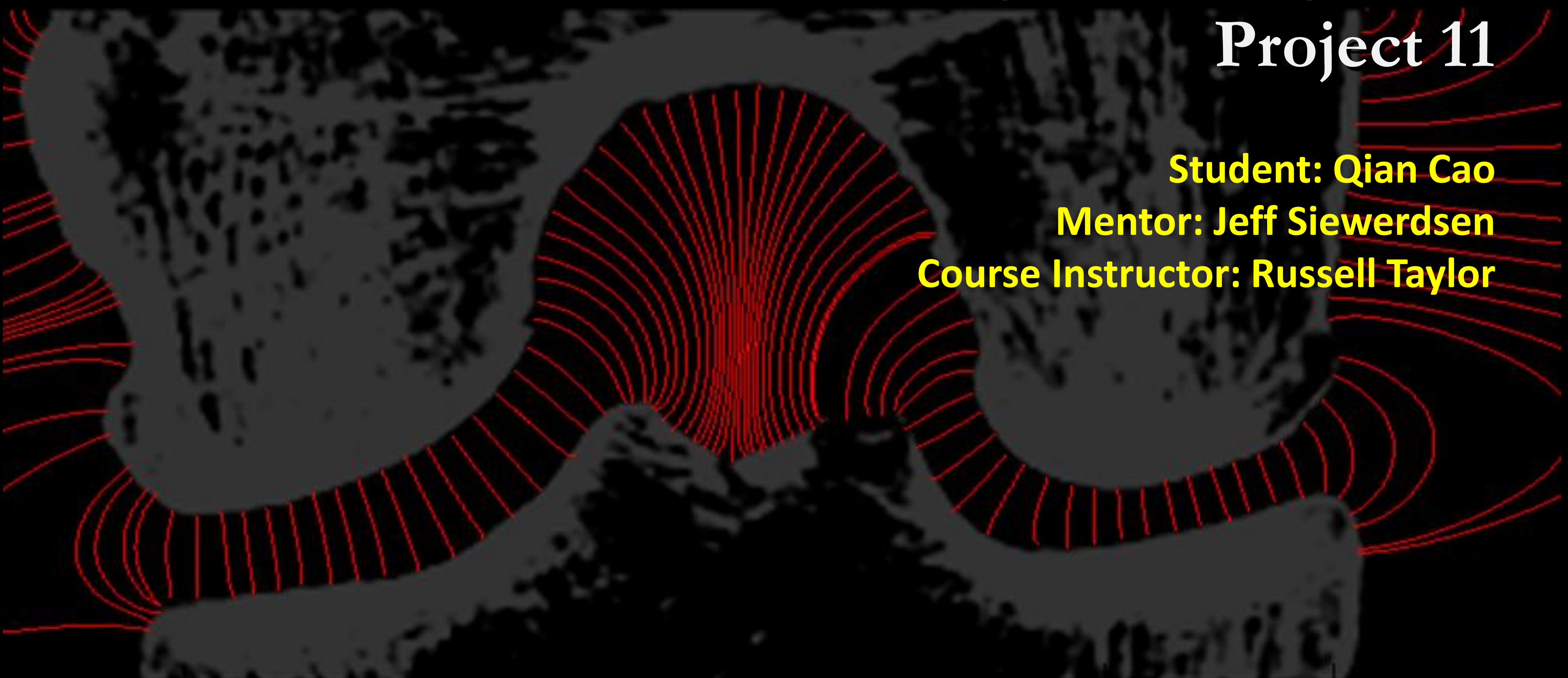
A Capacitor Model for Joint Space Quantification

Computer Integrated Surgery II

Project 11



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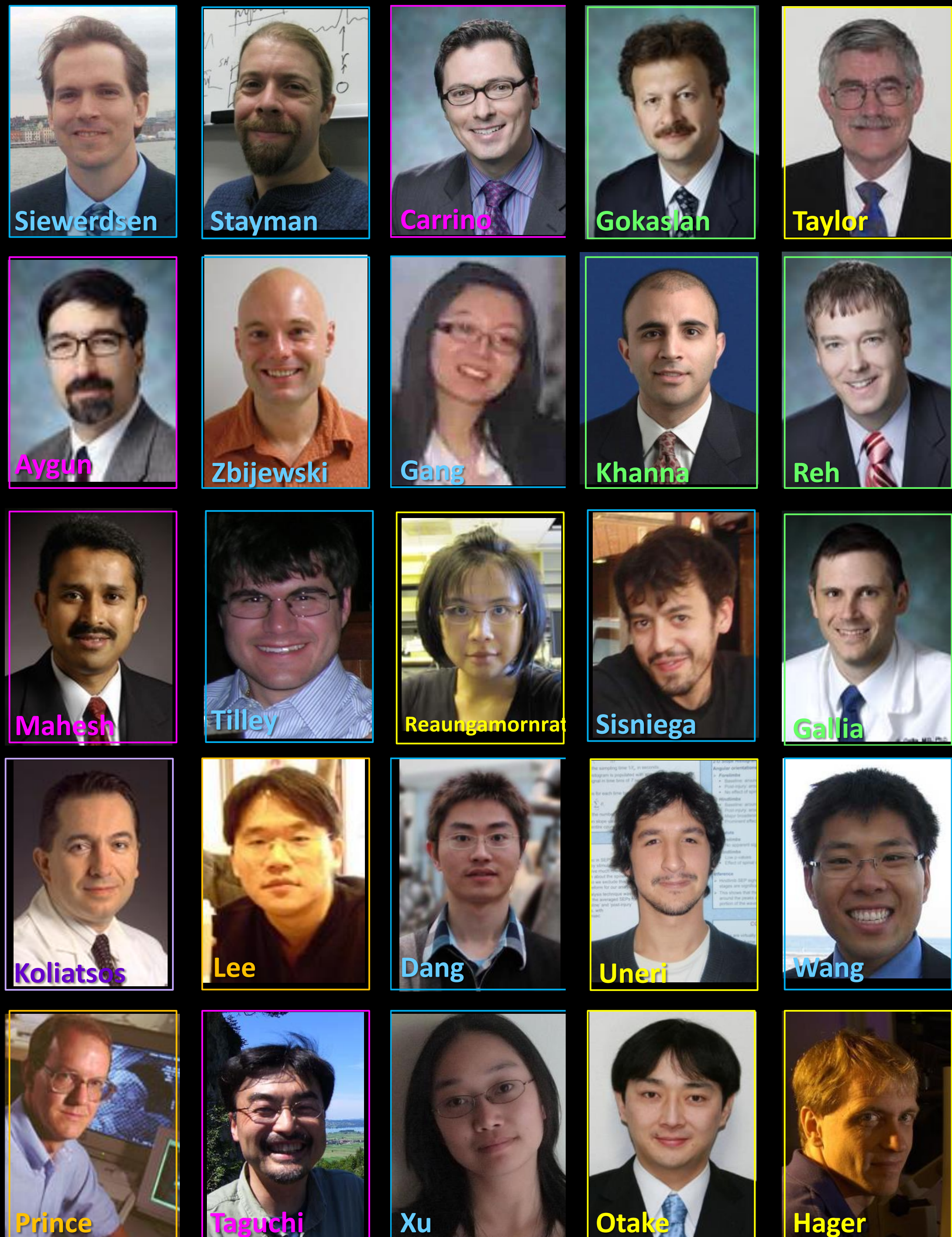
Course Instructor: Russell Taylor



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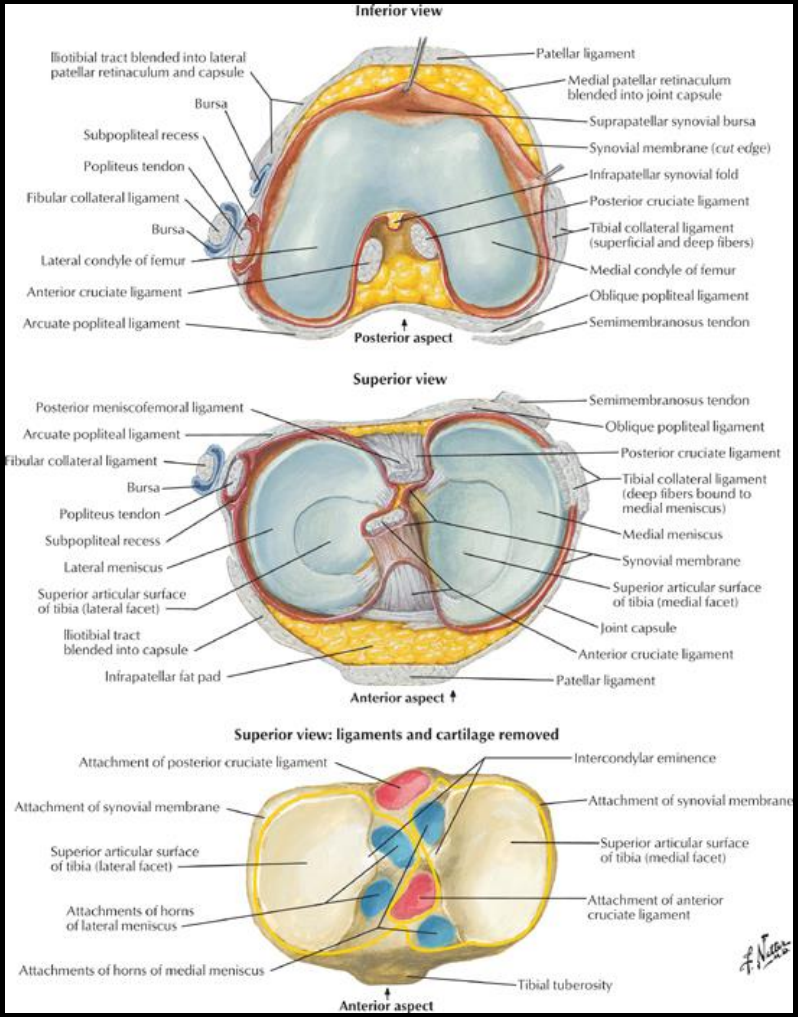
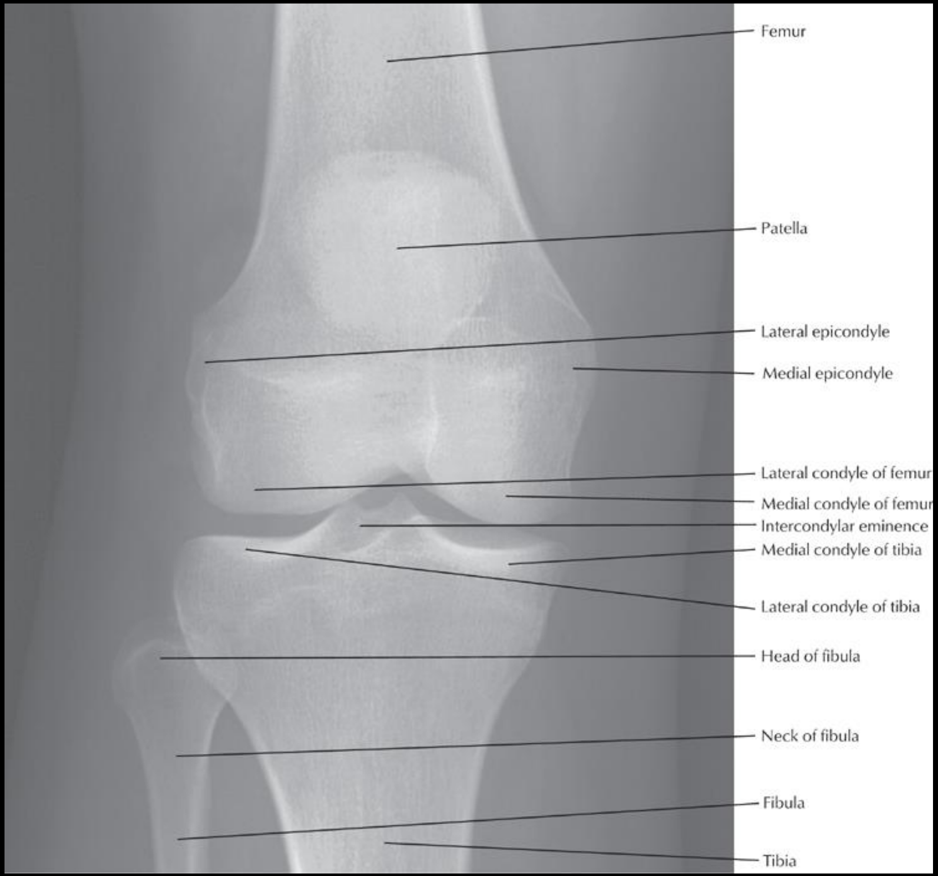




*Netter's *Atlas of Human Anatomy 5E*

Early Measurement Schemes

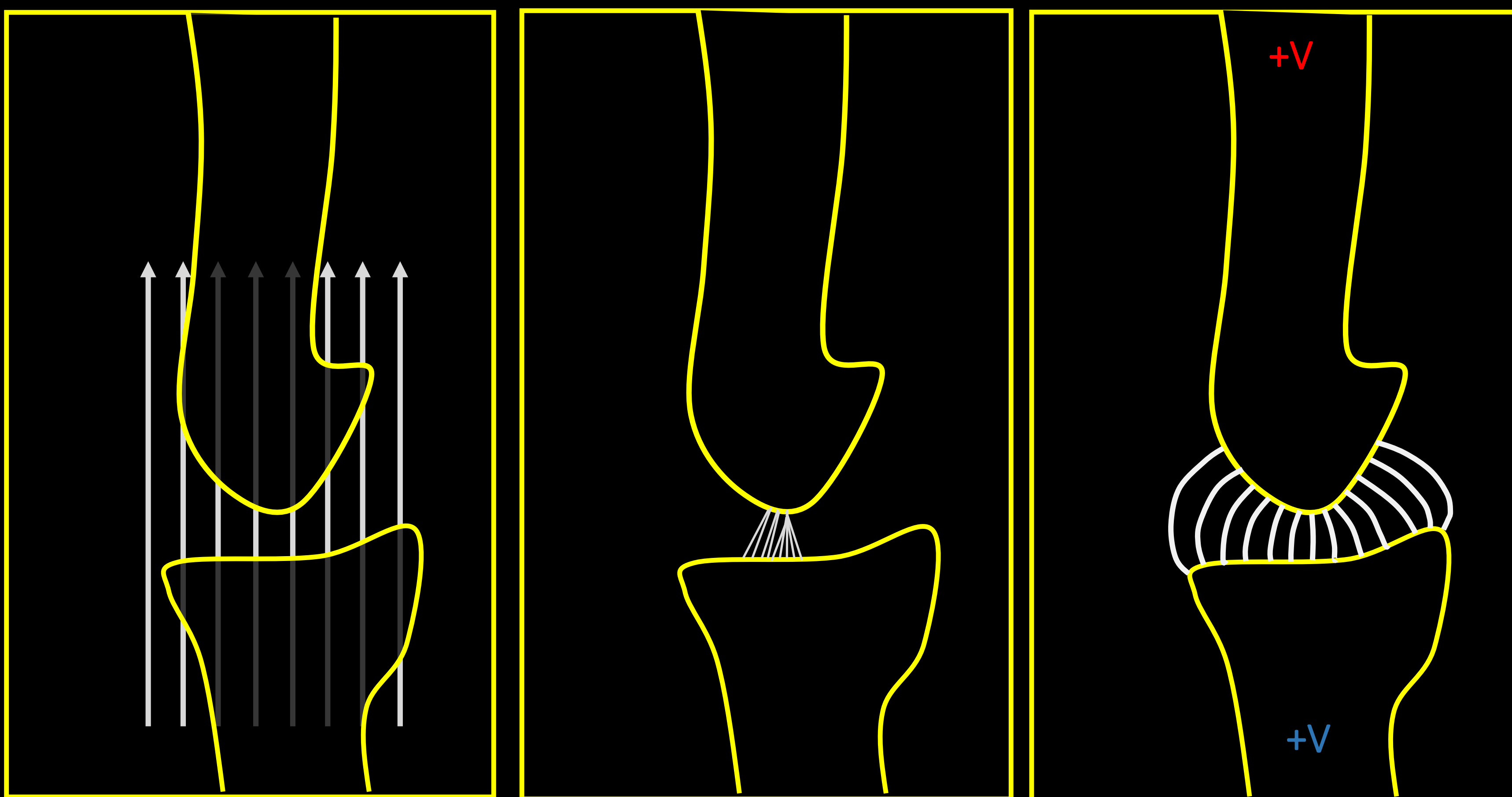
- Sharp-Larsen Score for Radiographs
  - Originally proposed in the 1970s to assess radiographs of hands and wrists.
  - Measures erosion and joint space narrowing on a semi-quantitative scale.
  - Many variants.
- Problems for Joint Space Width (JSW) Measurements
  - Standardization: beam direction, knee position, foot rotation.
  - Making use of volumetric data (map-making).

Schemes of 3D Joint Space Measurement

Longitudinal Axis

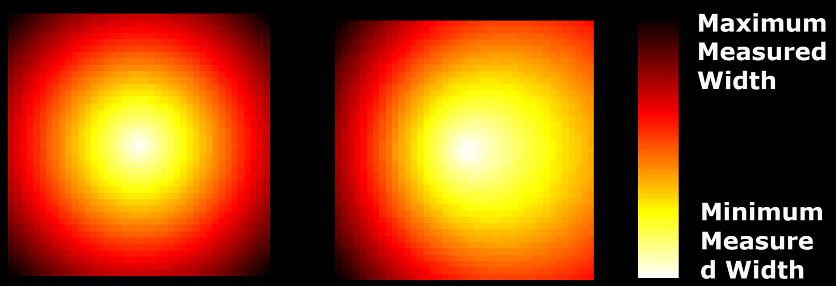
Closest Point

Electrostatic Model



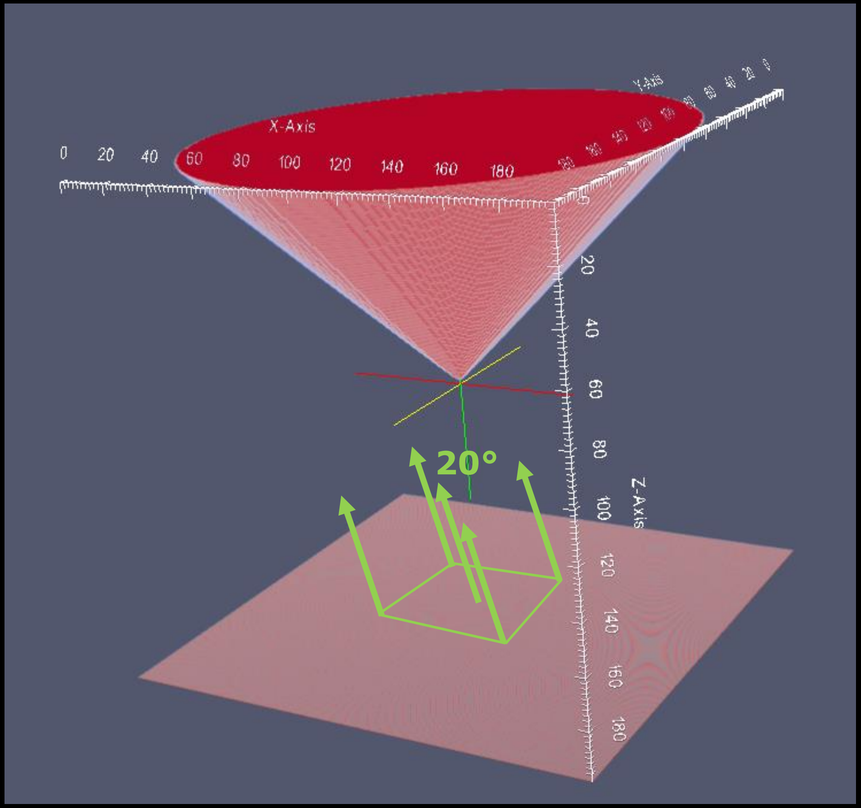
Schemes of 3D Joint Space Measurement

1. Longitudinal Axis



Projection along z.

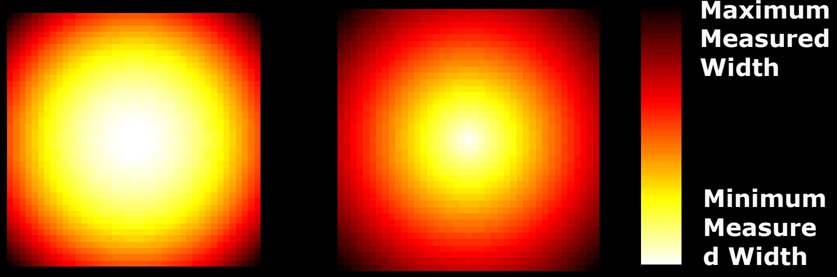
Angulated 20° about z axis.



1. Dependent on the selection of an arbitrary axis.
2. Sensitive to orientation of axis and bones.

Schemes of 3D Joint Space Measurement

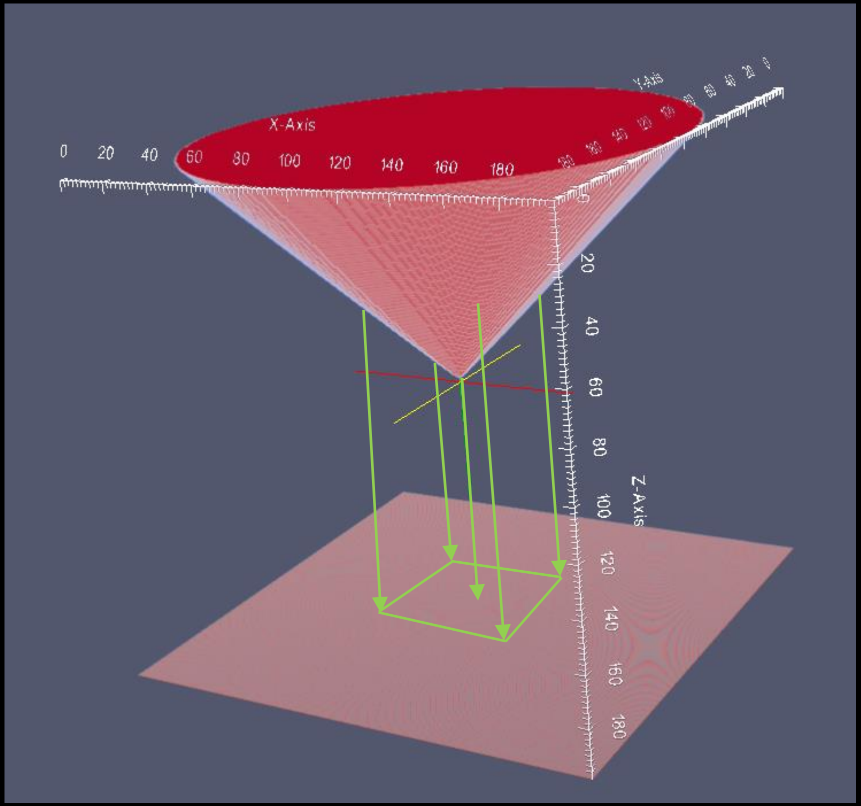
2. Closest Point



From plane to cone.

From cone to plane.

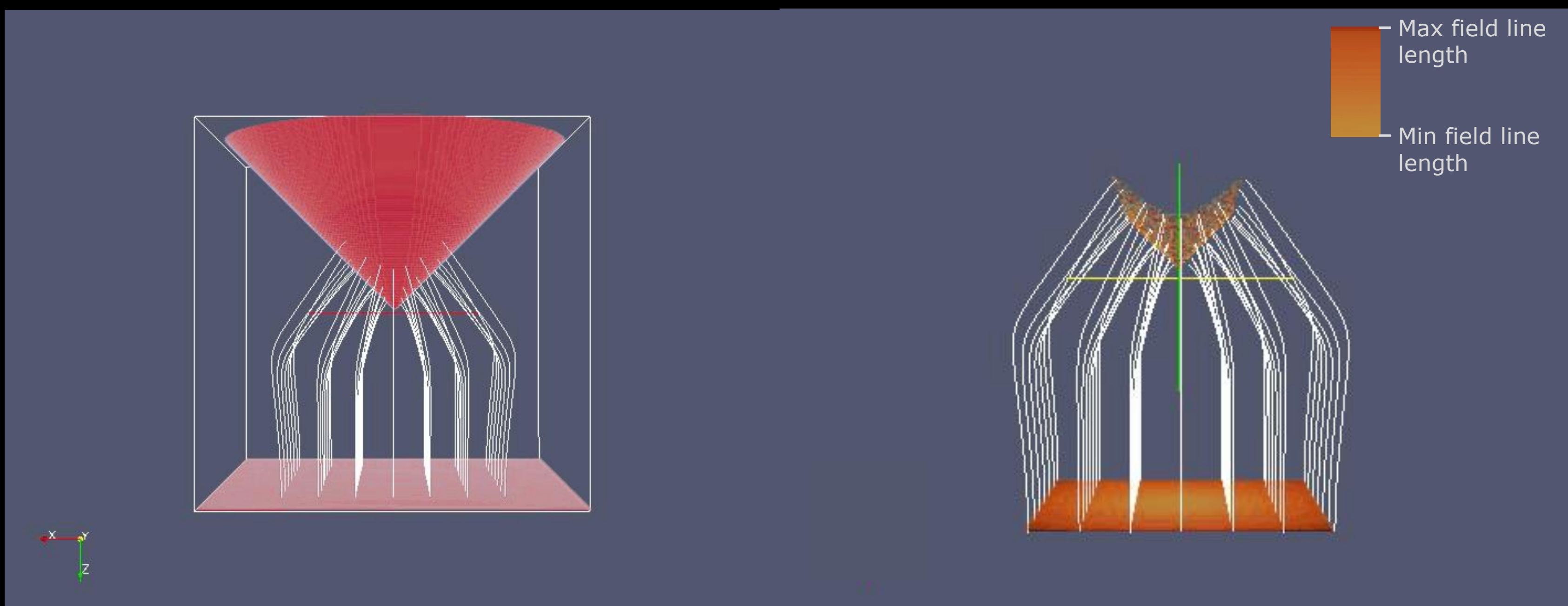
1. Mapping is not consistent.
2. Not all spatial information used.



Schemes of 3D Joint Space Measurement

3. Electrostatic Model

Trace electric field lines from one surface to another (e.g. femur to tibia). Then treat field line length as joint space width. Obtained mapping is one-to-one.



Orientation-invariant, non-arbitrary, non-degenerate, and continuous in the joint space.

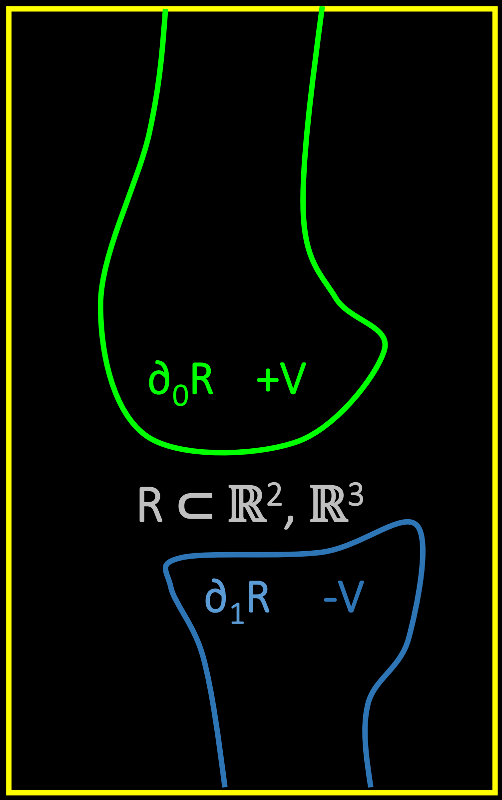
Specific Aims

1. Implement an algorithm that generates a joint space map (JSM) based on electrostatics.

2. Validate and test the algorithm on real knee volume data.

3. Provide thorough documentation and analysis of the algorithm for future biomechanical studies.

Capacitor Model – Problem Formulation



Gauss's law for electric fields in differential form:

$$\nabla \cdot \vec{\boldsymbol{E}} = \frac{\rho}{\varepsilon_0} \qquad (1)$$

ρ: Charge density [C/m^3]
ε_0: Electric permittivity of free space [C/(Vm)]

$$\vec{\boldsymbol{E}} = -\nabla\Phi \qquad (2)$$

Substitute (2) into (1),

$$\nabla^2\Phi = -\frac{\rho}{\varepsilon_0} \qquad (3)$$

Solve (3) for Φ. Assume $\forall x \in R, \rho(x)=0$, (3) becomes Laplace equation with boundary conditions:
$\Phi(\partial_0 R) = V_0$ and $\Phi(\partial_1 R) = V_1$

Capacitor Model – Jacobi Method

Consider Taylor's expansion for Φ in the x direction at (a,b,c):

$$\Phi(a+h,b,c) = \Phi(a,b,c) + \left.\frac{\partial\Phi}{\partial x}\right|_{(a,b,c)} h + \left.\frac{\partial^2\Phi}{\partial x^2}\right|_{(a,b,c)} \frac{h^2}{2!} + \left.\frac{\partial^3\Phi}{\partial x^3}\right|_{(a,b,c)} \frac{h^3}{3!} + O(h^3) \quad (1)$$

$$\Phi(a-h,b,c) = \Phi(a,b,c) - \left.\frac{\partial\Phi}{\partial x}\right|_{(a,b,c)} h + \left.\frac{\partial^2\Phi}{\partial x^2}\right|_{(a,b,c)} \frac{h^2}{2!} - \left.\frac{\partial^3\Phi}{\partial x^3}\right|_{(a,b,c)} \frac{h^3}{3!} + O(h^3) \quad (2)$$

Sum (1) and (2),

$$\left.\frac{\partial^2\Phi}{\partial x^2}\right|_{(a,b,c)} \approx \frac{\Phi(\mathrm{a+h,b,c}) - 2\Phi(a,b,c) + \Phi(\mathrm{a-h,b,c})}{h^2}$$

Similarly,

$$\left.\frac{\partial^2\Phi}{\partial y^2}\right|_{(a,b,c)} \approx \frac{\Phi(\mathrm{a,b+h,c}) - 2\Phi(a,b,c) + \Phi(\mathrm{a,b-h,c})}{h^2}$$

$$\left.\frac{\partial^2\Phi}{\partial z^2}\right|_{(a,b,c)} \approx \frac{\Phi(\mathrm{a,b,c+h}) - 2\Phi(a,b,c) + \Phi(\mathrm{a,b,c-h})}{h^2}$$

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0$$

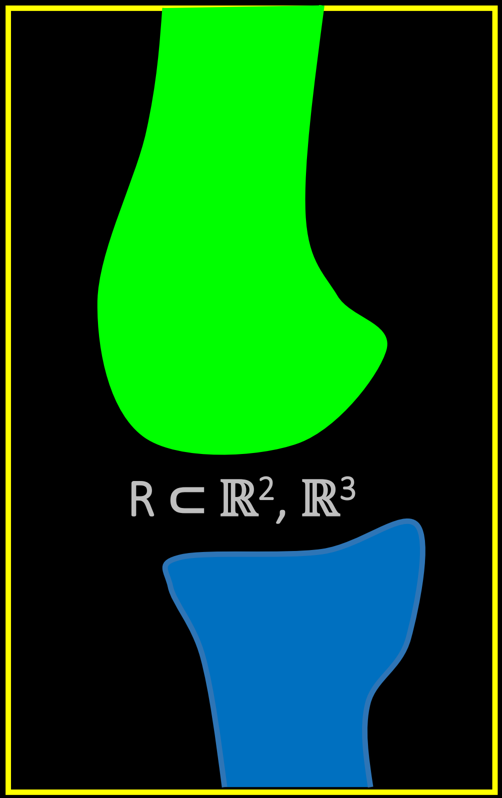
$$\approx \frac{\Phi(\mathrm{a+h,b,c}) + \Phi(\mathrm{a-h,b,c}) + \Phi(\mathrm{a,b+h,c}) + \Phi(\mathrm{a,b-h,c}) + \Phi(\mathrm{a,b,c+h}) + \Phi(\mathrm{a,b,c+h}) - 6\Phi(a,b,c)}{h^2}$$

Capacitor Model – Jacobi Method

Set h=1.

$$\Phi(a,b,c) \approx \frac{1}{6}[\Phi(a+1,b,c) + \Phi(a-1,b,c) + \Phi(a,b+1,c) + \Phi(a,b-1,c) + \Phi(a,b,c+1) + \Phi(a,b,c-1)]$$

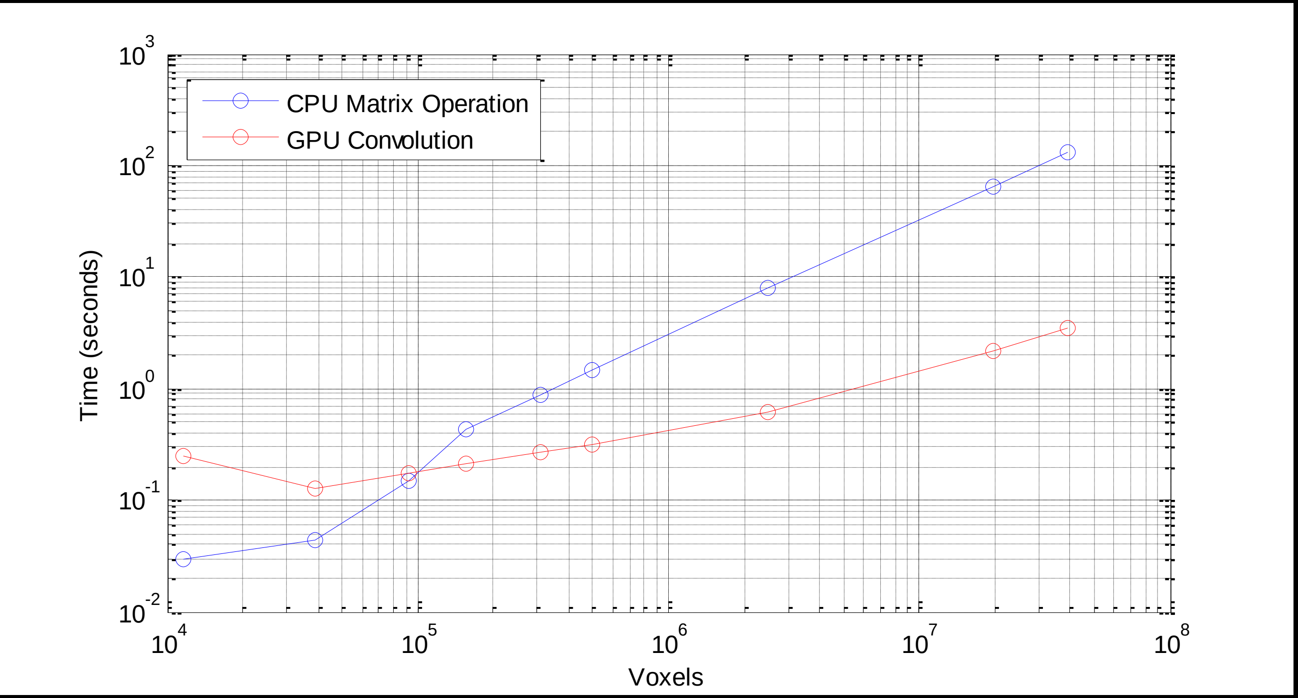
Each voxel is the average of neighboring voxels.



The Jacobi Method:

1. Make an initial guess.
2. Apply equation above to every voxel.
3. Iterate 2.

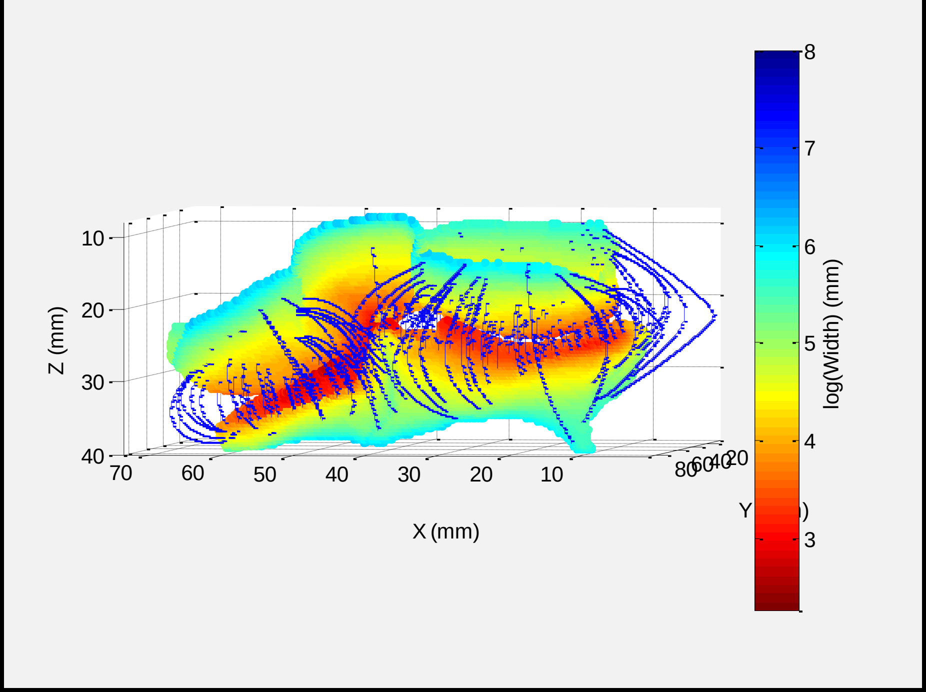
Capacitor Model – Implementation

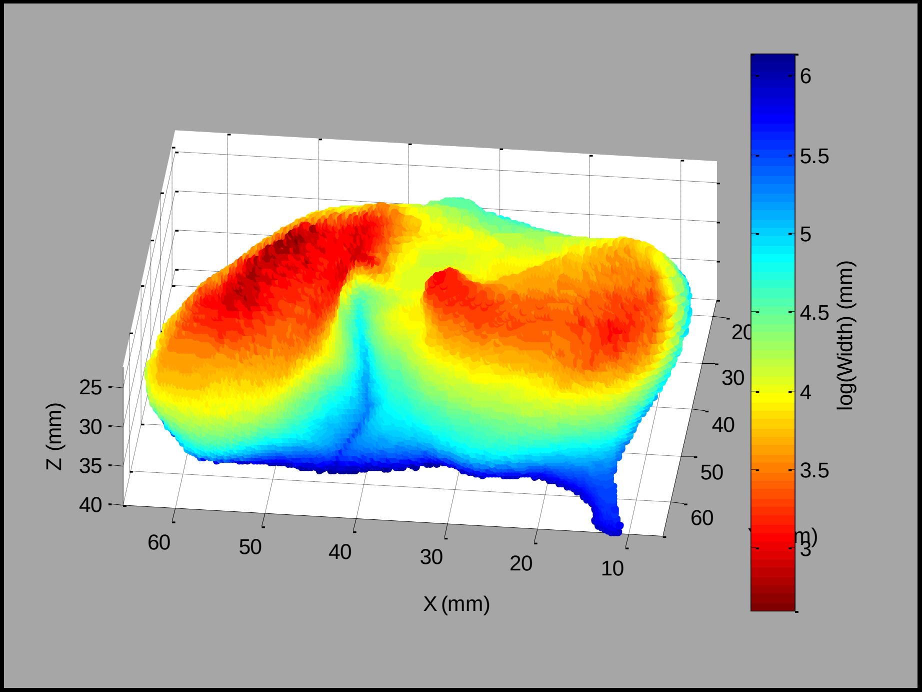


Implementation as convolution allows parallelizability.

*Benchmarking results on Intel Xeon E5405 2.00GHz and Nvidia GTX470 GPU.

Capacitor Model – Preliminary Results





Capacitor Model – More Work

1. Rate of convergence? Measure of error?

2. Boundary conditions.

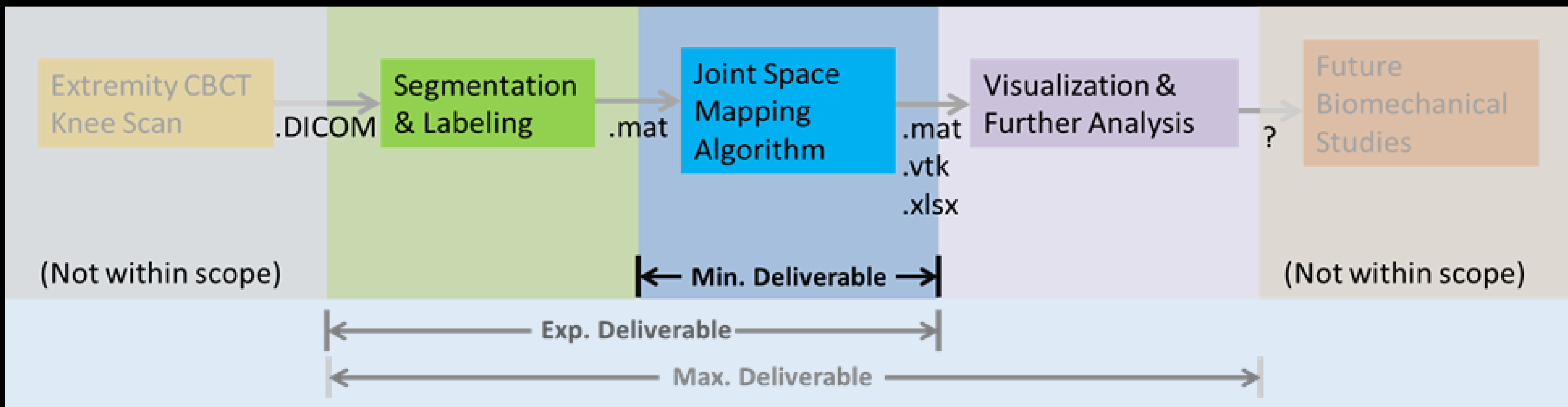
3. Ways to improve performance?
   - Expand higher order terms in the Taylor series and increase dimensions of convolution mask.
   - FFT → IFFT?

Segmentation

1. Application-specific segmentation method is a work in progress.

2. Preliminary segmentation algorithm uses:
   - Thresholding.
   - Connected-components analysis.
   - Morphological dilation and erosion.

Deliverables



Minimum Deliverable (Expected by 03/01/2014)

1. A set of prototyped MATLAB functions for joint space mapping using the capacitor model.
2. A set of prototyped MATLAB functions for segmentation.
3. Provide relevant documentation.

Expected Deliverable (Expected by 04/01/2014)

1. A set of validated and refined MATLAB functions for joint space mapping using the capacitor model.
2. A refined MATLAB function for segmentation.
3. Detailed analysis of algorithm performance (convergence characteristics, accuracy, speed etc)
4. Provide relevant documentation.

Maximum Deliverable (Expected by 05/01/2014)

1. MATLAB routines for visualization of the analysis results (volume rendering + GUI) in VTK and QT.
2. Detailed in-line and PDF documentation of all code.

Dependencies

1. Bi-weekly meeting with mentor (bi-weekly meeting scheduled with Prof. Siewerdsen).

2. CBCT knee volume test data (Two datasets available for algorithm testing and validation).

3. Computing resources.
   i) Up-to-date MATLAB w/ image processing and parallel computing toolboxes (R2013b).
   ii) CUDA-enabled graphics card (NVidia GTX470).
   iii) C++ IDE and compiler (Visual Studio 2008).
   iv) Visualization library (VTK).

4. Access to relevant literature (Lab database & JHU Library Website).

Timeline

Stage	Task	6-Jan	13-Jan	20-Jan	27-Jan	3-Feb	10-Feb	17-Feb	24-Feb	27-Feb	3-Mar	10-Mar	17-Mar	24-Mar	31-Mar	7-Apr	14-Apr	9-May
Preliminary Experimentation	Literature Review																	
	Problem Formulation																	
	Method Selection																	
	Synthesize Toy Data																	
	Experiment with Methods for Knee Volume Segmentation																	
	Prototype Algorithm for Joint Space Analysis																	
	Experiment with Visualization Toolchain (Volume Rendering, etc)																	
	Preliminary Profile and Benchmark																	
	Project Proposal Presentation																	
	Milestone #1																	
Algorithm Validation and Refinement	Refine and Test Segmentation Method																	
	Apply Joint Space Analysis Algorithm to Knee Data																	
	Algorithm: Consider alternative boundary conditions																	
	Algorithm: Characterize error																	
	Algorithm: Characterize Rate of Convergence																	
	Tweak, Debug, Profile, Document																	
	Project Checkpoint Presentation (TBA)																	
	Milestone #2																	
	Finalize Visualization Toolchain (MATLAB wrapper etc)																	
Documentation	Finalize documentation																	
	Milestone #3																	
	Final Presentation																	

Management Plan

1. Regular meetings with Prof. Siewerdsen (every other week).
2. Discuss and receive feedback from other students and post-docs in the group.
3. Guarantee 35 hours/week of work on average.

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Thanks!