{w}{1-w} \frac{M}{N}}$$
  - M-step: Solve for $\mathbf{B}, \mathbf{t}, \sigma^2$:
    - $N_{\mathbf{P}} = \mathbf{1}^T\mathbf{P}\mathbf{1}, \mu_{\mathbf{x}} = \frac{1}{N_{\mathbf{P}}}\mathbf{X}^T\mathbf{P}^T\mathbf{1}, \mu_{\mathbf{y}} = \frac{1}{N_{\mathbf{P}}}\mathbf{Y}^T\mathbf{P}\mathbf{1}$,
    - $\hat{\mathbf{X}} = \mathbf{X} - \mathbf{1}\mu_{\mathbf{x}}^T, \;\; \hat{\mathbf{Y}} = \mathbf{Y} - \mathbf{1}\mu_{\mathbf{y}}^T$,
    - $\mathbf{B} = (\hat{\mathbf{X}}^T\mathbf{P}^T\hat{\mathbf{Y}})(\hat{\mathbf{Y}}^T \mathrm{d}(\mathbf{P}\mathbf{1})\hat{\mathbf{Y}})^{-1}$,
    - $\mathbf{t} = \mu_{\mathbf{x}} - \mathbf{B}\mu_{\mathbf{y}}$,
    - $\sigma^2 = \frac{1}{N_{\mathbf{P}}D}(\mathrm{tr}(\hat{\mathbf{X}}^T \mathrm{d}(\mathbf{P}^T\mathbf{1})\hat{\mathbf{X}}) - \mathrm{tr}(\hat{\mathbf{X}}^T\mathbf{P}^T\hat{\mathbf{Y}}\mathbf{B}^T))$.
- The aligned point set is $\mathcal{T}(\mathbf{Y}) = \mathbf{Y}\mathbf{B}^T + \mathbf{1}\mathbf{t}^T$,
- The probability of correspondence is given by $\mathbf{P}$.

Fig. 3. Affine point set registration algorithm.

Taken from A. Myronenko and X.B. Song