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Group 8 Seminar Presentation: Critical Review

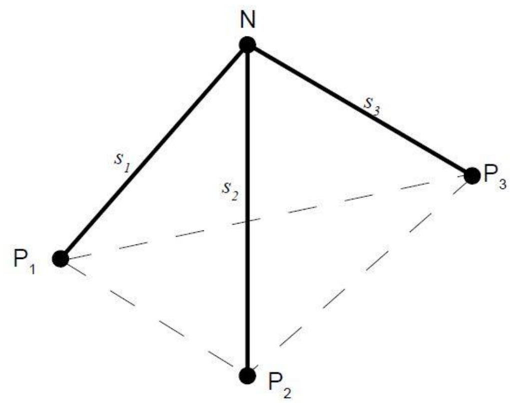
4/26/16

The article chosen for this review is “An Algebraic Solution to the Multilateration Problem” by Abdelmoumen Norrdine [1] presenting a solving approach for nonlinear multilateration problems. This article is chosen because the multilateration is one of the method using in Project 8, iPass: Photoacoustic Catheter Tracking. Project 8 aims to track a catheter using a stereo camera by applying laser spots on the surface. The reason for multilateration is to acquire the location of unknown point or the location of catheter tip in this project. Multilateration is the method of calculating locations of points by measurement of distances, and applying the theory of geometry of circles and spheres. The author presents a mathematic formulation and their implementation for sample tasks, and provides experimental results as well. The algorithm is useful to solve the multilateration problem and possible to implement in our current work for solving the nonlinear problems.

In general, multilateration is a navigation technique based on the measurement of the difference in distance to two stations at known locations. This measurement of the difference in distance between two stations normally turns out the results of infinite number of locations that satisfy this measurement. To locate the exact location, multiple measurements are required to produce more possible locations and then considered the intersection of those possible locations to be the solution. It is also noted that multilateration is a common technique in radio navigation systems.

In our case, this paper is relevant to formulate a nonlinear multilateration problem for use in iPASS. Since it is already known that with multiple Photoacoustic (PA) spots, we are dealing with a quadratic system that is able to be solved with at least three non-collinear PA spots. Furthermore, when there are multiple PA spots in the system, a minimization algorithm is required since there are no closed-form solution.

The author starts describing their method by giving the trilateration problem example as follows:



Given three reference points $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, $P_3(x_3, y_3, z_3)$ and the range measurements s_1, s_2, s_3. The determination of the coordinates (x, y, z) of the point N equals to finding the solutions to these quadratic equations

$$(x-x_1)^2+(y-y_1)^2+(z-z_1)^2=s_1^2$$
$$(x-x_2)^2+(y-y_2)^2+(z-z_2)^2=s_2^2$$
$$(x-x_3)^2+(y-y_3)^2+(z-z_3)^2=s_3^2$$

These equations can be rearranged in matrix representation

$$\begin{bmatrix} 1 & -2x_1 & -2y_1 & -2z_1 \\ 1 & -2x_2 & -2y_2 & -2z_2 \\ 1 & -2x_3 & -2y_3 & -2z_3 \end{bmatrix} \begin{bmatrix} x^2+y^2+z^2 \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} s_1^2-x_1^2-y_1^2-z_1^2 \\ s_2^2-x_2^2-y_2^2-z_2^2 \\ s_3^2-x_3^2-y_3^2-z_3^2 \end{bmatrix}$$

This can be also represented in the form

$$A\boldsymbol{x}=\boldsymbol{b}$$

with the constraint: $\boldsymbol{x} \in E$

where $E=\left\{\left(x_0, x_1, x_2, x_3\right)^T \in \mathbb{R}^4 \mid x_0 = x_1^2+x_2^2+x_3^2\right\}$

If three reference points $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, $P_3(x_3, y_3, z_3)$ do not lie on a straight line, then the general solution of $A\boldsymbol{x} = \boldsymbol{b}$ is $\boldsymbol{x} = \boldsymbol{x_p} + t\boldsymbol{x_h}$ where $\boldsymbol{x_p}$ is a particular solution of $A\boldsymbol{x} = \boldsymbol{b}$ and $\boldsymbol{x_h}$ is a solution of homogeneous system $A\boldsymbol{x} = \boldsymbol{0}$. To compute the particular solution and the solution of homogeneous system, the Gaussian elimination method is considered to find the vector $\boldsymbol{x_p}$ and $\boldsymbol{x_h}$**.** And also the pseudo inverse of the matrix A is one of the method to find the particular solution $\boldsymbol{x_p}$. After considering the determination of the parameter t, the solutions are

$$\boldsymbol{x_1} = \boldsymbol{x_p} + t_1 \cdot \boldsymbol{x_h}$$
$$\boldsymbol{x_2} = \boldsymbol{x_p} + t_2 \cdot \boldsymbol{x_h}$$

Where $\boldsymbol{x} = \left(x_0, x_1, x_2, x_3\right)^T$, $\boldsymbol{x_p} = \left(x_{p0}, x_{p1}, x_{p2}, x_{p3}\right)^T$, $\boldsymbol{x_h} = \left(x_{h0}, x_{h1}, x_{h2}, x_{h3}\right)^T$

The author then go on to describe their algorithm for multilateration problem example by adding the additional reference points and distances to the original problem and represent them as matrix representation as follows:

$$\begin{bmatrix} 1 & -2x_1 & -2y_1 & -2z_1 \\ 1 & -2x_2 & -2y_2 & -2z_2 \\ 1 & -2x_3 & -2y_3 & -2z_3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & -2x_n & -2y_n & -2z_n \end{bmatrix} \begin{bmatrix} x^2+y^2+z^2 \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} s_1^2 - x_1^2 - y_1^2 - z_1^2 \\ s_2^2 - x_2^2 - y_2^2 - z_2^2 \\ s_3^2 - x_3^2 - y_3^2 - z_3^2 \\ \vdots \\ s_n^2 - x_n^2 - y_n^2 - z_n^2 \end{bmatrix}$$

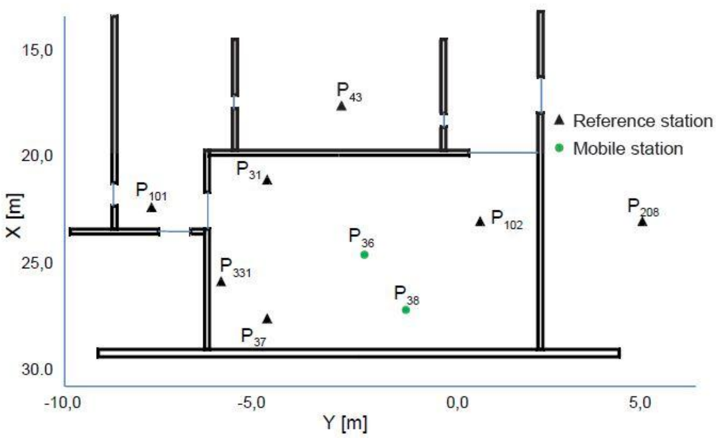
Again we can consider these matrix as the known form of $A\boldsymbol{x} = \boldsymbol{b}$ with the constraint $\boldsymbol{x} \in E$. The author proposes that the solution of $A\boldsymbol{x} = \boldsymbol{b}$ in the sense of least squares method is

$$\hat{\boldsymbol{x}} = \left(A^T A\right)^{-1} A^T \boldsymbol{b}$$

Then the recursive least square method is proposed by using the solution of $A\boldsymbol{x} = \boldsymbol{b}$ in the sense of least squares method as the first candidate. Next step that the author proposes is to select one of two solutions that we got from mentioned trilateration problem by picking the one that has location being closer to the first candidate as starting point of recursive method. Then x_0 is considered as the initial solution, and updated to x1 by every coming distance. Finally, the final solution is chosen when it minimizes the error square sum as this equation

$$\min_N \left\{ \left(\| N - P_1 \|^2 - s_1 \right)^2 + \ldots + \left(\| N - P_n \|^2 - s_n \right)^2 \right\}$$

After the algorithms were described, the author then go on to describe their experiments and experimental results from two samples using distance measurement between the stations with their developed positioning system.



Above figure illustrates the demonstration of the location of the mobile station located on points P_{36} and P_{38}. They tested their algorithms both of trilateration with three reference points and multilateration with six reference points. For trilateration problem they use P_{37}, P_{331}, and P_{102} as three reference points and P_{36} as the unknown point, then they measure the distance from each reference point to the unknown point where they know the true coordinate to compare with the result of their algorithm. Here is the summary of experimental result from the trilateration example:

- The true coordinate of the unknown point P_{36} are (24.34, -2.51, 1.13)
- The solutions of the trilateration problem are
    - N_1 = (24.35, −2.48, 1.67) and
    - N_2 = (24.31, −2.52, 1.54)

The six reference points for multilateration problem are P_{37}, P_{31}, P_{102}, P_{43}, P_{208}, and P_{101}. And P_{38} is used as the unknown point. The summary of their results are:

- The true coordinate of the unknown point P_{38} are (26.76, -1.34, 1.13)
- The solution of the multilateration problem is
    - N = (26.77, −1.34, 1.46)

While the experimental results showed that their algorithms were effective, it would have been more interesting to see the results of moving objects in dynamic scenarios. Additionally, the author showed the equations and steps related to their algorithm, but those are not well organized causing the reader might get lost. However, their algorithms showed that they are only based on linear algebraic method, which is understandable, and requires low complexity in computation because all represented variables are in the form of vector and matrix. Furthermore, there are no iterative solutions and any approximation method presented here. And the paper described simple samples on how their algorithm applied to, these easily make some senses to follow.

Since there is no the best method to find the approximation of location regarding to mentioned problems, the algorithm presented in this paper could be one interesting option to implement in our iPASS system. We expect to see promising results after applying this algorithm to our system in near future.

Reference:

[1] Norrdine, A. "An Algebraic Solution to the Multilateration Problem," *In Proceedings of the 15th International Conference on Indoor Positioning and Indoor Navigation*, 2012.