

Intraoperative Fiducial Tracking in TORS

CIS II Project #15

Paper Seminar Presentation

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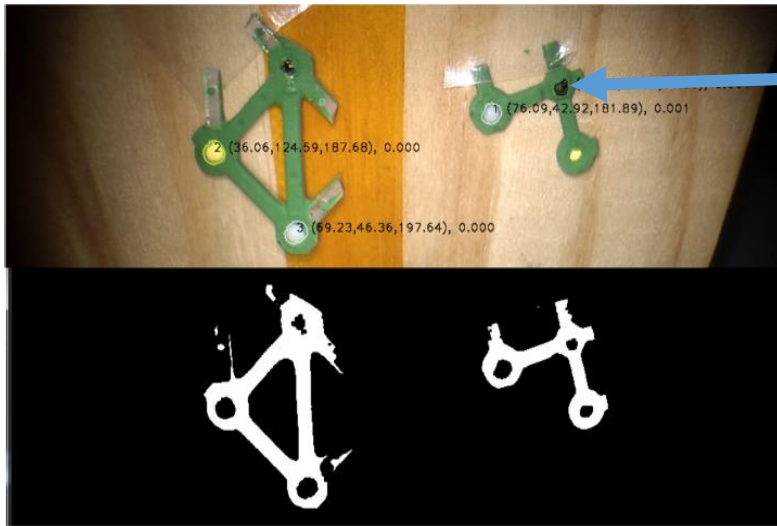
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Project Overview

- Project Goal:

The goal of this project is to design and implement an intraoperative fiducial tracking algorithm in TORS that can accurately track the fiducial under the endoscope.

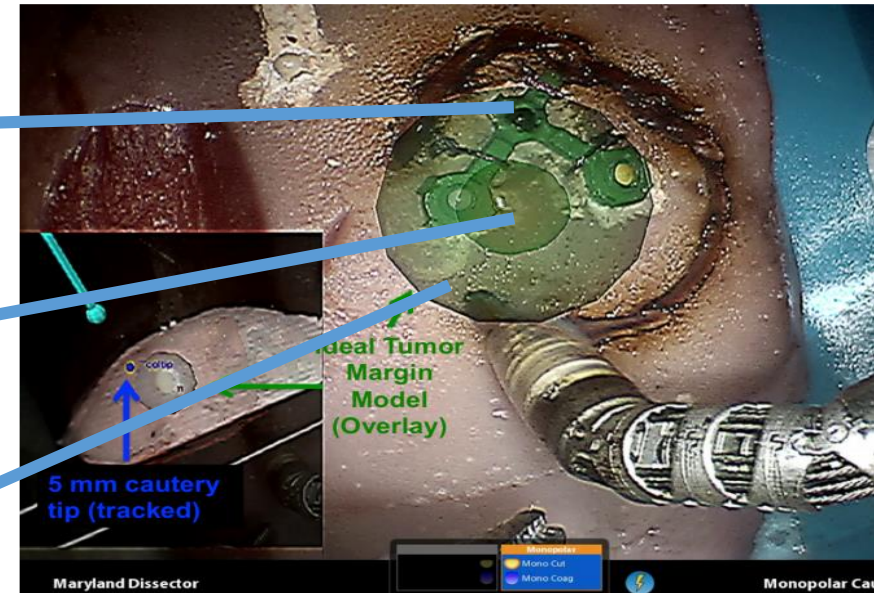


Images courtesy of Wen P. liu

Fiducial

Ideal virtual tumor estimated according to the position of the fiducial

Ideal virtual tumor margin



Project Overview (Continue)

- **Technical Summary**

Algorithms:

- Use both the color information and the edge detector to detect the frame
- Use the adjacency and color information to detect the fiducials from the frame
- For fiducial detection within the video and realtime camera, apply Kalman filter



Project Overview (Continue)

- **Technical Summary**

- Use both the color information and the edge detector to detect the frame



Problem: some noise → frame region hard to set → more difficult in fiducial detection

Paper Selected

- Tony F. Chan and Luminita A. Vese, “Active Contours Without Edges”, IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 10, NO. 2, FEBRUARY 2001
- Cited times: approximately 6178
- Provides a basic model of active contour method
- There have been a lot of adaptive models based on this one

Paper: Active Contours Without Edges

- Summary

- Proposed a new model for active contours to detect objects in a given image



its significance

- Can detect objects whose boundaries are not necessarily defined by gradient, using a particular segmentation of the image instead of the gradient,
Initial curve can be anywhere in the image

- Based on techniques of curve evolution, Mumford–Shah functional for segmentation and level sets



some background knowledge

Paper: Active Contours Without Edges

- Background Introduction: Active Contour Model (Snakes)
 - The basic idea is to evolve a curve, subject to constraints from a given image, in order to detect objects in that image
 - In the classical snakes and active contour models, an edge-detector is used, depending on the gradient of the image, to stop the evolving curve on the boundary of the desired object

Paper: Active Contours Without Edges

- Background Introduction: Active Contour Model (Snakes)

- Some notations:

Ω : a bounded open subset of R^2 ,

$\partial\Omega$: the boundary of Ω ,

u_0 : $\bar{\Omega} \rightarrow R$: a given image,

$C(s)$: $[0,1] \rightarrow R^2$: a parameterized curve, which would evolve to the contour

- Snake Model: $\inf_C J_1(C)$

$$J_1(C) = \alpha \int_0^1 |C'(s)|^2 ds + \beta \int_0^1 |C''(s)| ds$$

$$- \lambda \int_0^1 |\nabla u_0(C(s))|^2 ds.$$

Attracts the contour
toward the object

Control the smoothness of
the contour

Paper: Active Contours Without Edges

- Background Introduction: Active Contour Model (Snakes)

- A general edge detector depending on the gradient of the image:
is a positive and decreasing function g of the gradient, that $\lim_{z \rightarrow \infty} g(z) = 0$

an example:

$$g(|\nabla u_0(x, y)|) = \frac{1}{1 + |\nabla G_\sigma(x, y) * u_0(x, y)|^p}, \quad p \geq 1 \quad \text{it equals 0 at edges}$$

 Gaussian filter for smoothing

Paper: Active Contours Without Edges

- Background Introduction: Mumford–Shah functional

$$\begin{aligned} F^{\text{MS}}(u, C) = & \mu \cdot \text{Length}(C) \\ & + \lambda \int_{\Omega} |u_0(x, y) - u(x, y)|^2 dx dy \\ & + \int_{\Omega \setminus C} |\nabla u(x, y)|^2 dx dy \end{aligned}$$

μ, λ : positive parameters

Recall: Ω : a bounded open subset of R^2

u_0 : $\bar{\Omega} \rightarrow R$: a given image

C : a parameterized curve

What need to be got and which minimizes $F^{\text{MS}}(u, C)$

It is formed by smooth regions and with sharp boundaries, denoted here by C

Paper: Active Contours Without Edges

- Background Introduction: level sets
 - the level set method and in particular the motion by mean curvature have been used extensively in problems of curve evolution
 - The curve C is represented implicitly via a Lipschitz function ϕ , by $C = \{(x, y) | \phi(x, y) = 0\}$, and the evolution of the curve is given by the zero-level curve at time t of the function $\phi(t, x, y)$.

Paper: Active Contours Without Edges

- Background Introduction: level sets

- Evolving the curve C in normal direction with speed F amounts to solve the differential equation:

$$\frac{\partial \phi}{\partial t} = |\nabla \phi| F, \quad \phi(0, x, y) = \phi_0(x, y)$$

initial contour, when $t=0$

- When $F = \text{div}(\nabla \phi(x, y) / |\nabla \phi(x, y)|)$, which is the curvature of the 0 level-curve of ϕ passing through (x, y) , so:

$$\begin{cases} \frac{\partial \phi}{\partial t} = |\nabla \phi| \text{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right), & t \in (0, \infty), x \in \mathbb{R}^2 \\ \phi(0, x, y) = \phi_0(x, y), & x \in \mathbb{R}^2. \end{cases}$$

Paper: Active Contours Without Edges

- Background Introduction: level sets
 - A geometric active contour model based on the mean curvature motion is given by:

g : edge-function with $p=2$

ν : constant, which may be interpreted as a force pushing the curve toward the object, when the curvature becomes null or negative

$$\begin{cases} \frac{\partial \phi}{\partial t} = g(|\nabla u_0|) |\nabla \phi| \left(\operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) + \nu \right), \\ \text{in } (0, \infty) \times \mathbb{R}^2 \\ \phi(0, x, y) = \phi_0(x, y) \text{ in } \mathbb{R}^2 \end{cases}$$

Paper: Active Contours Without Edges

- The significance and reason of this new model:
 - The classical snakes and active contour models rely on the edge-function g , depending on the image gradient to stop the curve evolution, these models can detect only objects with edges defined by gradient
 - In practice, the discrete gradients are bounded and then the stopping function g is never zero on the edges, and the curve may pass through the boundary
 - If the image is very noisy, then the isotropic smoothing Gaussian has to be strong, which will smooth the edges too

Paper: Active Contours Without Edges

- The significance and reason of this new model:
 - The model proposed in this paper is not based on the gradient of the image for the stopping process. The stopping term is instead based on Mumford–Shah segmentation techniques
 - This model can detect contours both with or without gradient, for instance objects with very smooth boundaries or even with discontinuous boundaries
 - In addition, this model has a level set formulation, interior contours are automatically detected, and the initial curve can be anywhere in the image

Paper: Active Contours Without Edges

- The Model

- Some more notations:

ω : an open subset of Ω

$C = \partial\omega$: a curve

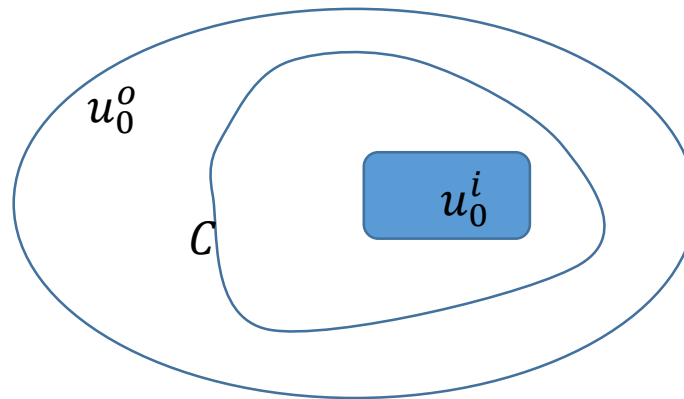
inside(C): the region of ω

outside(C): the region of $\Omega \setminus \omega$

Paper: Active Contours Without Edges

- The Model

- It's also to do minimization of an energy based-segmentation
- Assume that the image is formed by two regions of approximately piecewise-constant intensities, of distinct values u_0^i and u_0^o , and the object to be detected is represented by the region with the value u_0^i



Paper: Active Contours Without Edges

- The Model

$$F(c_1, c_2, C) = \underbrace{\mu \cdot \text{Length}(C) + \nu \cdot \text{Area}(\text{inside}(C))}_{\text{Regularizing term}} + \lambda_1 \int_{\text{inside}(C)} |u_0(x, y) - c_1|^2 dx dy + \lambda_2 \int_{\text{outside}(C)} |u_0(x, y) - c_2|^2 dx dy,$$

always $\lambda_1 = \lambda_2 = 1$, $\nu = 0$

c_1 and c_2 are the constants, depending on C , are the averages of u_0 inside C and respectively outside C

Paper: Active Contours Without Edges

- The Model

$$\begin{aligned} F(c_1, c_2, C) = & \mu \cdot \text{Length}(C) + \nu \cdot \text{Area}(\text{inside}(C)) \\ & + \lambda_1 \int_{\text{inside}(C)} |u_0(x, y) - c_1|^2 dx dy \\ & + \lambda_2 \int_{\text{outside}(C)} |u_0(x, y) - c_2|^2 dx dy, \end{aligned}$$

- For the model, it is to find C, c_1, c_2 that minimize F :

$$\inf_{c_1, c_2, C} F(c_1, c_2, C)$$

c_1 and c_2 are depending on C

Paper: Active Contours Without Edges

- Relates the Model to Mumford–Shah functional

$$F^{MS}(u, C) = \mu \cdot \text{Length}(C) + \lambda \int_{\Omega} |u_0(x, y) - u(x, y)|^2 dx dy + \int_{\Omega \setminus C} |\nabla u(x, y)|^2 dx dy$$

$$F(c_1, c_2, C) = \mu \cdot \text{Length}(C) + \nu \cdot \text{Area}(\text{inside}(C)) + \lambda_1 \int_{\text{inside}(C)} |u_0(x, y) - c_1|^2 dx dy + \lambda_2 \int_{\text{outside}(C)} |u_0(x, y) - c_2|^2 dx dy,$$

- minimal partition problem:

Restrict F^{MS} to piecewise constant functions u , i.e. $u = \text{constant } c_i$ on each connected component of $\Omega \setminus C \longrightarrow c_i = \text{average}(u_0)$ on each connected component

Paper: Active Contours Without Edges

- Relates the Model to Mumford–Shah functional

$$F^{MS}(u, C) = \mu \cdot \text{Length}(C) + \lambda \int_{\Omega} |u_0(x, y) - u(x, y)|^2 dx dy + \int_{\Omega \setminus C} |\nabla u(x, y)|^2 dx dy$$

$$F(c_1, c_2, C) = \mu \cdot \text{Length}(C) + \nu \cdot \text{Area}(\text{inside}(C)) + \lambda_1 \int_{\text{inside}(C)} |u_0(x, y) - c_1|^2 dx dy + \lambda_2 \int_{\text{outside}(C)} |u_0(x, y) - c_2|^2 dx dy,$$

- minimal partition problem:

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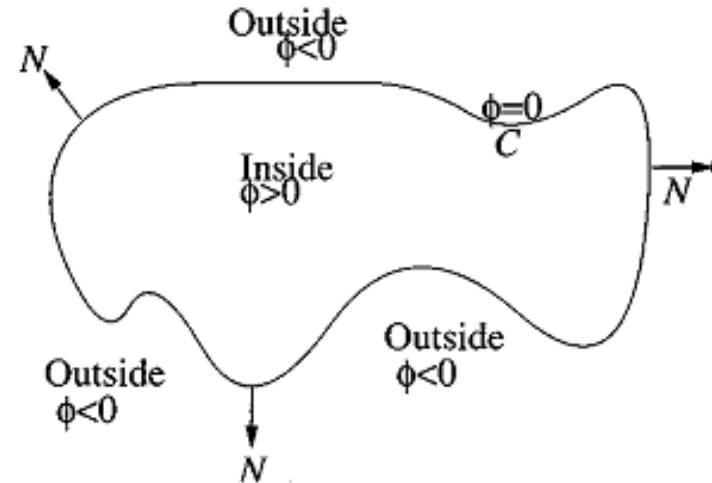
- The model proposed in the paper is a particular case of the minimal partition problem, which looks for the best approximation u of u_0 , as a function taking only two values

$$u = \begin{cases} c_1, & \text{when inside } C \\ c_2, & \text{when outside } C \end{cases}$$

Paper: Active Contours Without Edges

- Solve the Model using the level set method
 - In the level set method, C is represented by the zero level set of a Lipschitz function ϕ , such that

$$\begin{cases} C = \partial\omega = \{(x, y) \in \Omega: \phi(x, y) = 0\}, \\ \text{inside}(C) = \omega = \{(x, y) \in \Omega: \phi(x, y) > 0\} \\ \text{outside}(C) = \Omega \setminus \bar{\omega} = \{(x, y) \in \Omega: \phi(x, y) < 0\} \end{cases}$$



“Active Contours Without Edges”

Paper: Active Contours Without Edges

- Solve the Model using the level set method
 - Using the Heaviside function,

$$H(z) = \begin{cases} 1, & \text{if } z \geq 0 \\ 0, & \text{if } z < 0, \end{cases} \quad \delta_0(z) = \frac{d}{dz} H(z)$$

- the terms in the energy can be presented as:

$$\begin{aligned} \text{Length}\{\phi = 0\} &= \int_{\Omega} |\nabla H(\phi(x, y))| dx dy \\ &= \int_{\Omega} \delta_0(\phi(x, y)) |\nabla \phi(x, y)| dx dy, \end{aligned}$$

$$\text{Area}\{\phi \geq 0\} = \int_{\Omega} H(\phi(x, y)) dx dy,$$

$$\begin{aligned} \int_{\phi > 0} |u_0(x, y) - c_1|^2 dx dy \\ = \int_{\Omega} |u_0(x, y) - c_1|^2 H(\phi(x, y)) dx dy, \end{aligned}$$

$$\begin{aligned} \int_{\phi < 0} |u_0(x, y) - c_2|^2 dx dy \\ = \int_{\Omega} |u_0(x, y) - c_2|^2 (1 - H(\phi(x, y))) dx dy \end{aligned}$$

Paper: Active Contours Without Edges

- Solve the Model using the level set method

So,

$$\begin{aligned} F(c_1, c_2, \phi) &= \mu \int_{\Omega} \delta(\phi(x, y)) |\nabla \phi(x, y)| dx dy \\ &+ \nu \int_{\Omega} H(\phi(x, y)) dx dy \\ &+ \lambda_1 \int_{\Omega} |u_0(x, y) - c_1|^2 H(\phi(x, y)) dx dy \\ &+ \lambda_2 \int_{\Omega} |u_0(x, y) - c_2|^2 (1 - H(\phi(x, y))) dx dy \end{aligned}$$

$$c_1(\phi) = \frac{\int_{\Omega} u_0(x, y) H(\phi(x, y)) dx dy}{\int_{\Omega} H(\phi(x, y)) dx dy}$$

$$c_2(\phi) = \frac{\int_{\Omega} u_0(x, y) (1 - H(\phi(x, y))) dx dy}{\int_{\Omega} (1 - H(\phi(x, y))) dx dy}$$

Paper: Active Contours Without Edges

- Solve the Model using the level set method
 - Use a slightly regularized version of the functions H and δ_0 , denoted by H_ε and δ_ε , $\varepsilon \rightarrow 0$
 - Denote the associated regularized functional by F_ε ,

$$\begin{aligned} F_\varepsilon(c_1, c_2, \phi) &= \mu \int_{\Omega} \delta_\varepsilon(\phi(x, y)) |\nabla \phi(x, y)| dx dy \\ &+ \nu \int_{\Omega} H_\varepsilon(\phi(x, y)) dx dy \\ &+ \lambda_1 \int_{\Omega} |u_0(x, y) - c_1|^2 H_\varepsilon(\phi(x, y)) dx dy \\ &+ \lambda_2 \int_{\Omega} |u_0(x, y) - c_2|^2 (1 - H_\varepsilon(\phi(x, y))) dx dy \end{aligned}$$

- Minimizing F_ε with respect to ϕ ,

Paper: Active Contours Without Edges

- Solve the Model using the level set method
 - Minimizing F_ε with respect to ϕ , get

$$\frac{\partial \phi}{\partial t} = \delta_\varepsilon(\phi) \left[\mu \operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \nu - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2 \right] = 0 \text{ in } (0, \infty) \times \Omega,$$

$$\phi(0, x, y) = \phi_0(x, y) \text{ in } \Omega,$$

$$\frac{\delta_\varepsilon(\phi)}{|\nabla \phi|} \frac{\partial \phi}{\partial \vec{n}} = 0 \text{ on } \partial \Omega$$

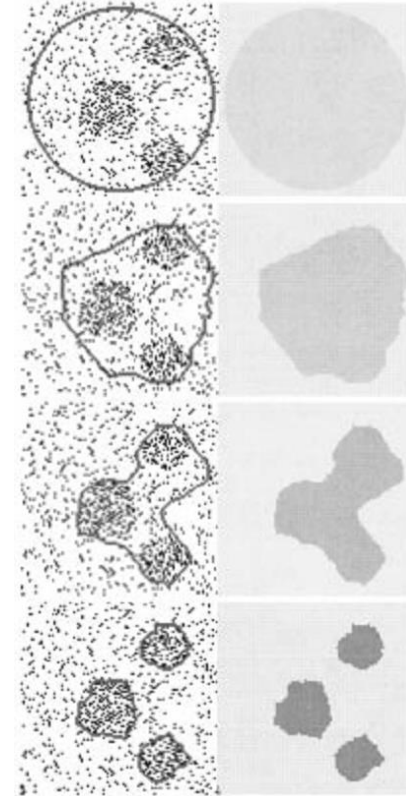
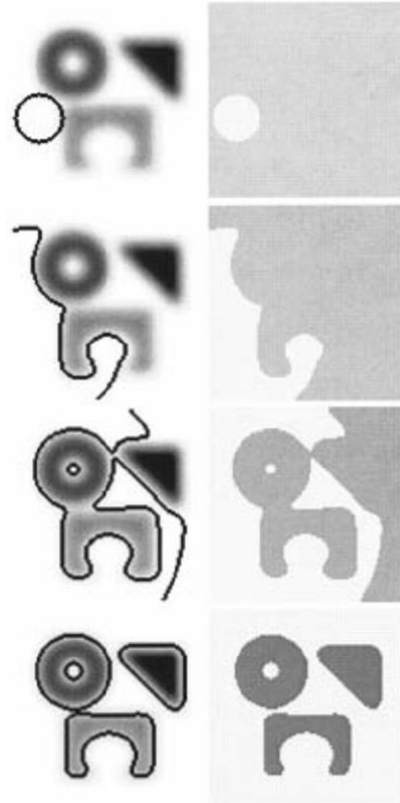
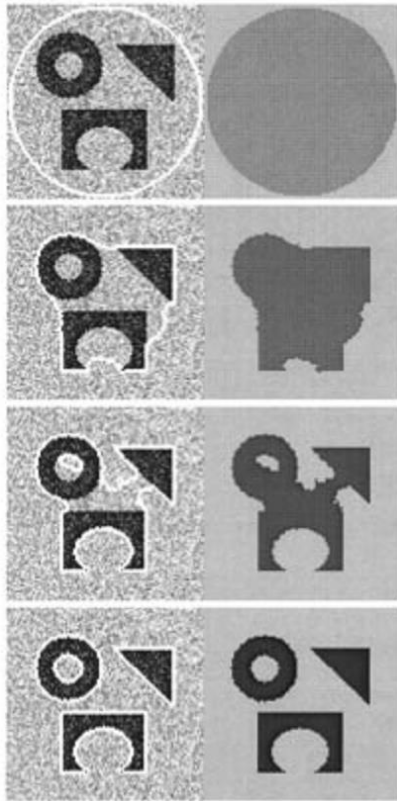
Normal derivative of ϕ at the boundary

Paper: Active Contours Without Edges

- Use the Model for contour detection:
 - Initialize ϕ^0 by ϕ_0
 - Compute $c_1(\phi^n)$ and $c_2(\phi^n)$
 - Solve the PDE in ϕ , to obtain ϕ^{n+1} (last slide equation)
 - Check whether the solution is stationary. If not, $n=n+1$ and repeat

Paper: Active Contours Without Edges

- Some experiments using the model



“Active Contours Without Edges”

Paper: Active Contours Without Edges

- Assessment of the paper

It's a great and very basic paper for object contour detection.

The model it describes can detect different kinds of object contours, and is irrelevant to the initial curve, according to the experimental results showed in the paper, which is of great use.

The paper is well written that even without reading its reference papers, it can still be understood.

Paper: Active Contours Without Edges

- Assessment of the paper
- As to my project, I can use it to detect the green frame
- Giving an initial input curve for the first image of the image stream, and apply the model to detect the frame contour
- Could help get rid of the isolated noisy points, and resulting in a more robust and efficient detection of the fiducials

Thank you

