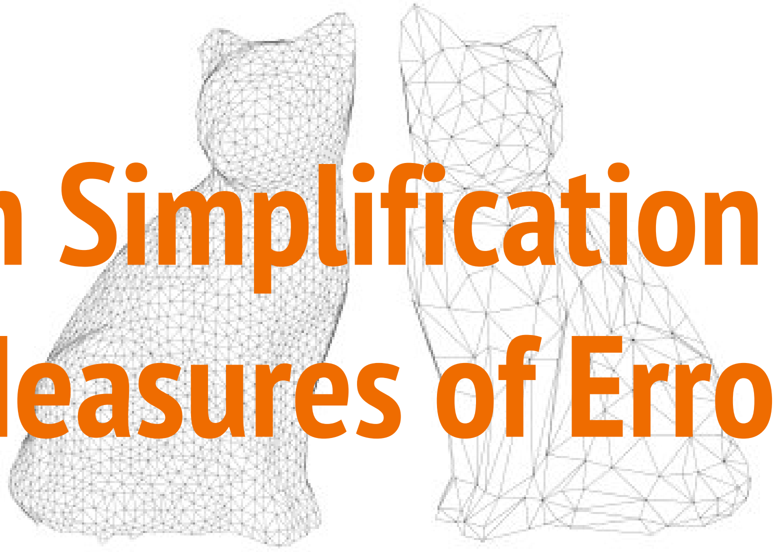
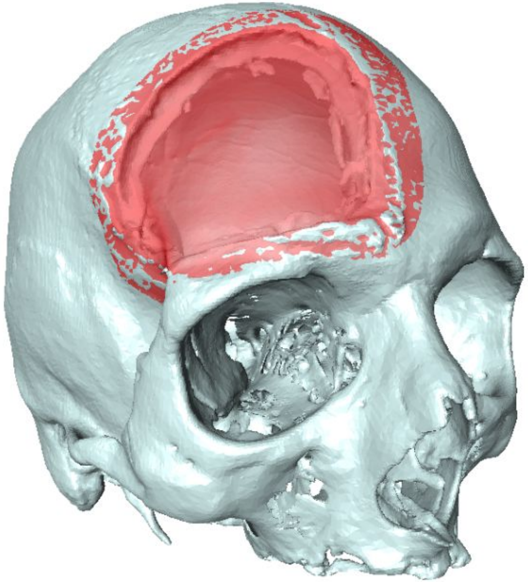


Mesh Simplification and Measures of Error

Erica Schwarz - Group #12

Project Review

Using ground truth models of cranioplasty defects in order to determine accuracy, evaluate robustness, and improve upon patient registration.



Today's Papers

Mesh Simplification:

Hoppe, Hugues, et al. "Mesh optimization." *Proceedings of the 20th annual conference on Computer graphics and interactive techniques.* ACM, 1993.

Measure of Error:

Aspert, Nicolas, Diego Santa Cruz, and Touradj Ebrahimi. "MESH: measuring errors between surfaces using the Hausdorff distance." ICME (1). 2002.

Significance

Mesh Simplification:

- Faster
- Cleaner
- Segmentation

Error Metrics:

- Accurate
- Rigorous
- Communicable

Introduction - Hoppe et al.

- Ease of manipulation is related to mesh complexity.
- Provides a method of mesh optimization
- Given a mesh M_0 represented by data points X, find a mesh M that has the same topological type as M_0 but has less vertices.
- Use an energy function to optimize the fit.
- Mesh optimization can be used for simplification, reconstruction, and segmentation.

Necessary Background - Hoppe et al.

Mesh Representation:

A mesh can be represented as simplicial complexes and vertices or M = (K, V)

- Simplicial complex: Space of unions between points, lines, and faces
  - Points: $\{i\} \in K$
  - Lines: $\{i, j\} \in K$
  - Faces: $\{i, j, k\} \in K$

Method - Hoppe et al.

Energy function:

$$E(K, V) = E_{dist}(K, V) + E_{rep}(K) + E_{spring}(K, V)$$

Minimize

Method - Hoppe et al.

Energy function:

mesh

$$E(K, V)$$

Method - Hoppe et al.

Energy function:

$$E(K, V) = E_{dist}(K, V)$$

minimize distance

$$\sum_{i=1}^{n} d^2(\mathbf{x}_i, \phi_V(|K|))$$

Method - Hoppe et al.

Energy function:

$c_{rep}m$

minimize number of vertices, edges, and faces

$$E(K, V) = E_{dist}(K, V) + E_{rep}(K)$$

Method - Hoppe et al.

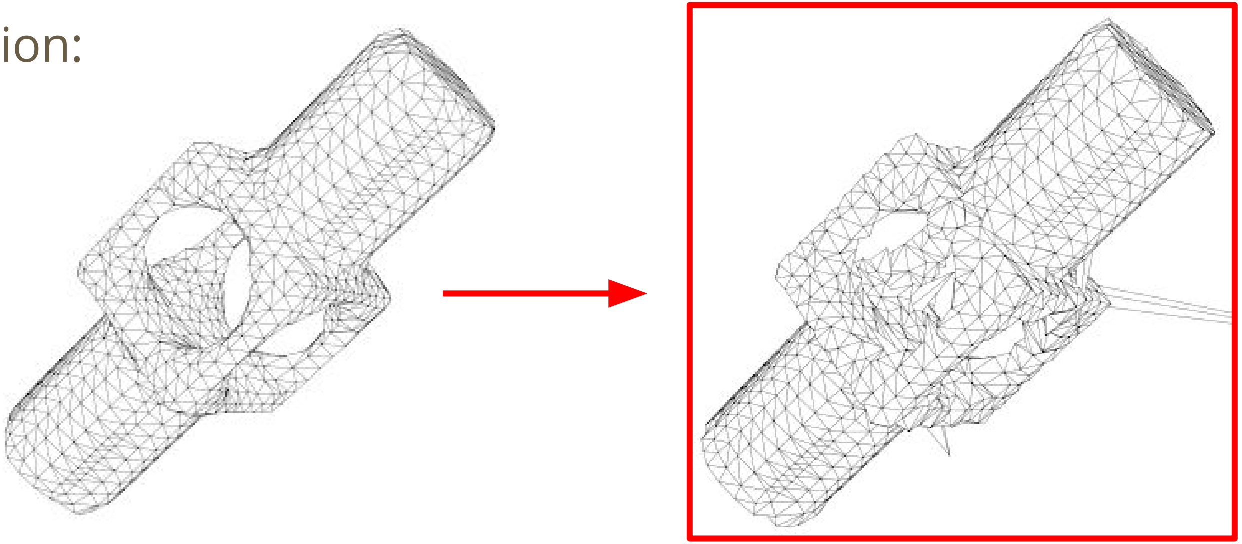
Energy function:

$$E(K, V) = E_{dist}(K, V) + E_{rep}(K)$$

Is this enough?

Method - Hoppe et al.

Energy function:



No!

(lack of minimum)

Method - Hoppe et al.

$$\sum_{\{j,k\}\in K} \kappa \|\mathbf{v}_j - \mathbf{v}_k\|^2$$

keeps the mesh compact
ensures minimum
can change with iteration

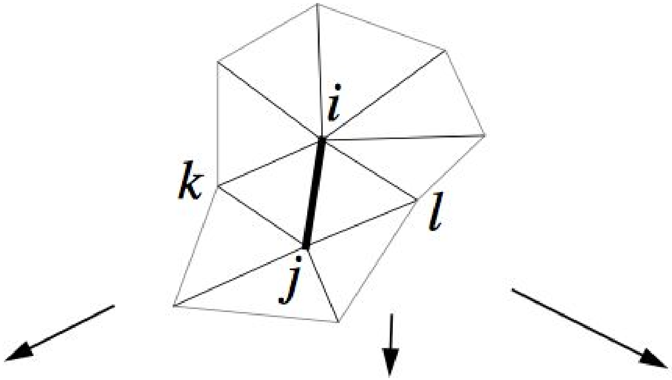
Energy function:

$$E(K, V) = E_{dist}(K, V) + E_{rep}(K) + E_{spring}(K, V)$$

Solution

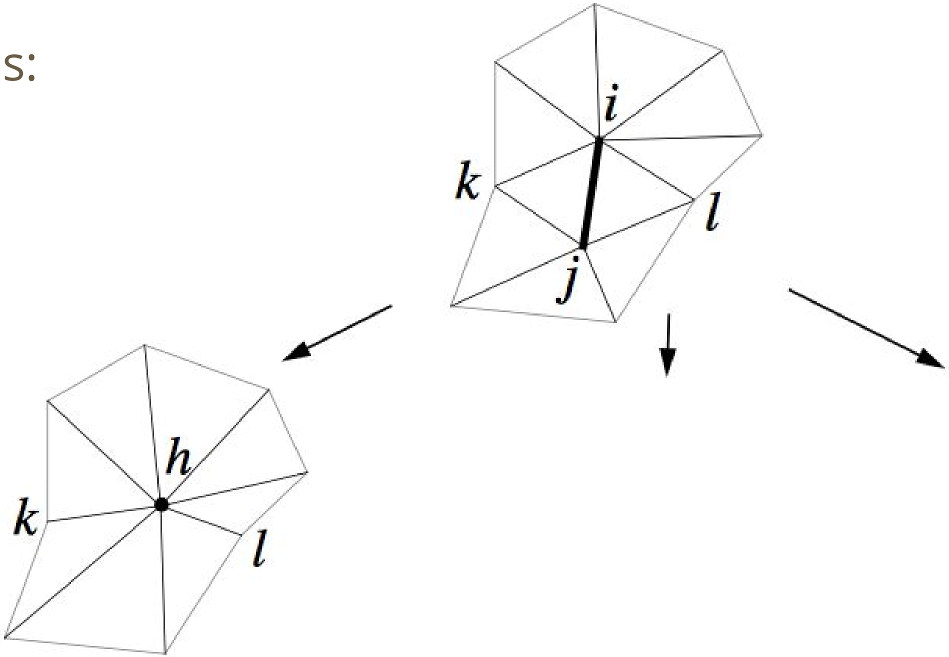
Method - Hoppe et al.

Legal moves:



Method - Hoppe et al.

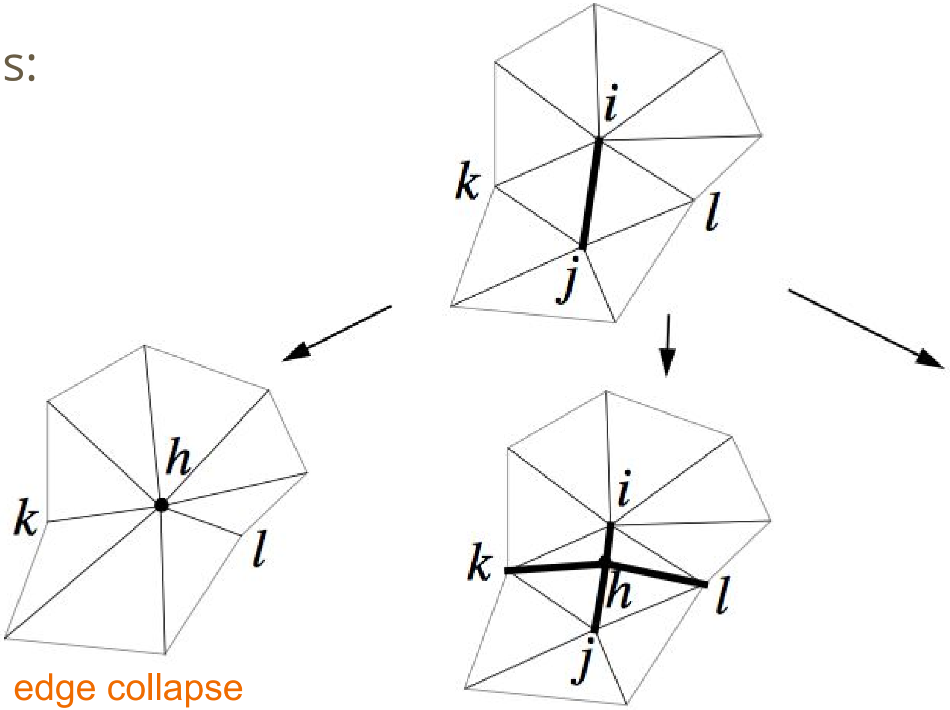
Legal moves:



edge collapse

Method - Hoppe et al.

Legal moves:

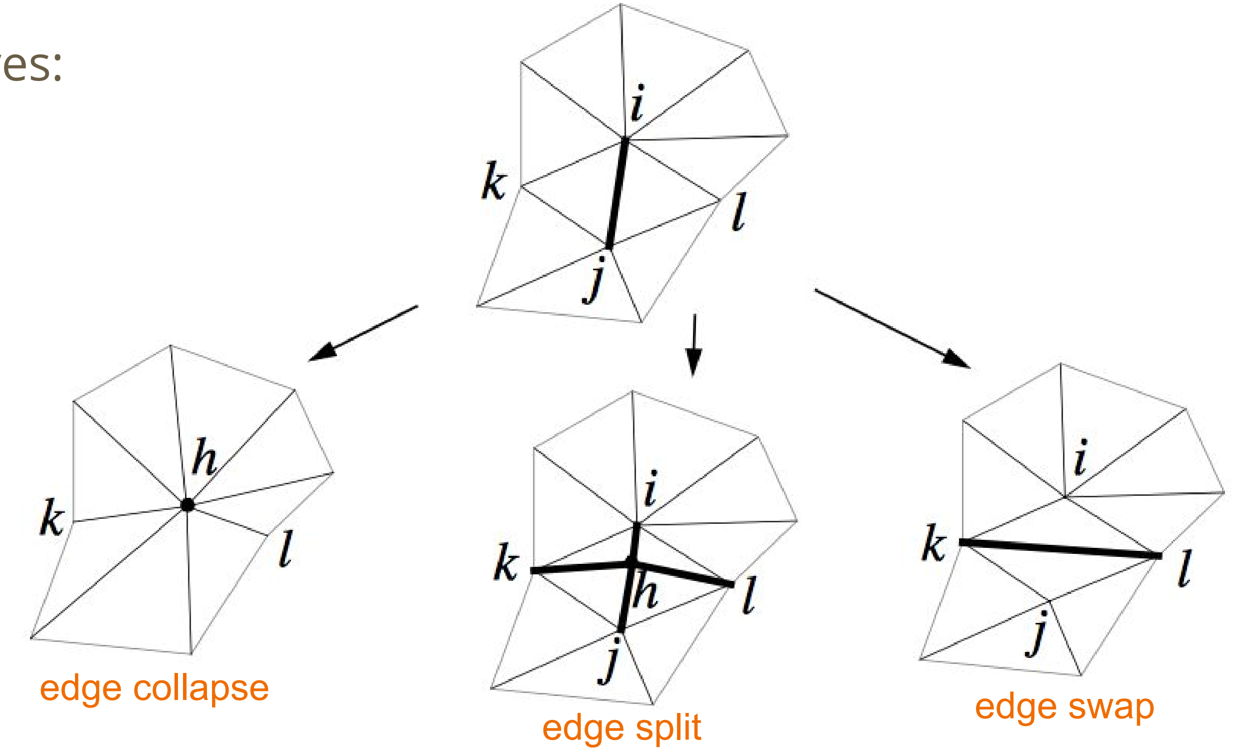


edge collapse

edge split

Method - Hoppe et al.

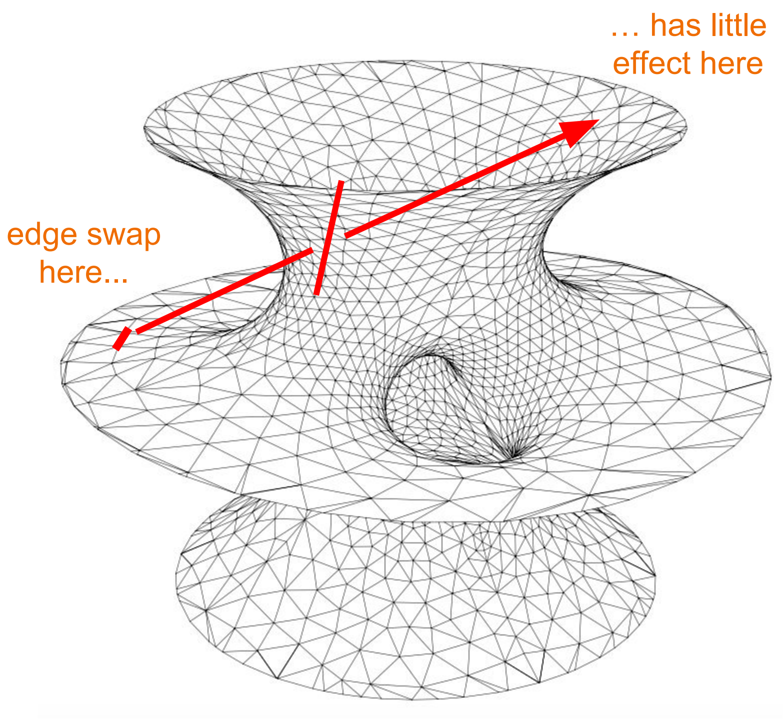
Legal moves:



Method - Hoppe et al.

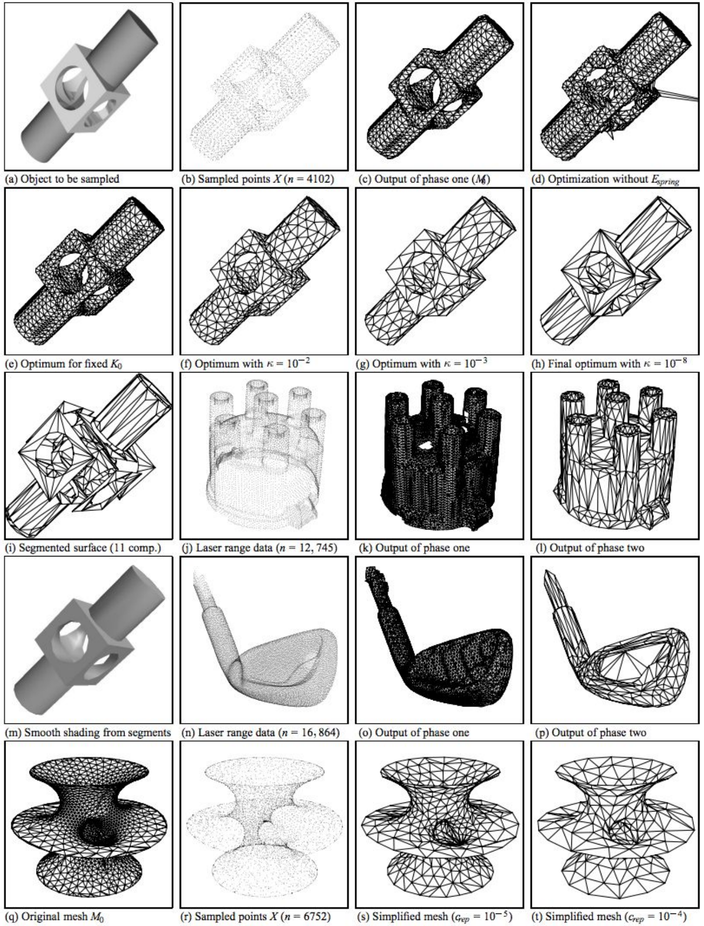
Localization:

- Algorithm over the whole mesh is inefficient
- However, changes in local area don't affect distant vertices



Results - Hoppe et al.

Fig.	#vert. m	#faces	#data n	Parameters c_{rep}	Parameters κ	Resulting energies E_{dist}	Resulting energies E	time (min.)
7c	1572	3152	4102	-	-	8.57×10^{-2}	-	-
7e	1572	3152	4102	10^{-5}	10^{-2}	8.04×10^{-4}	4.84×10^{-2}	1.5
7f	508	1024	4102	10^{-5}	10^{-2}	6.84×10^{-4}	3.62×10^{-2}	(+3.0)
7g	270	548	4102	10^{-5}	10^{-3}	6.08×10^{-4}	6.94×10^{-3}	(+2.2)
7h	163	334	4102	10^{-5}	*varied*	4.86×10^{-4}	2.12×10^{-3}	17.0
7k	9220	18272	12745	-	-	6.41×10^{-2}	-	-
7l	690	1348	12745	10^{-5}	*varied*	4.23×10^{-3}	1.18×10^{-2}	47.0
7o	4059	8073	16864	-	-	2.20×10^{-2}	-	-
7p	262	515	16864	10^{-5}	*varied*	2.19×10^{-3}	4.95×10^{-3}	44.5
7q	2032	3832	-	-	-	-	-	-
7s	487	916	6752	10^{-5}	*varied*	1.86×10^{-3}	8.05×10^{-3}	9.9
7t	239	432	6752	10^{-4}	*varied*	9.19×10^{-3}	4.39×10^{-2}	10.2



Assessment - Hoppe et al.

Pros

- Versatile
- Uses data representation analogous to our project
- Recovers sharp edges
- Allows for addition and deletion of points
- Optimization framework allows for increasing levels of accuracy
- Could potentially be used for segmentation
- Well laid out

Cons

- Discusses a lot of barycentric concepts that aren't needed for main concept
- Implies E_{spring} is somewhat arbitrary
- Does not discuss locality in depth
- Shown test cases have ideal starting meshes vs. more realistic noisy models
- Could include more optimization terms
- Limited discussion of distance metric
- Time tradeoff

Introduction - Aspert et al.

- Error between meshes are important for gauging accuracy
- Common methods are mean square error and total square error
- However, a variation on the Hausdorff distance may provide a more accurate measure of error
- In addition, the paper outlines an efficient method for finding distance

Necessary Background - Aspert et al.

- Hausdorff distance
  - Euclidean norm

$$d(p, \mathcal{S}') = \min_{p' \in \mathcal{S}'} \|p - p'\|_2$$

  - Max distance

$$d(\mathcal{S}, \mathcal{S}') = \max_{p \in \mathcal{S}} d(p, \mathcal{S}')$$

Necessary Background - Aspert et al.

- Hausdorff distance
  - Euclidean norm

$$d(p, \mathcal{S}') = \min_{p' \in \mathcal{S}'} \|p - p'\|_2$$

  - Max distance

$$d(\mathcal{S}, \mathcal{S}') = \max_{p \in \mathcal{S}} d(p, \mathcal{S}')$$

Not symmetrical!

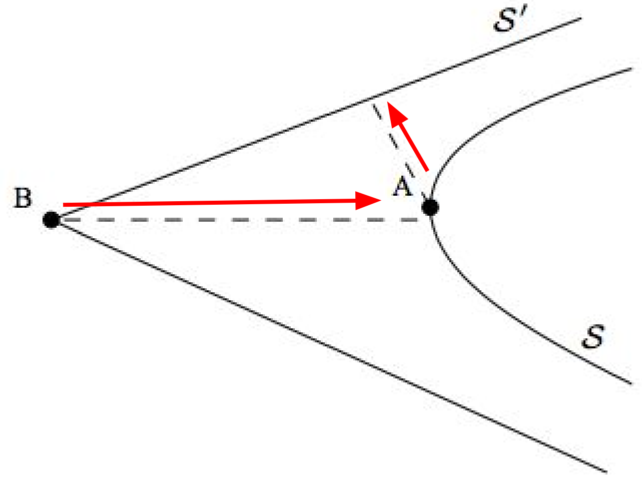
$$d(\mathcal{S}, \mathcal{S}') \neq d(\mathcal{S}', \mathcal{S})$$

Method - Aspert et al.

Forward and backward distance:

- Takes the max of the two distances

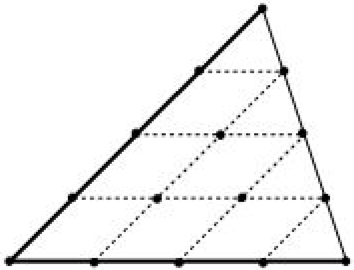
  [d(S, S'), d(S', S)]



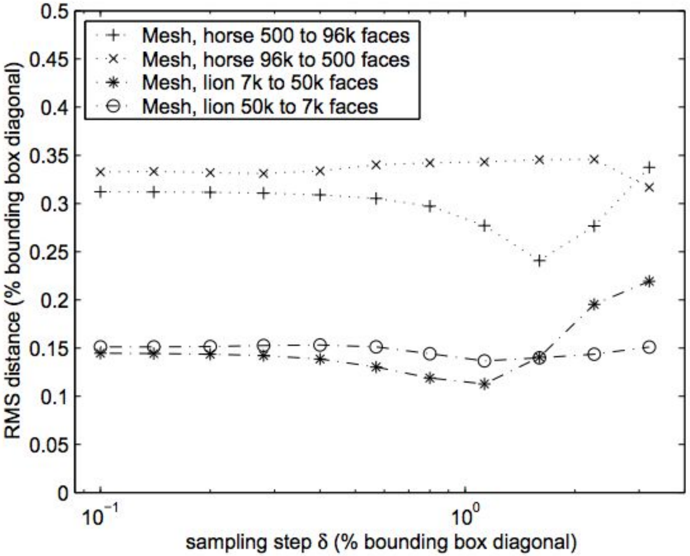
Method - Aspert et al.

Grid sampling:

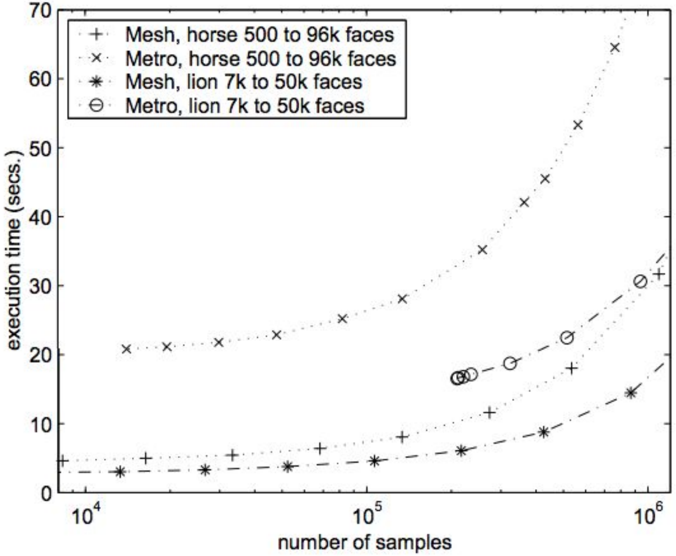
- Allows for discrete representation of surface integral
- Greatly reduces computation time



Results - Aspert et al.



Difference in metrics of error



Time less than analogous standard

Assessment - Aspert et al.

Pros

- Takes more topologies into consideration
- More robust to unusual corners and curves
- Gives “upper bound” on error estimate

Cons

- More complicated
- Application dependent
- Grid pattern cannot exhaust surface

Conclusion

Hoppe et al.

- The Hoppe et al. method may provide a way to simplify our mesh
- Could also provide a revised segmentation method

Aspert et al.

- The Aspert et al. method provides a useful method gauging accuracy.
- May provide a faster, more accurate way of finding distance.
- Especially important considering we are trying to minimize all gaps.

Questions?