Recapitulation of the project

• Pose \( T = [R, t] \) of the surgical tool in optical tracker coordinates?

**Conventional physical rigid body (PRB)**

**Virtual rigid body (VRB)**

How do the two types of rigid body compare?
Designing Optically Tracked Instruments for Image-Guided Surgery

J.B. West, C.R. Maurer,

IEEE Transactions in Med. Imaging, 2004, 23(5)

Goal of the paper

• Overview of registration errors in optical tracking system
• Theoretical prediction, simulation and experimental evaluation

Key points & significance

Key points

• Theoretical model predicts that:
  1. Tracking error of a tooltip ↑ with
     ↑ measurement error of fiducial positions — internal to hardware/software
     ↑ distance of the fiducials from the tooltip
     ↓ distance of the fiducials from each other
     ↓ number of fiducials
  2. Error accumulates in quadrature with multiple registration
• Model prediction matches well with experimental results

Significance

• Convenient means to estimate the accuracy from the given fiducial design
• General guideline for fiducial marker design
Background

Optical tracking
• Purpose: Identifies a surgical tool in the pre/intraoperative image or other frames.
• Composition:
  - Optical position sensor (OPS): measures the fiducial position
  - Fiducial marker
    - Active: emits light
    - Passive: reflective / non-reflective (MicronTracker)

Registration
Transformation between two coordinate frames
• Point-to-point registration
  - Find \( T = [R, t] \) between corresponding \( N \) points \( \{x_i\} \) and \( \{y_i\} \) that minimizes:
    \[
    d^2 = \frac{1}{N} \sum_{i=1}^{N} |y_i - (Rx_i + t)|^2
    \]
  - Quaternion implementation for CIS I

Circumstances in optical tracking
• Single registration
  - Between the optical position sensor (OPS) and the surgical instrument
• Two (or more) registrations
  - Introduction of coordinate reference frame (CRF)
  - Fiducials attached to the patient to track the tool with respect to the patient
  - Allows repositioning the OPS and the patient to maintain line of sight
Accomplishment details

1. Theory
   1) Types of error in optical tracking system
   2) Statistical prediction of the registration errors
   3) Statistical prediction of composite error with multiple registration

2. Numerical simulation

3. Experimental evaluation

Theory - 1. Types of errors in optical tracking

1) Fiducial Localization Error (FLE)
   • Distance between the actual and measured fiducial positions in optical position sensor (OPS) coordinate
   • Value reported by the manufacturers (~0.4mm Polaris)

For the jth out of N fiducials

\[
FLE_j \equiv \|f^*_p - f_{pj}\| \\
FLE \equiv \text{rms}[FLE_1, \ldots, FLE_N] = \sqrt{\frac{1}{N} \sum_{j=1}^{N} FLE_j^2}
\]

\(f^*_p\) : actual  
\(f_{pj}\) : measured  
\(\text{rms} \) : root-mean-squared
2) **Fiducial Registration Error (FRE)**

- Distance between the actual and registered fiducial positions in OPS coordinate
- Assumption: actual fiducial positions are known in the instrument coordinate ($f_i^*$)

For the $j$th out of $N$ fiducials

$$FRE_j \equiv \left\| T_{ip}^* \cdot f_i^* - T_{ip} \cdot f_{ij} \right\| = \left\| T_{ip}^* \cdot f_{ij} - f_{pj} \right\|$$

$FRE \equiv rms[FRE_1, \cdots, FRE_N]$  

3) **Target Registration Error (TRE)**

- Distance between the actual and registered target or tooltip position in OPS coordinate
- Assumption: actual tooltip position is known in the instrument coordinate ($p_i^*$)

$$TRE \equiv \left\| p_{p}^* - p_p \right\| = \left\| T_{ip}^* \cdot p_i^* - T_{ip} \cdot p_i \right\| = \left\| T_{ip}^* \cdot p_i^* - p_p \right\|$$
Theory - 2. Statistical prediction of registration errors

1. Theory
   1) Types of error
   2) Statistical prediction
   3) Composite error

Simulation

Experiment

1. Theory

2. Statistical prediction of registration errors

\[ \langle FLE^2 \rangle = 3\sigma^2 \]

\( \sigma^2 \) variance of random noise

\[ \langle FRE^2 \rangle = \frac{N - 2}{N} \langle FLE^2 \rangle \]  (Sibson, R., 1979)

Remark: FRE does not depend on spatial distribution of fiducials

\[ \langle TRE^2(r) \rangle = \frac{\langle FLE^2 \rangle}{N} \left( 1 + \frac{1}{3} \sum_{k=1}^{3} \frac{d_k^2}{f_k^2} \right) \]  (Fitzpatrick, J., 2001)

\( d_k \): distance of tooltip from each principal axis of the instrument

\( f_k \): rms distance of the fiducials from each principal axis of the instrument

Remark: TRE depends on number and spatial distribution of fiducial markers

\[ \begin{array}{c|c|c|c} 
 k=1 & 2 & 3 \\
 \hline 
 d_k & \rho & \rho & 0 \\
 f_k & A/2 & A/2 & 0 \\
\end{array} \]

Example:

Smaller TRE \iff Larger \( A \): distance between fiducials
Smaller \( \rho \): distance from tooltip

Figure and table from (West, 2004)
Theory - 3. Composite error with multiple registration

- Tracking the tooltip in the patient (coordinate reference) frame
  \[ p_c = T_{pc} \cdot T_{ip} \cdot p_i \] : Two registration transformations

- What is the composite error?

\[ TRE_{ip,pc}(p_c) = p_c - p_c^* \]

\[ = T_{pc} \cdot p_p - T_{pc}^* \cdot p_p^* \] vector form

\[ = T_{pc} \cdot p_p - T_{pc} \cdot (p_p - TRE_{ip}(p_p)) \]

\[ = TRE_{pc}(p_c) + T_{pc} \cdot TRE_{ip}(p_p) \]

\[ \langle TRE_{ip,pc}^2(p_c) \rangle = \langle TRE_{pc}^2(p_c) \rangle + \langle TRE_{ip}^2(p_p) \rangle + 2\langle TRE_{pc}^2(p_c) \cdot TRE_{ip}^2(p_p) \rangle \]

Orthogonal, zero-mean

\[ \langle TRE_{ip,pc}^2(p_c) \rangle = \langle TRE_{ip}^2(p_p) \rangle + \langle TRE_{pc}^2(p_c) \rangle : \text{Quadrature sum of each TRE} \]

1. Theory
   1) Types of error
   2) Statistical prediction
   3) Composite error
2. Simulation
3. Experiment

- Spatial map of \( \langle TRE_{ip}^2(p_p) \rangle \)

- Moving the tool does not change the instrument fiducial configuration with respective to tooltip
- Independent of spatial location of tooltip.

Figure modified from (West, 2004)
**Theory - 3. Composite error with multiple registration**

- Spatial map of $\langle T^2_{PC}(p_c) \rangle$

*Moving the tool does change the CRF fiducial configuration with respective to the tooltip.*

*Dependent of spatial location of tooltip!*

Figure modified from (West, 2004)

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**Theory - 3. Composite error with multiple registration**

$\langle T^2_{IP}(p_p) \rangle + \langle T^2_{PC}(p_c) \rangle = \langle T^2_{IPPC}(p_c) \rangle$

Figure modified from (West, 2004)
### Simulation

1. Choose arbitrary actual values $c_i, f_i, T_{ic}, T_{ip}$
2. Compute actual values $p_i, f_i, c_i, p_p$ by adding noise with $\sigma = \frac{FLE}{\sqrt{3}}$

where FLE values are provided by the manufacturers of OPS.
3. Simulate OPS measurements $c_p, p_p$ by
4. Compute transformations $T_{pc}, T_{ip}$ from "measured" values
5. The actual and measured values pointer tool in CRF are given by $p_c = T_{pc}(p_i), p_c = T_{ip}(p_i)$
6. $(TRE_{ip,pc}(p_c)) = ||p_c - p_c||^2$
7. Loop 1-6 100000 times, and calculate rms value of the acquired TREs.
8. Perform simulation at various locations with respective to CRF

### Experiment

- **Goal**
  - Experimentally measure $(TRE_{ip,pc}(p_c))$ for a set of instrument and CRF fiducial configurations
- **Setup**
  1. OPS, CRF, surgical instrument
  2. Drilled 4x4 grid of divots (0.025 mm accuracy)
  3. Divot grid coplanar, rigidly attached to CRF
  4. Pivot calibration of tooltip
- **Process**
  For given fiducial configuration,
  1. Tool tip is placed at each divot
  2. Registration between OPS and divot grid ($T_{dp}$), and its FRE is calculated
  3. Evaluate $TRE_{m} = \sqrt{\frac{N}{N-2}FRE}$

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**Notes:**
- Apr. 17th, 2014. David Lee (dslee@cis.jhu.edu)
- Evaluation and optimization of virtual rigid body
- Designing Optically Tracked Instruments for Image-Guided Surgery
### Experiment

- **Experimental combinations**
  1. Two values of \( \rho \) (tool fiducial configuration)
  2. Two values of \( r \) (CRF fiducial configuration)
  3. Four values of \( d \) (CRF fiducial configuration)
  4. Three types of experiments

  At each divot,
  - A. Tool orientation, divots grid and CRF positions are constant.
  - B. Tool orientation is random, divots grid and CRF positions are constant.
  - C. Tool orientation, divots grid and CRF positions are constant.

- **Five types of OPS(fiducial marker**
  1. Optotrack 3020 / Active (infrared)
  2. Polaris / Active (infrared)
  3. Polaris / Passive (retroreflective)

### Simulation

- Difference with prediction < 0.6%

### Experiment

<table>
<thead>
<tr>
<th>Instrument</th>
<th>CRF</th>
<th>( d ) (mm)</th>
<th>Theory</th>
<th>Exp. A</th>
<th>Exp. B</th>
<th>Exp. C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = 50 ) mm</td>
<td>( r = 28 ) mm</td>
<td>100</td>
<td>0.36</td>
<td>0.22 (38%)</td>
<td>0.35 (35%)</td>
<td>0.22 (39%)</td>
</tr>
<tr>
<td>( B = 25 ) mm</td>
<td>200</td>
<td>0.62</td>
<td>0.23 (63%)</td>
<td>0.37 (40%)</td>
<td>0.58 (69%)</td>
<td></td>
</tr>
<tr>
<td>( \rho = 75 ) mm</td>
<td>300</td>
<td>0.89</td>
<td>0.39 (56%)</td>
<td>0.54 (40%)</td>
<td>1.37 (54%)</td>
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</tr>
<tr>
<td>( 400 )</td>
<td>1.17</td>
<td>0.66 (49%)</td>
<td>1.09 (-7%)</td>
<td>1.03 (-13%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A = 50 ) mm</td>
<td>( r = 28 ) mm</td>
<td>100</td>
<td>0.49</td>
<td>0.25 (48%)</td>
<td>0.28 (42%)</td>
<td>0.51 (5%)</td>
</tr>
<tr>
<td>( B = 25 ) mm</td>
<td>200</td>
<td>0.70</td>
<td>0.34 (51%)</td>
<td>0.56 (-19%)</td>
<td>1.00 (43%)</td>
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</tr>
<tr>
<td>( \rho = 150 ) mm</td>
<td>300</td>
<td>0.95</td>
<td>0.61 (56%)</td>
<td>0.59 (-38%)</td>
<td>1.50 (58%)</td>
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</tr>
<tr>
<td>( 400 )</td>
<td>1.22</td>
<td>0.72 (41%)</td>
<td>0.63 (48%)</td>
<td>1.77 (45%)</td>
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<td></td>
</tr>
<tr>
<td>( A = 50 ) mm</td>
<td>( r = 57 ) mm</td>
<td>100</td>
<td>0.26</td>
<td>0.15 (45%)</td>
<td>0.12 (-56%)</td>
<td>0.39 (47%)</td>
</tr>
<tr>
<td>( B = 25 ) mm</td>
<td>200</td>
<td>0.36</td>
<td>0.14 (62%)</td>
<td>0.25 (59%)</td>
<td>0.28 (45%)</td>
<td></td>
</tr>
<tr>
<td>( \rho = 75 ) mm</td>
<td>300</td>
<td>0.49</td>
<td>0.19 (-66%)</td>
<td>0.51 (6%)</td>
<td>0.53 (10%)</td>
<td></td>
</tr>
<tr>
<td>( 400 )</td>
<td>0.62</td>
<td>0.43 (-30%)</td>
<td>0.49 (-21%)</td>
<td>0.78 (27%)</td>
<td></td>
<td></td>
</tr>
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<td>100</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Considerable variability, but overall matches well
Relevance

- Theoretical support for virtual rigid body (VRB)'s usefulness
  Compared to conventional rigid bodies,
  - VRB can be projected farther from each other
  - VRB are projected closer to the tooltip

- Analysis and design considerations
  - Originally, primary interest was in FRE.
  - FLE and TRE should be kept in mind.
  - FLE: Detection of physical vs. projected light checkerboard
  - TRE: TRE, but not FRE, depends on fiducial configuration

\[
\langle FLE^2 \rangle = 3\sigma^2
\]
\[
\langle FRE^2 \rangle = \frac{N-2}{N} \langle FLE^2 \rangle \quad \text{(Sibson, R., 1979)}
\]
\[
\langle TRE^2(r) \rangle = \frac{\langle FLE^2 \rangle}{N} \left(1 + \frac{1}{3} \sum_{k=1}^{3} \frac{d_k^2}{r_k} \right) \quad \text{(Fitzpatrick, J., 2001)}
\]

Picture of laparoscope taken from (Sánchez-Margallo, 2013)

Assessment

- Great summary of error in optical tracking
  - Classification
  - High-level theory, example applications, experimental evaluations

- Difficulty in following the experiment
- Qualitative analysis of experimental data regarding TRE and fiducial configurations

- Suggestions for further works
  - More test cases of instrument fiducial configurations
    - Combinations of A and B
    - Slanted distribution
  - Details and verification of rotational tooltip error
    \[
    \Delta \theta_k = \frac{FLE}{\sqrt{AN/k}}
    \]
  - Fully passive fiducial markers (MicronTracker)
Further readings

- Statistical details

- Experimental evaluations

Questions?