



NSF Engineering Research Center  
for Computer Integrated Surgical  
Systems and Technology



LABORATORY FOR  
**Computational  
Sensing + Robotics**  
THE JOHNS HOPKINS UNIVERSITY

## Notes for PA5: Deformable Registration to a Statistical Shape Model



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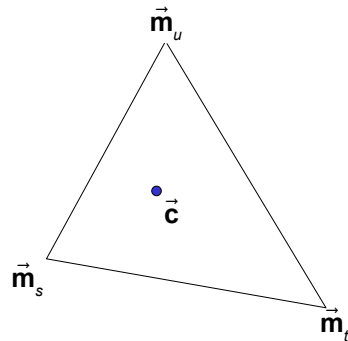
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## Barycentric Coordinates of Deforming Triangle

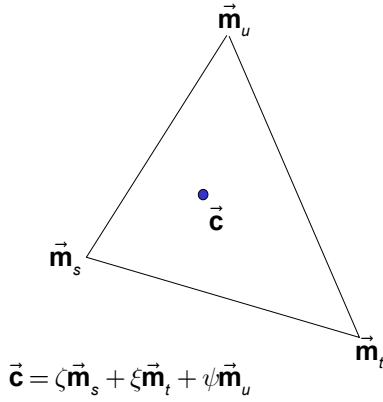


$$\vec{c} = \zeta \vec{m}_s + \xi \vec{m}_t + \psi \vec{m}_u$$



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## Barycentric Coordinates of Deforming Triangle



$$\vec{m}_s = \vec{m}_{0,s} + \sum_{m=1}^{N_{\text{modes}}} \lambda_m^{(t)} \vec{m}_{m,s}$$

$$\vec{m}_t = \vec{m}_{0,t} + \sum_{m=1}^{N_{\text{modes}}} \lambda_m^{(t)} \vec{m}_{m,t}$$

$$\vec{m}_u = \vec{m}_{0,u} + \sum_{m=1}^{N_{\text{modes}}} \lambda_m^{(t)} \vec{m}_{m,u}$$

$$\vec{q}_{m,k} = \zeta_k \vec{m}_{m,s} + \xi_k \vec{m}_{m,t} + \psi_k \vec{m}_{m,u}$$

$$\vec{c}_k^{(t)} = \vec{q}_{0,k} + \sum_{m=1}^{N_{\text{modes}}} \lambda_m^{(t)} \vec{q}_{m,k}$$

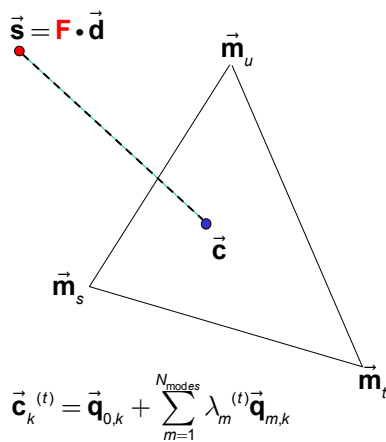
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## Deformable Registration to SSM



Step 1 For sample points, find closest matches to current mesh

Step 2 Solve  $\mathbf{F} \cdot \vec{d}_k \approx \vec{q}_{0,k} + \sum_{m=1}^{N_{\text{modes}}} \lambda_m^{(t)} \vec{q}_{m,k}$   
for  $\mathbf{F}$  and/or  $\lambda_m^{(t)}$

Step 3 If change the shape parameters then update bounding boxes

Step 4 Iterate to convergence

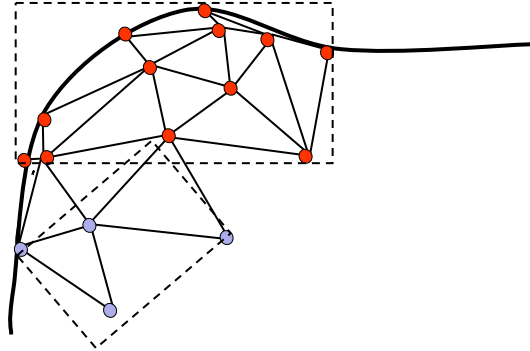
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## Updating Bounding Boxes



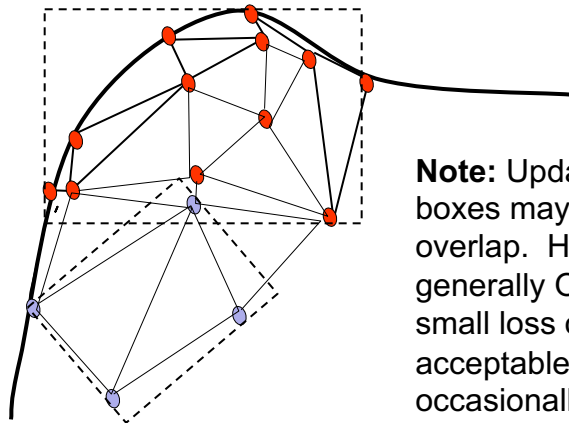
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## Updating Bounding Boxes



**Note:** Updated bounding boxes may increase overlap. However, this is generally OK, since some small loss of efficiency is acceptable. You can occasionally rebuild the whole tree if it becomes an issue.

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