“A Constrained Optimization Approach to Virtual Fixtures”

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Seminar Presentation
Group 2: Synthetic Tracked Aperture Ultrasound Imaging

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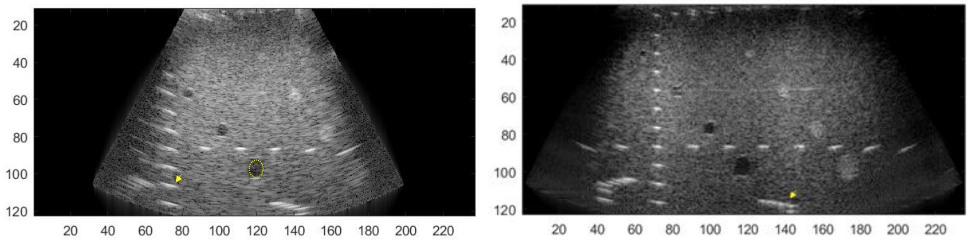
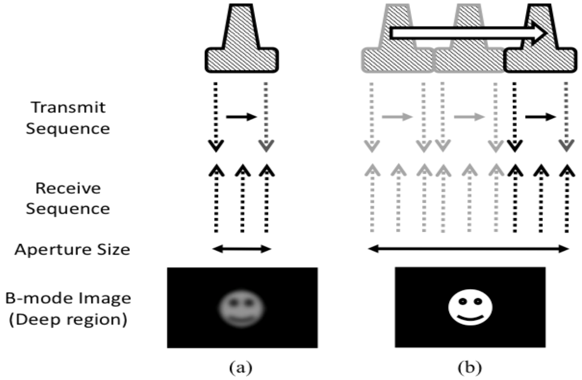
STRATUS Overview

- Aperture size of the ultrasound transducer limits image quality
- Synthetic tracked aperture imaging shows improvement
- Goal: bring system from autopilot to co-robotic freehand using virtual fixtures and force control

Use the UR5 to guide a sonographer to scan a specific trajectory for a higher quality ultrasound image





Paper Selection

M. Li, A. Kapoor, and R. H. Taylor, “A constrained optimization approach to virtual fixtures,” in IROS, 2005, pp. 1408–1413.

- Fundamental to our understanding of VFs
- Desired formulation of geometric constraints
- Ease of implementation in current system
- Access to Dr. Taylor (!!!)

Virtual Fixtures

In general:

- Augment motion commands from the user, thus enhancing precision, stability, and patient safety

In our case:

- Ensure that correct path is scanned
- Ensure that any other area is not scanned
- Limit joint velocities
- Control force applied on patient

How:

- Constrained optimization approach

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Constrained Optimization Approach

$$\underset{\frac{\Delta\vec{q}}{\Delta t}}{\operatorname{argmin}} \left\| W \left(\frac{\Delta\vec{x}}{\Delta t} - \frac{\Delta\overrightarrow{x_d}}{\Delta t} \right) \right\|$$

$$s.t. \quad H \frac{\Delta\vec{x}}{\Delta t} \geq \vec{h}$$

$$\frac{\Delta\vec{x}}{\Delta t} = J \frac{\Delta\vec{q}}{\Delta t}$$

Δx– computed incremental end effector motion

Δx_d– desired incremental end effector motion

Δq– desired incremental joint motion

Δt- small time interval

W- diagonal weighting matrix

H- constraint coefficient matrix

h- constraint vector

J- Jacobian matrix

“Move Along a Line” Constraint

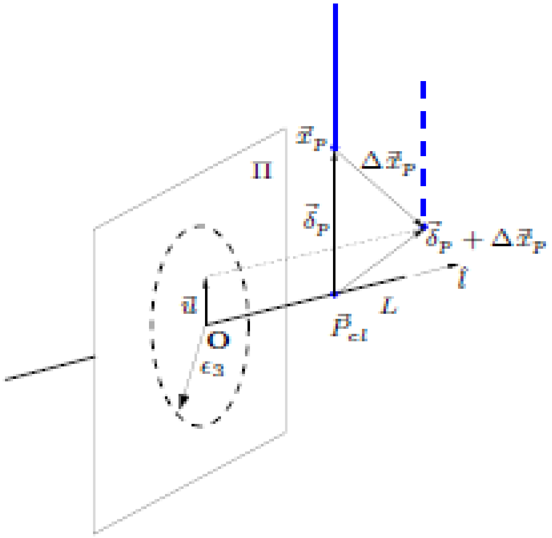
1. Define line $L = \overrightarrow{L_0} + \hat{l} * s$
2. Calculate closest point on line P_{cl}
3. Calculate error $\overrightarrow{\delta_p} = \vec{x}_p - P_{cl}$
4. Project error onto plane perpendicular to L

$$R = [\widehat{v_1} \quad \widehat{v_2} \quad \hat{l}] \quad \widehat{v_1} = \frac{\hat{l} \, x \, \hat{l'}}{\|\hat{l} \, x \, \hat{l'}\|} \quad \widehat{v_2} = \frac{\hat{l} \, x \, \widehat{v_1}}{\|\hat{l} \, x \, \widehat{v_1}\|}$$

5. Require projection to be within error range, approximated by n-dim polygon

$$[(R * [\cos(\alpha_i); \quad \sin(\alpha_i); \quad 0])^T \quad 0 \quad 0 \quad 0] \cdot (\overrightarrow{\delta_p} + \Delta\vec{x}) \leq \varepsilon$$

$$\alpha_i = \frac{2 * \pi * i}{n}$$



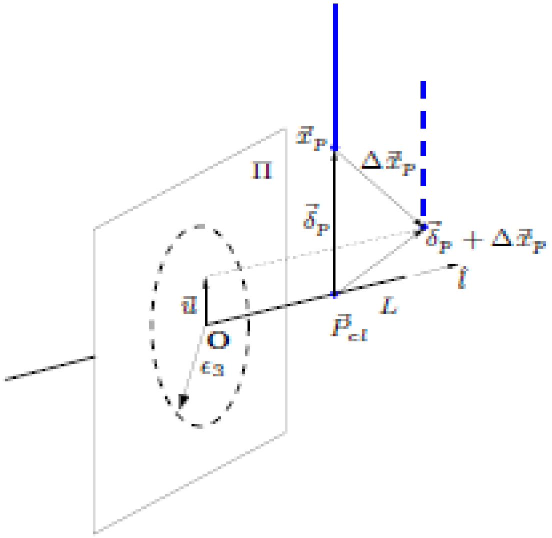
“Move Along a Line” Constraint, cont’d

$$[(R * [\cos(\alpha_i); \quad \sin(\alpha_i); \quad 0])^T \quad 0 \quad 0 \quad 0] \cdot (\vec{\delta_p} + \Delta\vec{x}) \leq \varepsilon$$

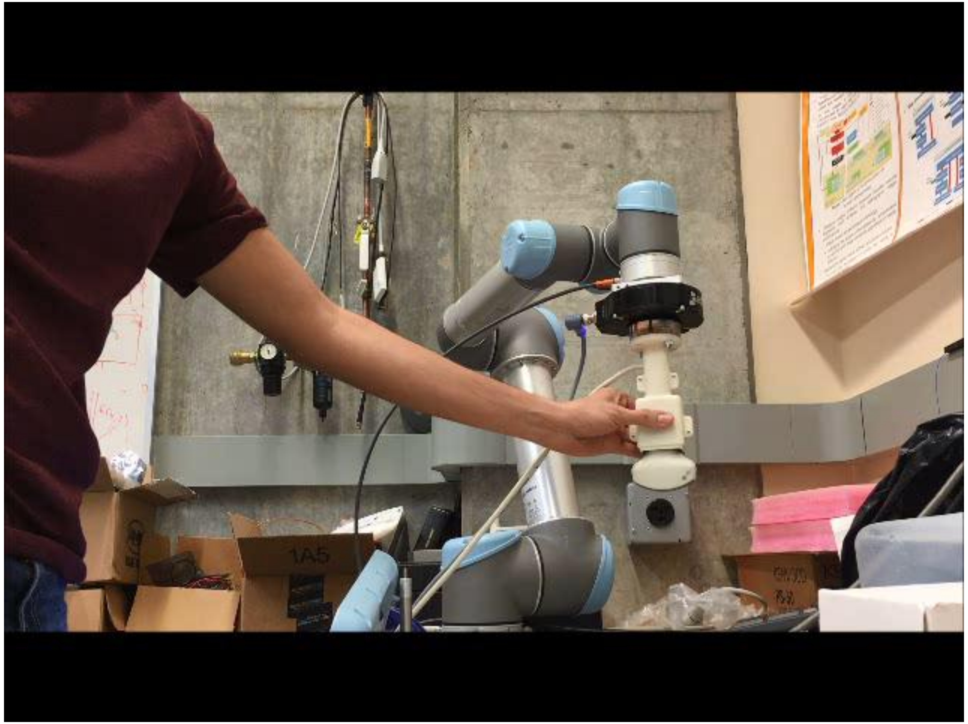
Rewrite in form $H\frac{\Delta\vec{x}}{\Delta t} \geq \vec{h}$

$$H = \begin{bmatrix} (-R * [\cos(\alpha_1); & \sin(\alpha_1); & 0])^T & 0 & 0 & 0 \\ & \vdots & & & & \\ (-R * [\cos(\alpha_n); & \sin(\alpha_n); & 0])^T & 0 & 0 & 0 \end{bmatrix}$$

$$\vec{h} = \begin{bmatrix} \varepsilon \\ \vdots \\ \varepsilon \end{bmatrix} - H * \vec{\delta}.$$

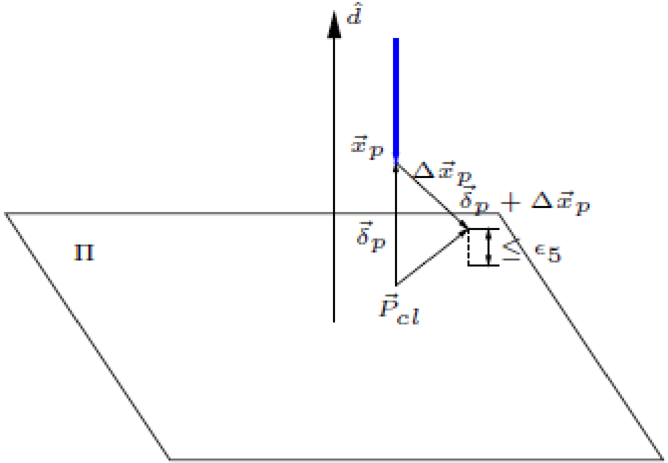


Demonstration of "Move Along a Line"



Plane Related Case

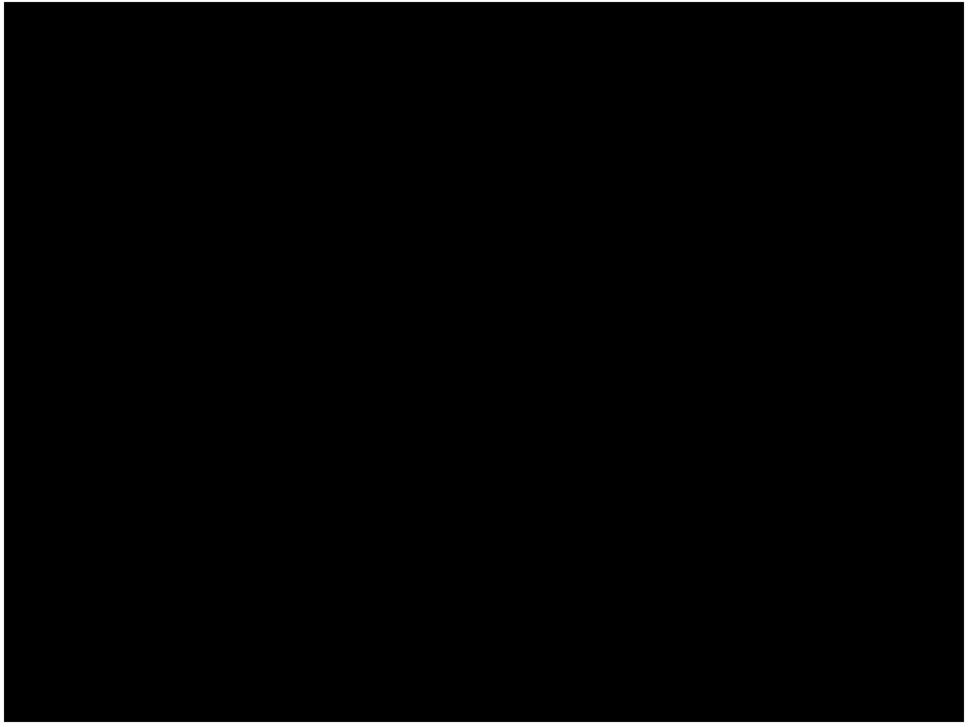
- Extension of "move along a line"
- Multiple applications
  - We restrict movement to within a plane



1. Define normal to plane $\hat{d}^{\,t}$
2. Calculate closest point on plane P_{cl}
3. Calculate error $\overrightarrow{\delta_p} = \vec{x}_p - P_{cl}$
4. Define H and h

$$H = \begin{bmatrix} \hat{d}^{\,t}, 0, 0, 0 \\ -\hat{d}^{\,t}, 0, 0, 0 \end{bmatrix}, \ \vec{h} = \begin{bmatrix} 0 \\ -\epsilon_5 \end{bmatrix} - H\vec{\delta}.$$

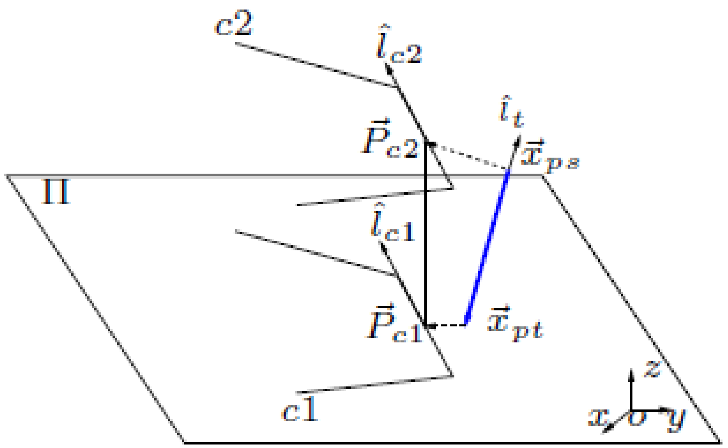
Demonstration of Plane Related Case



Experiment #1

- Follow a curve with a fixed tool orientation with respect to the curve
  - Follow tangent direction of 5^{th} degree b-spline

- "Move along a line" constraints:
  - Tool tip frame
  - Tool shaft frame

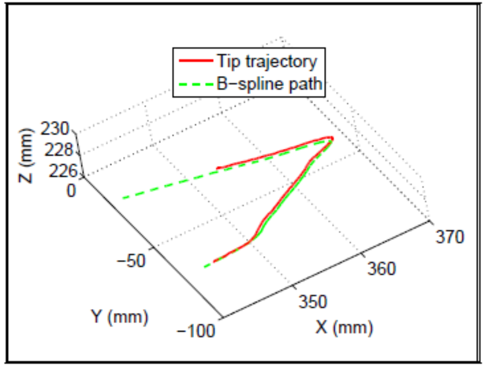


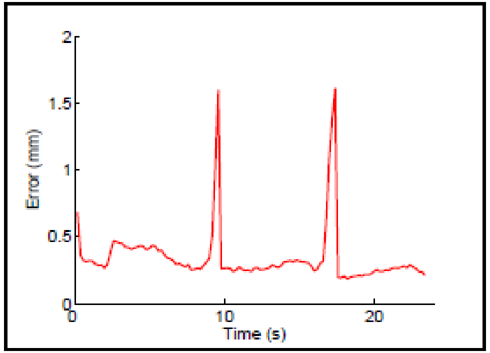
$$\underset{\Delta\vec{q}}{\operatorname{argmin}} \left\| W \left(J\Delta\vec{q} - k\vec{f} \right) \right\|$$

$$s.t. \quad \begin{bmatrix} H_t & 0 \\ 0 & H_s \end{bmatrix} \begin{bmatrix} J_t \\ J_s \end{bmatrix} \Delta\vec{q} \geq \begin{bmatrix} \vec{h_t} \\ \vec{h_s} \end{bmatrix}$$

Experimental Results

- Error: distance from the actual tool tip position to the spline
  - Optical tracker and LEDs
- Average error of 5 trials: 0.32 +- .19mm
- Source of error:
  - sharp turns where the tangent direction changed dramatically
  - communication delays between the optical tracker and the robot





Assessment

Pros:

- Straightforward
- Helpful figures
- Necessary geometric constraints for STRATUS system
- Easy implementation into current STRATUS system
- Versatile

Cons:

- Typos
- Unclear in parts
- Lack of wider range of experimentation
  - Always $n = 8$; $\varepsilon = 0.001$
  - Effects of varying these?
- Weighting matrix never used or explained

Overall: extremely useful, good results, wide ranging applications

Questions?

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