Evaluation and Optimization of Virtual Rigid Body

Project 14

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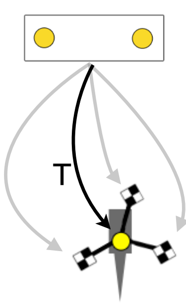
Recapitulation of the project

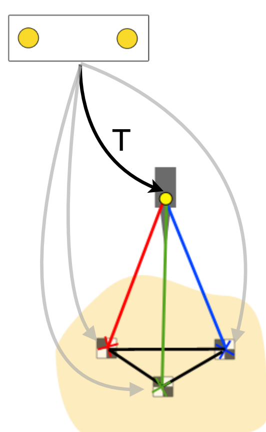
•Pose (T = [R, t]) of the surgical tool in optical tracker coordinates?

Conventional physical rigid body (PRB)

Optical Tracker
(MicronTracker)



Virtual rigid body (VRB)



How do the two types of rigid body compare?

Designing Optically Tracked Instruments for Image-Guided Surgery

J.B. West, C.R. Maurer,

IEEE Transactions in Med. Imaging, 2004, 23(5)

Goal of the paper

- Overview of registration errors in optical tracking system
- Theoretical prediction, simulation and experimental evaluation

Key points

- Theoretical model predicts that:
  1. Tracking error of a tooltip ↑ with
     - ↑ measurement error of fiducial positions — internal to hardware/software
     - ↑ distance of the fiducials from the tooltip
     - ↓ distance of the fiducials from each other
     - ↓ number of fiducials

     fiducial design
  2. Error accumulates in quadrature with multiple registration
- Model prediction matches well with experimental results

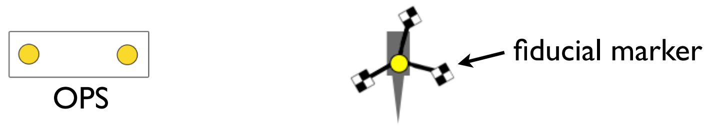
Significance

- Convenient means to estimate the accuracy from the given fiducial design
- General guideline for fiducial marker design

Background

Optical tracking

- Purpose: Identifies a surgical tool in the pre/intraoperative image or other frames.
- Composition:
  - Optical position sensor (OPS): measures the fiducial position
  - Fiducial marker
    - Active: emits light
    - Passive: reflective / non-reflective (MicronTracker)



Registration

Transformation between two coordinate frames

- Point-to-point registration
  - Find T=[R,**t**] between corresponding N points $\{\boldsymbol{x}_i\}$ and $\{\boldsymbol{y}_i\}$ that minimizes :

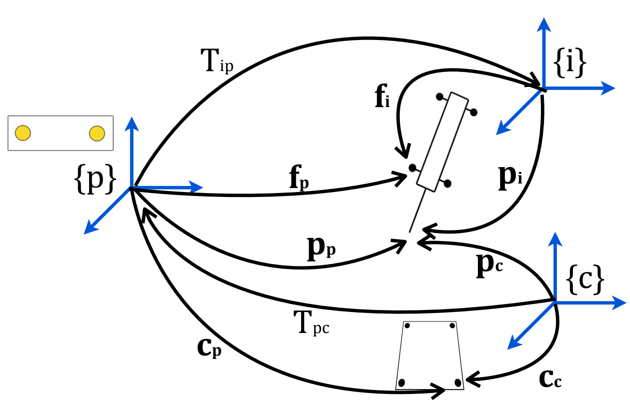
$$d^2 = \frac{1}{N}\sum_{i=1}^{N} |\boldsymbol{y}_i - (R\boldsymbol{x}_i + \boldsymbol{t})|^2$$

  - Quaternion implementation for CIS I

Background

Circumstances in optical tracking

- Single registration
  - between the optical position sensor (OPS) and the surgical instrument
- Two (or more) registrations
  - Introduction of coordinate reference frame (CRF)
  - Fiducials attached to the patient to track the tool with respect to the patient
  - Allows repositioning the OPS and the patient to maintain line of sight



f : instrument fiducial position
p : instrument tip position
c : CRF fiducial position
{p} : OPS coordinate
{i} : instrument coordinate
{c} : CRF coordinate

Accomplishment details

1. Theory
   1) Types of error in optical tracking system
   2) Statistical prediction of the registration errors
   3) Statistical prediction of composite error with multiple registration

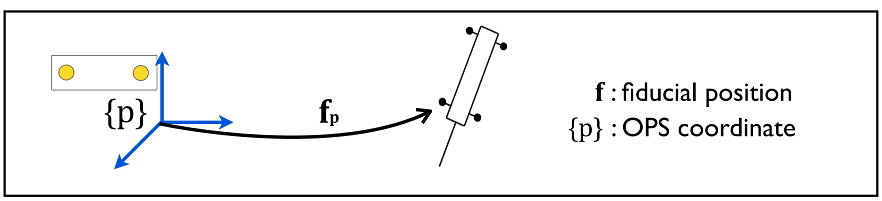
2. Numerical simulation

3. Experimental evaluation

Theory - 1. Types of errors in optical tracking

1. Theory
   1) Types of error
   2) Statistical prediction
   3) Composite error
2. Simulation
3. Experiment

1) <u>Fiducial Localization Error</u> (FLE)
- Distance between the actual and measured fiducial positions in optical position sensor (OPS) coordinate
- Value reported by the manufacturers (~0.4mm Polaris)



For the *j*th out of *N* fiducials

$$FLE_j \equiv \left\| \boldsymbol{f}^*_{pj} - \boldsymbol{f}_{pj} \right\|$$

$$FLE \equiv rms[FLE_1, \cdots, FLE_N] = \sqrt{\frac{1}{N}\sum_{j=1}^{N} FLE_j^2}$$

[]* : actual
[] : measured
rms : root-mean-squared

Theory - 1.Types of errors in optical tracking

1. Theory
1) Types of error
2) Statistical prediction
3) Composite error
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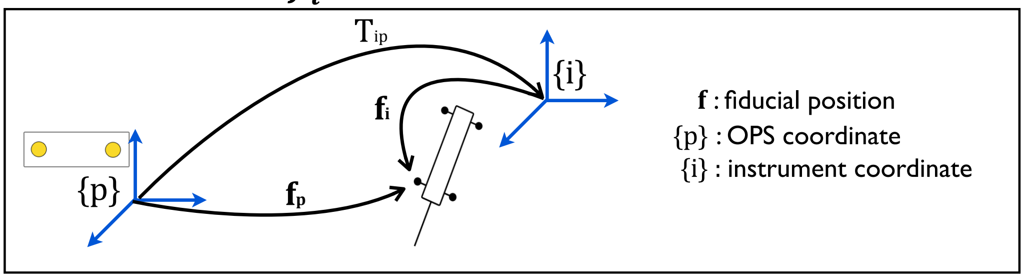
3. Experiment

2) <u>Fiducial Registration Error</u> (FRE)

- Distance between the actual and registered fiducial positions in OPS coordinate
- Assumption: actual fiducial positions are known in the instrument coordinate ($\boldsymbol{f}_{\boldsymbol{i}}^{*}$)



For the jth out of N fiducials

$$FRE_j \equiv \left\|T_{ip}^{*} \cdot \boldsymbol{f}_{\boldsymbol{ij}}^{*} - T_{ip} \cdot \boldsymbol{f}_{\boldsymbol{ij}}^{*}\right\| = \left\|T_{ip}^{*} \cdot \boldsymbol{f}_{\boldsymbol{ij}}^{*} - \boldsymbol{f}_{\boldsymbol{pj}}\right\|$$

$$FRE \equiv rms[FRE_1, \cdots, FRE_N]$$

[]* : actual
[] : measured
rms : root-mean-squared

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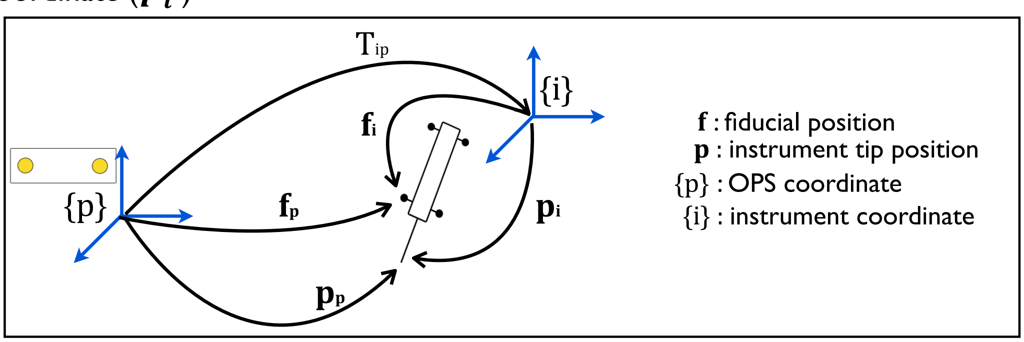
Theory - 1.Types of errors in optical tracking

1. Theory
1) Types of error
2) Statistical prediction
3) Composite error
2. Simulation
3. Experiment

3) <u>Target Registration Error</u> (TRE)

- Distance between the actual and registered target or tooltip position in OPS coordinate
- Assumption: actual tooltip position is known in the instrument coordinate ($\boldsymbol{p}_{\boldsymbol{i}}^{*}$)



$$TRE \equiv \left\|\boldsymbol{p}_{\boldsymbol{p}}^{*} - \boldsymbol{p}_{\boldsymbol{p}}\right\| = \left\|T_{ip}^{*} \cdot \boldsymbol{p}_{\boldsymbol{i}}^{*} - T_{ip} \cdot \boldsymbol{p}_{\boldsymbol{i}}^{*}\right\| = \left\|T_{ip}^{*} \cdot \boldsymbol{p}_{\boldsymbol{i}}^{*} - \boldsymbol{p}_{\boldsymbol{p}}\right\|$$

[]* : actual
[] : measured

Theory - 2. Statistical prediction of registration errors

$\langle\cdot\rangle$: expected value

$$\langle FLE^2 \rangle = 3\sigma^2$$

σ^2: variance of random noise

$$\langle FRE^2 \rangle = \frac{N-2}{N}\langle FLE^2 \rangle$$

(Sibson, R., 1979)

- Remark: FRE does *not* depend on spatial distribution of fiducials

$$\langle TRE^2(\boldsymbol{r}) \rangle = \underbrace{\frac{\langle FLE^2 \rangle}{N}\left(1}_{\text{translational}} + \underbrace{\frac{1}{3}\sum_{k=1}^{3}\frac{d_k^2}{f_k^2}}_{\text{rotational}}\right)$$

(Fitzpatrick, J., 2001)

d_k : distance of tooltip from each principal axis of the instrument

f_k : rms distance of the fiducials from each principal axis of the instrument

- Remark: TRE depends on number and spatial distribution of fiducial markers

Theory - 2. Statistical prediction of registration errors

$$\langle TRE^2(\boldsymbol{r}) \rangle = \frac{\langle FLE^2 \rangle}{N}\left(1 + \frac{1}{3}\sum_{k=1}^{3}\frac{d_k^2}{f_k^2}\right)$$

(Fitzpatrick, J., 2001)

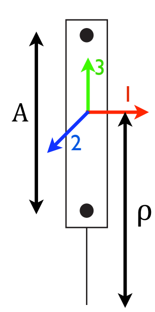
d_k : distance of tooltip from each principal axis of the instrument

f_k : rms distance of the fiducials from each principal axis of the instrument

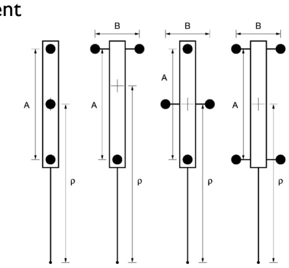
- Example:

	k=1	2	3
d_k	ρ	ρ	0
f_k	A/2	A/2	0

$$\langle TRE^2(\boldsymbol{r}) \rangle = \frac{\langle FLE^2 \rangle}{2}\left(1 + \frac{1}{3}\cdot\frac{4\rho^2}{A^2}\right)$$



Smaller TRE ⟵ Larger A : distance between fiducials
Smaller ρ : distance from tooltip



Case	Configuration	N	Rotational Component
a	Line	2	$\frac{4}{3}\frac{\rho^2}{A^2}\langle \mathrm{FLE}^2\rangle$
b	Line	3	$\frac{4}{3}\frac{\rho^2}{A^2}\langle \mathrm{FLE}^2\rangle$
c	Triangle	3	$\frac{8+3\xi^2}{2(4+3\xi^2)}\frac{\rho^2}{A^2}\langle \mathrm{FLE}^2\rangle$
d	Cross	4	$\frac{2(2+\xi^2)}{3(1+\xi^2)}\frac{\rho^2}{A^2}\langle \mathrm{FLE}^2\rangle$
e	Rectangle	4	$\frac{2+\xi^2}{3(1+\xi^2)}\frac{\rho^2}{A^2}\langle \mathrm{FLE}^2\rangle$

Figure and table from (West, 2004)

Theory - 3. Composite error with multiple registration

- Tracking the tooltip in the patient (coordinate reference) frame

  $\boldsymbol{p_c} = \boldsymbol{T_{pc}} \cdot \boldsymbol{T_{ip}} \cdot \boldsymbol{p_i}$:Two registration transformations

- What is the composite error?

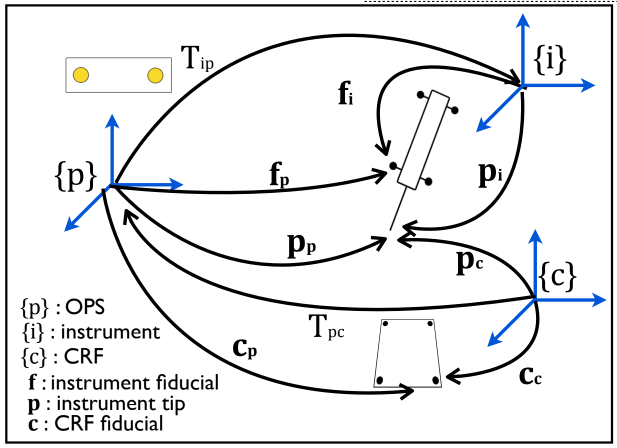
$$\boldsymbol{TRE_{ip,pc}}(\boldsymbol{p_c}) = \boldsymbol{p_c} - \boldsymbol{p_c^*}$$

$$= T_{pc} \cdot \boldsymbol{p_p} - T_{pc}^* \cdot \boldsymbol{p_p^*} \quad \text{vector form}$$

$$= T_{pc} \cdot \boldsymbol{p_p} - T_{pc}^* \cdot \left((p_p - \boldsymbol{TRE_{ip}}(\boldsymbol{p_p})\right)$$

$$= \boldsymbol{TRE_{pc}}(\boldsymbol{p_c}) + T_{pc}^* \cdot \boldsymbol{TRE_{ip}}(\boldsymbol{p_p})$$

$$\langle TRE_{ip,pc}^2(\boldsymbol{p_c})\rangle = \langle TRE_{pc}^2(\boldsymbol{p_c})\rangle + \langle TRE_{ip}^2(\boldsymbol{p_p})\rangle + \underbrace{2\langle TRE_{pc}^2(\boldsymbol{p_c}) \cdot TRE_{ip}^2(\boldsymbol{p_p})\rangle}_{\text{Orthogonal, zero-mean}}$$



$$\langle TRE_{ip,pc}^2(\boldsymbol{p_c})\rangle = \boxed{\langle TRE_{ip}^2(\boldsymbol{p_p})\rangle} + \boxed{\langle TRE_{pc}^2(\boldsymbol{p_c})\rangle}$$: Quadrature sum of each TRE

Theory - 3. Composite error with multiple registration

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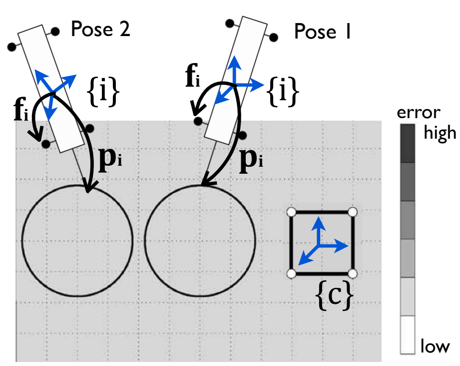
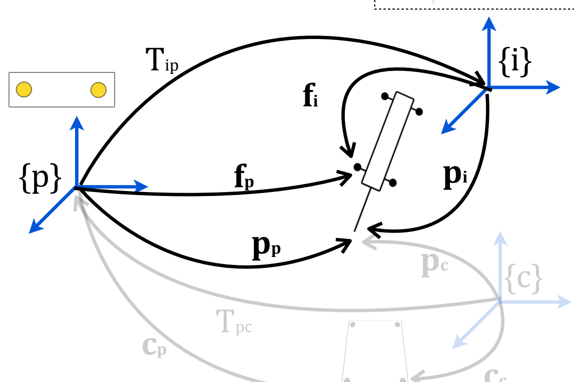
- Spatial map of $\boxed{\langle TRE_{ip}^2(\boldsymbol{p_p})\rangle}$





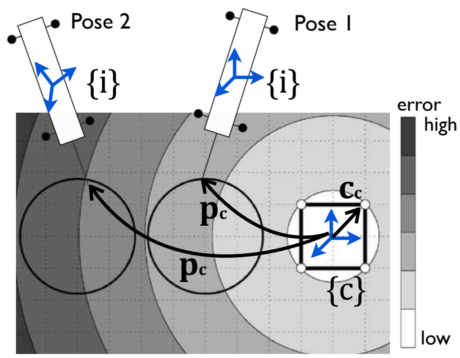
- Moving the tool does not change the instrument fiducial configuration with respective to tooltip
- Independent of spatial location of tooltip.

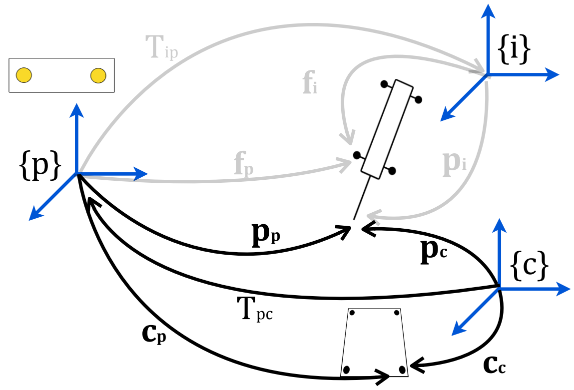
{p} : OPS
{i} : instrument
{c} : CRF
f : instrument fiducial
p : instrument tip
c : CRF fiducial

Figure modified from (West, 2004)

Theory - 3. Composite error with multiple registration

- Spatial map of $\boxed{\langle TRE^2_{pc}(\boldsymbol{p_c})\rangle}$



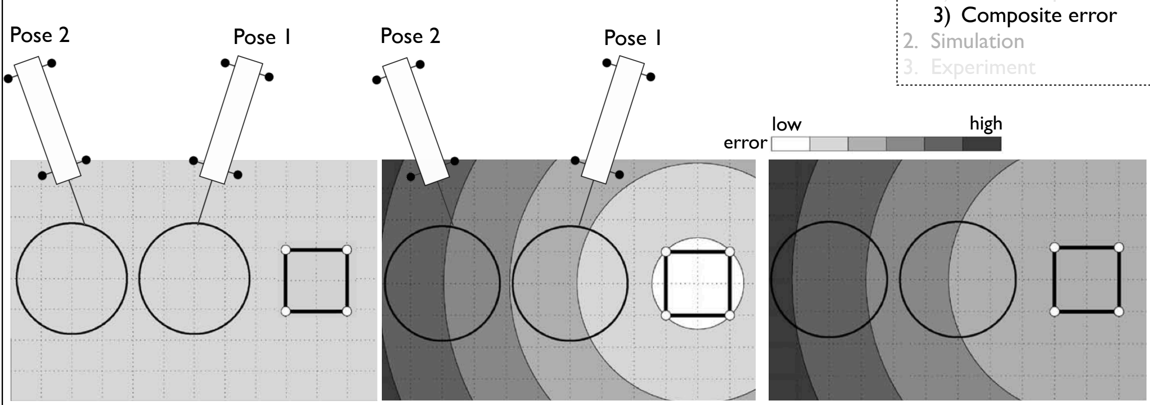


- Moving the tool <u>does</u> change the CRF fiducial configuration with respective to the tooltip.
- <u>Dependent</u> of spatial location of tooltip!

{p} : OPS
{i} : instrument
{c} : CRF
f : instrument fiducial
p : instrument tip
c : CRF fiducial

Figure modified from (West, 2004)

Theory - 3. Composite error with multiple registration



$$\langle TRE^2_{ip}(\boldsymbol{p_p})\rangle \quad + \quad \langle TRE^2_{pc}(\boldsymbol{p_c})\rangle \quad = \quad \langle TRE^2_{ip,pc}(\boldsymbol{p_c})\rangle$$

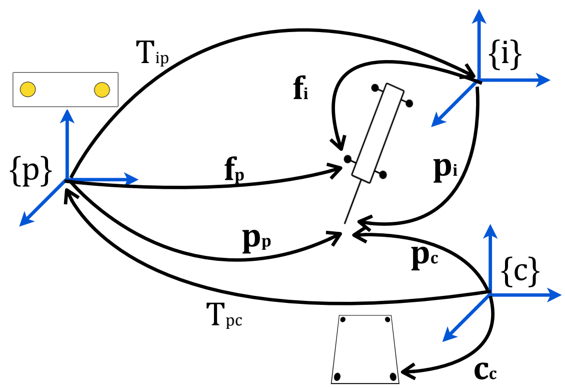
Figure modified from (West, 2004)

Simulation

1. Choose arbitrary actual values $\boldsymbol{c}_c^*, \boldsymbol{f}_i^*, T_{ic}^*, T_{cp}^*$
2. Compute actual values $\boldsymbol{p}_c^*, \boldsymbol{f}_c^*, \boldsymbol{c}_p^*, \boldsymbol{p}_p^*, \boldsymbol{f}_p^*$
3. Simulate OPS measurements $\boldsymbol{c}_p, \boldsymbol{p}_p$ by adding noise with
   $$\sigma = \frac{FLE}{\sqrt{3}}$$
   where FLE values are provided by the manufacturers of OPS.
4. Compute transformations T_{pc}, T_{ip} from "measured" values
5. The actual and measured values pointer tool in CRF are given by
   $$\boldsymbol{p}_c^* = T_{ic}^*(\boldsymbol{p}_i^*), \boldsymbol{p}_c = T_{pc} \cdot T_{ip}(\boldsymbol{p}_i^*)$$
6. $\langle TRE_{ip,pc}^2(\boldsymbol{p}_c)\rangle = \|\boldsymbol{p}_c - \boldsymbol{p}_c^*\|^2$
7. Loop 1-6 100000 times, and calculate rms value of the acquired TREs.
8. Perform simulation at various locations with respective to CRF

f : instrument fiducial position
p : instrument tip position
c : CRF fiducial position

{p} : OPS coordinate
{i} : instrument coordinate
{c} : CRF coordinate

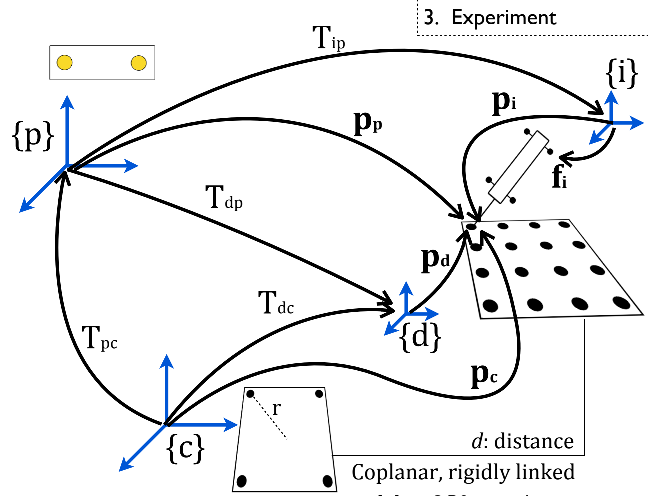


Experiment

- Goal
  - Experimentally measure $\langle TRE_{ip,pc}^2(\boldsymbol{p}_c)\rangle$ for a set of instrument and CRF fiducial configurations
- Setup
1. OPS, CRF, surgical instrument
2. Drilled 4x4 grid of divots (0.025 mm accuracy)
3. Divot grid coplanar, rigidly attached to CRF
4. Pivot calibration of tooltip

- Process

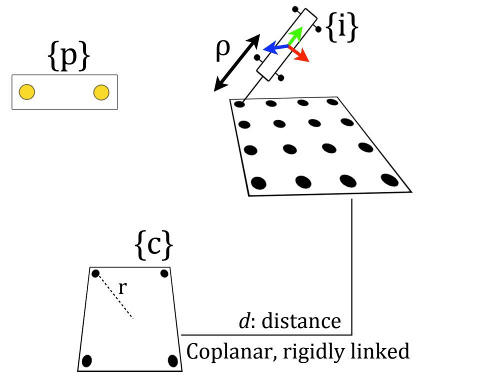
For given fiducial configuration,
1. Tooltip is placed at each divot
2. Registration between OPS and divot grid (T_{dp}), and its FRE is calculated
3. Evaluate $TRE_m = \sqrt{\frac{N}{N-2}}FRE$



{p} OPS coordinate
{i} instrument coordinate
{c} CRF coordinate
{d} divot grid coordinate

Experiment

- Experimental combinations
1. Two values of ρ (tool fiducial configuration)
2. Two values of r (CRF fiducial configuration)
3. Four values of d (CRF fiducial configuration)
4. Three types of experiments
   At each divot,
   A. Tool orientation, divots grid and CRF positions are constant.
   B. Tool orientation is random, divots grid and CRF positions are constant.
   C. Tool orientation, divots grid and CRF positions are constant.
5. Three types of OPS/fiducial marker
   1. Optotrack 3020 / Active (infrared)
   2. Polaris / Active (infrared)
   3. Polaris / Passive (retroreflective)

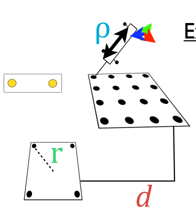


{p} OPS coordinate
{i} instrument coordinate
{c} CRF coordinate
{d} divot grid coordinate

Result

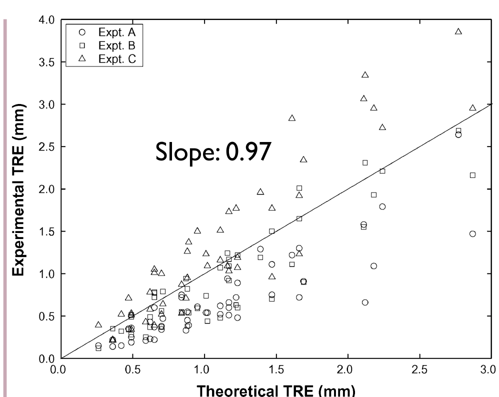
- Simulation
  - difference with prediction < 0.6%
- Experiment

Instrument	CRF	d (mm)	rms[TRE] (mm) Theory	Expt. A	Expt. B	Expt. C
$A = 50$ mm	$r = 28$ mm	100	0.36	0.22 (-38%)	0.35 (-3%)	0.22 (-39%)
$B = 25$ mm		200	0.62	0.23 (-63%)	0.37 (-40%)	0.58 (-6%)
$\rho = 75$ mm		300	0.89	0.39 (-56%)	0.54 (-40%)	1.37 (54%)
		400	1.17	0.60 (-49%)	1.09 (-7%)	1.03 (-13%)
$A = 50$ mm	$r = 28$ mm	100	0.49	0.25 (-48%)	0.28 (-42%)	0.51 (5%)
$B = 25$ mm		200	0.70	0.34 (-51%)	0.56 (-19%)	1.00 (43%)
$\rho = 150$ mm		300	0.95	0.61 (-36%)	0.59 (-38%)	1.50 (58%)
		400	1.22	0.72 (-41%)	0.63 (-48%)	1.77 (45%)
$A = 50$ mm	$r = 57$ mm	100	0.26	0.15 (-45%)	0.12 (-56%)	0.39 (47%)
$B = 25$ mm		200	0.36	0.14 (-62%)	0.35 (-5%)	0.21 (-42%)
$\rho = 75$ mm		300	0.49	0.19 (-60%)	0.51 (6%)	0.53 (10%)
		400	0.62	0.43 (-30%)	0.49 (-21%)	0.78 (27%)
$A = 50$ mm	$r = 57$ mm	100	0.42	0.15 (-65%)	0.32 (-23%)	0.52 (23%)
$B = 25$ mm		200	0.49	0.36 (-26%)	0.49 (0%)	0.34 (-31%)
$\rho = 150$ mm		300	0.59	0.21 (-64%)	0.25 (-58%)	0.43 (-27%)
		400	0.70	0.38 (-46%)	0.37 (-47%)	0.52 (-26%)



Experimental setups and TRE

TRE↑ ⟵ ↑d ↓r ↑ρ



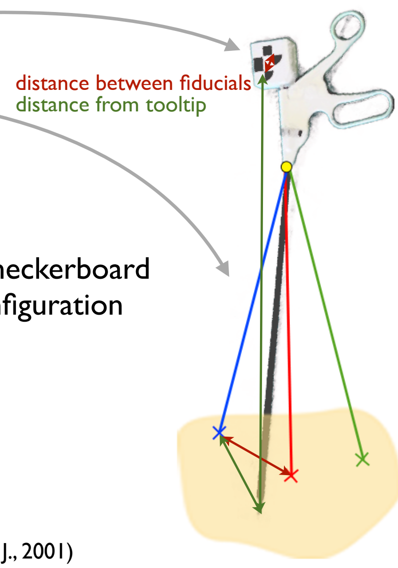
Comparison between theory and experiment

Considerable variability, but overall matches well

Table and figure taken from (West, 2004)

Relevance

- Theoretical support for virtual rigid body (VRB)'s usefulness
  Compared to conventional rigid bodies,
  - VRB can be projected farther from each other
  - VRB are projected closer to the tooltip
- Analysis and design considerations
  - Originally, primary interest was in FRE.
  - FLE and TRE should be kept in mind.
    - FLE : Detection of physical vs. projected light checkerboard
    - TRE : TRE, but not FRE, depends on fiducial configuration



$$\langle FLE^2 \rangle = 3\sigma^2$$

$$\langle FRE^2 \rangle = \frac{N-2}{N}\langle FLE^2 \rangle$$ (Sibson, R., 1979)

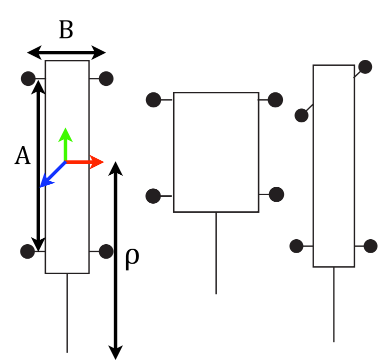
$$\langle TRE^2(\boldsymbol{r}) \rangle = \frac{\langle FLE^2 \rangle}{N}\left(1 + \frac{1}{3}\sum_{k=1}^{3}\frac{d_k^2}{f_k^2}\right)$$ (Fitzpatrick, J., 2001)

Picture of laparoscope taken from (Sánchez-Margallo, 2013)

Assessment

- Great summary of error in optical tracking
  - Classification
  - High-level theory, example applications, experimental evaluations
- Difficulty in following the experiment
- Qualitative analysis of experimental data regarding TRE and fiducial configurations
- Suggestions for further works
  - More test cases of instrument fiducial configurations
    - Combinations of A and B
    - Slanted distribution
  - Details and verification of rotational tooltip error

    $$\Delta\theta_k = \frac{FLE}{\sqrt{3N}f_k}$$
  - Fully passive fiducial markers (MicronTracker)



Further readings

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- Statistical details
  - R.Sibson, "Studies in the robustness of multidimensional scaling : Perturbational analysis of classical scaling", J. Roy. Statist. Soc. B, vol. 41, pp. 217–229, 1979.
  - J. M. Fitzpatrick and J. B. West, "Distribution of target registration error in rigid-body point-based registration", IEEE Trans. Med. Imag., vol. 20, pp. 917–927, Sept. 2001

- Experimental evaluations
  - J.Hummel et. al., "Design and application of an assessment protocol for electromagnetic tracking systems", Med. Phys. vol. 32(7), pp. 2371-2379, July 2005
  - J.A.Sánchez-Margallo, "Technical Evaluation of a Third Generation Optical Pose Tracker for Motion Analysis and Image-Guided Surgery", CLIP 2012, pp.75-82, 2013

Questions?