**Critical Review: 3D Forward and Back-Projection for X-Ray CT Using Separable Footprints**

Long, Yong, Jeffrey A. Fessler, and James M. Balter. "3D Forward and Back-Projection for X-Ray CT Using Separable Footprints." *IEEE Transactions on Medical Imaging IEEE Trans. Med. Imaging* 29.11 (2010): 1839-850. Web.

**Project Overview**

 Aim of our project is to develop a surgeon-friendly user-interface (UI) that simulates X-ray projections of a mobile C-arm. The mobile C-arm, contributed by its flexibility in source positioning, is a widely used in orthopedic surgeries or diagnostic imaging. However, fine placement of the C-arm to display a preferred fluoroscopic view during surgical procedures requires multiple fluoroscopic images to be taken and is often time consuming. Our UI aims to reduce this burden in placement of the C-arm by providing simulated fluoroscopic image from source positions defined in simulation.

Our project is consisted of a number of elements including generation of digitally reconstructed radiograph (DRR), defining relative position between C-arm and CT data, and geometric calibration of the C-arm in a virtual space. The paper selected here is particularly related to generation of DRR of preoperative CT data by forward-projection.

**Paper Selection**

The paper selected, titled “*3D Forward and Back-Projection for X-Ray CT Using Separable Footprints*”, proposes a novel way to approximate voxel footprint functions as 2D separable functions, thereby reducing computational burden of integration over detector cells and enhancing accuracy compared to other methods to approximate the footprint function. For our purpose to provide DRRs to help surgeons to find their preferred fluoroscopic view, for smooth integration with surgical workflows, DRRs should be ideally generated in real time. Therefore a simple trilinear interpolation method is used to get a forward projection. However, on instances where a surgeon found his/her preferred view and wishes to see higher-quality DRR of the view, forward-projection algorithm discussed in this paper could be implemented to generate the DRR, with higher accuracy and longer computation time.

**Problem Statement**

 Iterative methods for 3D image reconstruction in CT has a number of advantages over conventional methods like filtered back-projections. The method provides images with better quality and reduced dose. However, the iterative method is computationally more expensive than conventional methods because of its iterative forward and backprojection processes, especially in 3D image reconstructions. There were many approaches to reduce this complexity, including approaches to enhance speed of integration of footprint functions. This paper proposes approximation of voxel footprint function as 2D separable functions. The paper explains background information, technical explanation of the methods with reasoning behind, and experiments to evaluate the methods. Rest of this review paper will be covered to briefly explain backgrounds of cone-beam (CB) geometry and projection, the methods for 2D separable footprints (SF), and comparison between SF methods to other methods such as distance-driven (DD) projector approach.

**Backgrounds: Cone-Beam (CB) geometry and projection**



Figure 1. Geometry of cone-beam with flat detector (left), and diagram of azimuthal and polar angle $φ$ and $θ$ of ray (right)

The paper is written based on a CB system with a flat detector. This section of the paper provides necessary technical backgrounds in CB geometries and projections. Not all technical explanation on the paper is listed in this section.

Figure 1 explains parameters used to explain geometries of the CB system. In the system, $\vec{p}\_{0}$ denotes a source position, $\vec{p}\_{1}$ is a projection point on the 2D detector, $\vec{e}$ is the direction vector of a ray from $\vec{p}\_{0}$ to $\vec{p}\_{1}$. β is the angle of the source position counterclockwise from y axis. (s, t) denotes the detector coordinates. *Ds0* and *D0d* is distance from source to rotation center and distance from rotation center to detector each.

Discretized object represented by superimposed basis function $β\_{0}$ on $N\_{1} × N\_{2} × N\_{3}$ Cartesian grid is:

$$f\left(\vec{x}\right)=\sum\_{\vec{n}}^{}f\left[\vec{n}\right]β\_{0}(\left(\vec{x}-\vec{c}\left[\vec{n}\right]\right)./\vec{∆})$$

where sum is for all the lattices $\vec{n}=(n\_{1}, n\_{2}, n\_{3})$. $\vec{c}\left[\vec{n}\right]=(c\_{1}\left[\vec{n}\right],c\_{2}\left[\vec{n}\right],c\_{3}\left[\vec{n}\right])$ is a center of $\vec{n}$th basis function. $β\_{0}(\vec{x})$ is a common basis function, which is superimposed on the Cartesian grid. $\vec{∆}$ denotes grid spacing and operator ‘./’ represent elementwise division.

Then, projection *yβ*[s*k*, t*l*] of 3D discretized object $f\left[\vec{n}\right]$ is defined as:

$$y\_{β}\left[s\_{k},t\_{l}\right]=\sum\_{\vec{n}}^{}a\_{β}\left[s\_{k},t\_{l};\vec{n}\right]f(\vec{n})$$

Where $a\_{β}\left[s\_{k},t\_{l};\vec{n}\right]=F(s\_{k},t\_{l} β,\vec{n})$ denotes a blurred footprint function. This blurred footprint can be decomposed into system blur function *h* and CB footprint *q* of the basis function, as:



Where basis footprint function $q(s\_{k},t\_{l} β,\vec{n})$, which is CB footprint of basis function $β\_{0}$, is defined as



The paper chooses to use cubic voxels, so footprints of a cubic voxel is defined as multiples of three *rect* functions in x, y, z directions. Exact calculation of the footprint function is described in the paper yet it is computationally expensive.



Figure 2. True footprint function (1st row) of voxels centered at origin (left column), [100, 150, 15] (mid column), and [93, 93, 93] (right column) with s and t profiles (2nd, 3rd row)

**Approximation methods**

1. Separable footprints (SF)

It is computationally expensive to compute the exact footprint/blurred footprint functions during forward or backprojections. So the author of the paper approximated the system footprint function from a simulation result of CB geometry. “True” footprint function was computed for voxels centered at origin, [100, 150, 15], and [93, 93, 93], which is provided in fig. 2. Shapes of the true footprint functions suggest that the functions are 2D separable in s and t direction. The paper proposed two ways of separating the footprint functions. One is to separate it to a trapezoid function, in transaxial direction, and to a rectangular function, in axial direction. A method with this approximation is named SF-TR. The second method is to approximate separable footprints as trapezoid functions in both axial and transaxial direction, named SF-TT. The primary motivation for using the two ways of approximation was difference of footprint profile based on axial angle. The right side of the figure 2 display a voxel centered at (93, 93, 93), where polar and azimuthal angles are 11.7 and 11.5. With higher axial angle, profile of the footprint function in t is more trapezoidal than rectangular. The second approximation method was developed to accurately approximate these types of footprint functions.

2. Amplitude functions

 The paper defines approximation of a footprint of a cubic basis as a multiple of amplitude function and the SF. The paper proposed three different ways to compute the amplitude function, each named A1, A2, and A3. While the amplitude function could be separated in terms of two different ray angles $l=l\_{φ}l\_{θ}$, A3 uses voxel-dependent approach and computes amplitude function of each voxel for each projection angles and thus is most computationally complex. A2 method is similar but computes amplitude of each detector cell position (s,t) for $l\_{θ}$. Amplitude based on azimuthal angle is still computed in voxel-based way. A1 method is completely ray-dependent, and computes both $l\_{φ} and l\_{θ}$ of each detector cell based on projection angle of a ray for each detector cell. The paper stated A1 and A2 method have similar speed with similar accuracies, where A3 method is slower yet more accurate.

**Experimental Setup and Results**



Figure 3. Maximum error comparison between DD, SF-TR, and SF-TT forward projection methods with a voxel centered at origin and a voxel

The study evaluated the SF methods by comparing results from both SF-TR and SF-TT methods with different amplitude methods with results of DD methods. They compared accuracy and computation time of forward/back projection and quality of reconstruction image of each method.

The maximum error of each method was computed by subtracting approximated blurred footprint function from exact blurred footprint function, which is generated by linearly averaging 1000 x 1000 analytical line integrals of rays sampled over each detector cell [1]. In this comparison, combination of SF-TT method with A3 amplitude method is much slower than others with not much significant increase of accuracy and thus is not listed. Footprint functions of a 1x1x1 voxel at origin or a voxel at
(100, 150, -100) were compared (Fig. 3). For a voxel centered at origin, maximum errors from SF methods were lower than that of DD method and the errors of A1 amplitude method was slightly higher than the errors of A2 and A3 method, by 3.4x10-4. As rectangular and trapezoidal approximation results similar profile for a voxel centered at origin, error curves generated by both SF methods with same amplitude method overlap. However, in case of a voxel centered at (100, 150, -100), error curves of SF\_TT method showed the lowest maximal error. As the voxel has a larger axial angle, the trapezoidal model more accurately approximates the footprint function.



Table 1. Comparison of computation time for each method in forward and back projection

Table 1 compares speed of each method in forward and backprojections. Forward and backprojections of 512x512x128 volume with sampling spacing 0.5mm in x, y, z direction was used. Elapsed time was measured as an average of 5 projector runs on 8-core Sun Fire X2270 server. SF-TR method is as fast as DD method, whereas SF-TR method is about 2 times slower than DD or SF-TR method. Comparison between SF-TR-A1, A2 methods and SF-TR-A3 method also shows that A3 method is about 50% slower than the other two methods.

 Another experiment was performed to evaluate quality of reconstructed images. The study compared full FOV images of 3D Shepp-Logan digital phantom reconstructed by SF-TR-A1 method and DD method. Figures of the FOV images are not provided in this review as differences are hard to see when printed. The FOV images are reconstructed with coarse resolution of 0.98 × 0.98 × 0.63 mm3 with sampling rate of 256 x 256 x 64. Maximum/Root mean square (RMS) errors of the two images were similar, but one key difference was artifact introduced at top and bottom of DD FOV image. The paper proposed poor sampling of off-axis slices introduced cone-beam artifact. Also, ROI (50 x 50 x 20) images with higher resolution (0.24×0.24×0.31 mm3) are provided for both methods.



Figure 4. Reconstructed ROI Image (top) and errors (bottom) for SF-TR (left column) and DD (right column) method.

Comparison of reconstructed ROI images from SF-TR method and DD method shows better image quality of SF-TR method. The paper stated that rectangle approximation in transaxial direction of DD method could have resulted the artifacts on ROI DD images. They also stated when full FOV images are reconstructed with finer resolution, the aliasing effect is removed, mainly because difference of rectangular and transaxial approximation decreases with a higher resolution.

Two SF methods are not compared to each other, while trapezoidal approximation in SF-TT method is expected to provide the better approximation especially for voxel with larger axial angles. The two method did not show obvious differences in visual or in maximum or RMS errors. The paper states CB artifacts introduced by poor sampling dominates other errors, such as error from rectangular approximation of footprint in axial direction for voxel with high axial angles.

**Discussion**

The results proposed by the paper is significant as it resolves complexity associated with computational burden in exact calculation of the footprint functions in accurate, simple ways. The method presented in this paper also has a wide possibility of implementation, such as implementation of this method with a system with spherically symmetric basis function.

Although they are not all included in this review, the paper contains a good amount of background information for an individual unfamiliar with CB geometry and projection could understand the proposed method. Reasoning and numerical basis the proposed approaches are well explained, leave almost no room for ambiguity. Method of validation of their approach is well illustrated as well, with detailed depiction of materials used for the validation. For instance, although not presented in this review for simplicity, the study mentions details of their system setting such as types of processor used, implemented routines, dimensions of FOV and ROI, and specification of the digital phantom they used. Also, the study well explains how the ‘ground truth’ data were established for each evaluation method. Figures and tables are well explained and all have meanings. The study also addresses possible future works to be done, such as approximation of footprint function for spherically symmetric basis function or implementation of GPU programming techniques. One possible improvement that could be done is assessing qualities of images reconstruction with higher resolution to test difference of SF-TR and SF-TT images. Using digital phantom with higher resolution and increasing sampling rate might have introduced visual differences between images generated by the two methods.

**Conclusion**

The forward and back projection methods for X-ray CT in the paper show good accuracy and speed. Although the computation time of forward projection is too long for the purpose of our project to provide DRRs ideally in real time, accuracy of the method makes it worth to be implemented in the UI we are developing as a future work. Other paper with acceleration of the SF method with GPU reports reduction of computation time to 20.9s for a single-turn (360**°**) with 984 views for 3D object of size 512 x 512 x 640, which makes implementation of the SF approach in our project more plausible. If the method could be applied to our module, it would provide additional option for a surgeon to observe a particular fluoroscopic view with a higher accuracy, in sacrifice of computation time.

**References**

[1] Long, Yong, Jeffrey A. Fessler, and James M. Balter. "3D Forward and Back-Projection for X-Ray CT Using Separable Footprints." *IEEE Transactions on Medical Imaging IEEE Trans. Med. Imaging* 29.11 (2010): 1839-850. Web.

[2] M. Wu and J. A. Fessler. “GPU acceleration of 3D forward and backward projection using separable footprints for X-ray CT image reconstruction”. *Proc. Intl. Mtg. on Fully 3D Image Recon. in Rad. and Nuc. Med*, pages 56–9, 2011