Programming Assignments 1 & 2 601.455 and 601/655 Fall 2021
Please also indicate which section(s) you are in (one of each is OK)

Score Sheet

Name 1	
Email	
Other contact information (optional)	
Name 2	
Email	
Other contact information (optional)	
Signature (required)	I (we) have followed the rules in completing this assignment _____ _____

| Grade Factor		
Program (40)		
Design and overall program structure	20	
Reusability and modularity	10	
Clarity of documentation and programming	10	
Results (20)		
Correctness and completeness	20	
Report (40)		
Description of formulation and algorithmic approach	15	
Overview of program	10	
Discussion of validation approach	5	
Discussion of results	10	
TOTAL	100	

Instructions

- You may work alone or in groups of 2 people. I am <u>opposed</u> to groups of three or more.

  - Contact the TA if you are having trouble finding a partner.

  - In the past, a team consisting of one person who is a bit stronger in math with one who is a bit stronger

- Except in extraordinary circumstances, both members of the group will receive the same grade, but your report should indicate who did what.

- You may use C, C++, Python, or MATLAB for the assignment, or some other language with permission of the TA.

- You may use standard numerical libraries and packages for least-squares problems and Cartesian coordinate transformations.

  - One such library is that associated with Numerical Recipes in C. There is also a numerical library I wrote for the ERC. There is a large collection of C++ libraries for medical robotics and computer-integrated surgery, described at https://github.com/jhu-cisst/cisst/wiki. The code is at https://github.com/jhu-cisst/cisst. These libraries are supported by engineering staff at the CISST ERC, as part of our open-source "Surgical Assistant Workstation (SAW)" project.

  - Generally, you are free to use such external libraries or packages, but you **MUST** provide citations for any libraries or packages that you use.

  - If you do use software from our web site, remember that it is copyrighted and part of the CIS ERC infrastructure.   It is mostly open source, but respect any rules and copyright for this software and any other software you use.

  - You should **not** use software that essentially solves the programming problem. E.g., you can use packages for Cartesian transformation, numerical solvers, and the like, but do not use canned subroutines for solving point-cloud-to-point-cloud registration transformations or for performing calibrations. When in doubt, talk to the TA or instructor.

- You must <u>cite</u> the source of any software that you do not write yourselves.

- You may not use the results of previous years' programming assignments or the work of other students on this year's assignments.

- Your programs (and the reports) should clearly indicate authorship of the code and also should cite any libraries that you use, including source.

- Do <u>not</u> go out and try to find a package that solves this particular problem. Numerical libraries are one thing, but using someone's registration or calibration code is entirely another matter.

- Note that these rules are intended to supplement the general instructions discussed at the opening session of the class, not to replace them. If you have questions, see me or the TA.

- You should hand in a report containing the following:

  - A narrative report (typically about 5-8 pages long) summarizing

    - The mathematical approach taken

    - The algorithmic steps followed

- An overview of the structure of the computer program, sufficient to enable someone with reasonable skill (the grader) to understand your approach and follow your code.

- The steps taken to verify that the program is working correctly. Typically, this would take the form of a discussion of the results using the debugging examples.

- A tabular summary of the results obtained for unknown data

- A short discussion for the results of running your program. This certainly includes the tabular summary above, but may also include a discussion of convergence if you adopt an iterative process or of difficulties if you suspect that your answer is wrong.

- A short statement of who did what.

o A well structured and well documented program listing

o TAR or ZIP file containing:

- A PDF file for your report

- A directory called "PROGRAMS", containing (at least) all source files for your program, together with a file "README.TXT", containing the names of all the source files, together with a 1-line description of each file. You should **also** include an executable program and instructions for using it. For this, you should place a copy of the input data files in a place that your executable program can reach it when we copy your PROGRAMS directory to a local temporary folder. This means that you should use relative path names like "../Input Data/pa1-debug-a-calbody.txt" or else provide a command line option enabling us to point at the directory.

- Another directory, called "OUTPUT", containing the program output files in the specified format and with the specified name (see below).

The report and TAR or ZIP file containing your code should be uploaded to Gradescope like your Homework submission. There should be **two** files, one is your report in PDF format, and the other is a TAR or ZIP file containing your code, README, and output data.

Description of the Problem Scenario

This problem concerns calibration, simple registration, and tracking for a stereotactic navigation system that uses an electromagnetic positional tracking device.
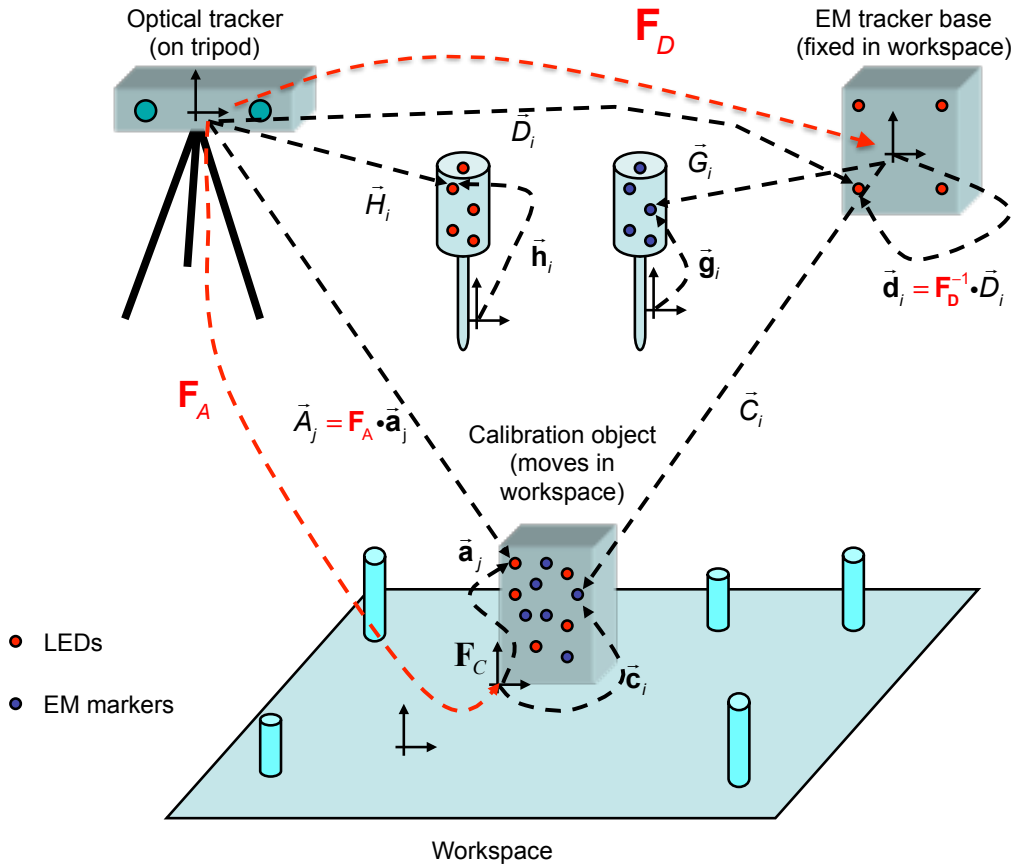
Equipment and Calibration

You have been given an electromagnetic (EM) tracking system that is capable of measuring the 3D position of small markers relative to a measuring base unit. The measurements are subject to some uncharacterized distortions, but you have been given a calibration object containing N_C markers at known positions $\vec{c}_i$ for i=1$\cdots N_C$ relative to the calibration object coordinate system $\mathbf{F}_C$. The calibration object also has N_A Optical LED markers at known positions $\vec{\mathbf{a}}_j$ on the calibration object. N_D other optical markers are placed at known positions $\vec{\mathbf{d}}_j$ on the base unit of the electromagnetic tracking system. The optical tracking system is able to read the positions of the optical markers to very high accuracy. For the purpose of this exercise, we will assume no geometric error from the optical tracker. We will denote the positions of the calibration object LEDs relative to the optical tracker as $\vec{A}_j$ and those of the electromagnetic base markers as $\vec{D}_j$. We will designate the measured positions of the electromagnetic tracker markers on the calibration

object, corresponding to the $\vec{\mathbf{c}}_i$ as $\vec{C}_i$. In addition to its distortion, which is large (up to several mm) but very repeatable, the sensor system is subject to a certain amount of random noise ν, which varies from zero (on a good day) to a value up to about 0.3 mm.

The calibration object will be placed at various positions in the workspace, and the positions of the optical markers and the electromagnetic markers will be measured. Thus, each "sample frame" of calibration data will consist of the following information: $[, \vec{D}_1, \cdots, \vec{D}_{N_D}, \vec{A}_1, \cdots, \vec{A}_{N_A}, \vec{C}_1, \cdots, \vec{C}_{N_C}]$.

The workspace also has one or more dimpled calibration posts placed at fixed, though unknown, positions and orientations with respect to the electromagnetic tracking system. You have been given two pointer probes. One equipped with N_H LED markers at unknown, though fixed positions $\vec{\mathbf{h}}_i$. The other is equipped with N_G electromagnetic markers, again at unknown, though fixed locations $\vec{\mathbf{g}}_i$. You are given the data from "pivot calibrations" of each probe using a dimpled post. Each frame of data from the optical probe calibration consists of measurements $[\vec{D}_1, \cdots, \vec{D}_{N_D}, \vec{H}_1, \cdots, \vec{H}_{N_H}]$ and each frame of data from the electromagnetic probe calibration consists of values $[\vec{G}_1, \cdots, \vec{G}_{N_G}]$.

Note: To some extent, this represents an unrealistic scenario. One is not likely to have <u>both</u> an optically-tracked probe and an electromagnetically tracked probe, although it is perfectly plausible to have an optical tracking system for performing once-only calibrations. The main reason for including this element in our problem scenario is specifically to enable me to ask you to program a pivot calibration step with a non-distorted tracking system. Also, during system development it may indeed be plausible to have both optical and EM tracked probes. In fact, the calibration body itself may be a pointer containing both types of markers.

Stereotactic Navigation after calibration

After the calibration steps described above are performed, the system is ready for use. We assume that N_B fiducial landmarks have been selected that can be located accurately in preoperative CT images and by pointing with the EM probe. Assume that the locations of these landmarks in the CT images are given by $\vec{\mathbf{b}}_j$ for $j = 1, \cdots, N_B$.

The surgical procedure is as follows:

1. The patient's anatomy is fixed in space relative to the base of the EM tracking system.

2. The tip of the EM pointer probe is positioned on each of the fiducial landmarks $\vec{\mathbf{b}}_j$ and a "frame" of EM data ($[\vec{G}_1, \cdots, \vec{G}_{N_G}]$) i.e., is taken. The corresponding position $\vec{B}_j$ of the pointer tip relative to the EM base coordinate system is computed.

3. The transformation between EM and CT coordinates is computed (i.e., $\vec{\mathbf{b}}_j \cong \mathbf{F}_{reg} \bullet \vec{B}_j$)

4. Subsequently, the system reads successive frames of EM data $[\vec{G}_1, \cdots, \vec{G}_{N_G}]$, computes the corresponding position of the tip in CT coordinates, and displays the corresponding CT data.

Assignment 1: Basic transformations & pivot calibration

The main focus of assignment 1 is development of mathematical tools and subroutines that you may use in subsequent assignments. The specific goals are:

1. Develop (or develop proficiency with) a Cartesian math package for 3D points, rotations, and frame transformations.

2. Develop a 3D point set to 3D point set registration algorithm

3. Develop a "pivot" calibration method.

4. Given a distortion calibration data set, as described above, compute the "expected" values $\vec{C}_i^{(\text{expected})}$ for the $\vec{C}_i$:

   a. For each calibration data frame $[,\vec{D}_1,\cdots,\vec{D}_{N_D},\vec{A}_1,\cdots,\vec{A}_{N_A},\vec{C}_1,\cdots,\vec{C}_{N_C}]$, compute the transformation $\mathbf{F}_D$ between optical tracker and EM tracker coordinates. I.e., compute a frame $\mathbf{F}_D$ such that $\vec{D}_j = \mathbf{F}_D \bullet \mathbf{d}_j$.

   b. Similarly, compute a transformation $\mathbf{F}_A$ between calibration object and optical tracker coordinates. I.e., $\vec{A}_j = \mathbf{F}_A \bullet \mathbf{a}_j$.

   c. Given $\mathbf{F}_D$ and $\mathbf{F}_A$, compute $\vec{C}_i^{(\text{expected})} = \mathbf{F}_D^{-1} \bullet \mathbf{F}_A \bullet \mathbf{c}_i$.

   d. Output $\vec{C}_i^{(\text{expected})}$ (see file formats below)

   You will use this code in Assignment 2 to calibrate for the distortion.

5. Apply the EM tracking data to perform a pivot calibration for the EM probe and determine the position relative to the EM tracker base coordinate system of the dimple in the calibration post. The suggested procedure is as follows.

   a. Use the first "frame" of pivot calibration data to define a local "probe" coordinate system and use this to compute $\mathbf{g}_j$ One simple method is as follows. First compute the midpoint of the observed points

   $$\vec{G}_0 = \frac{1}{N_G}\sum \vec{G}_j$$

   Then translate the observations relative to this midpoint. I.e., compute
   $$\mathbf{g}_j = \vec{G}_j - \vec{G}_0$$

   There are alternative methods, many of which involve rotating $\mathbf{g}_j$. But this isn't particularly critical. Your pivot calibration will determine a tip coordinates $\vec{t}_G$ defined in the same probe coordinate system. I.e., if $\mathbf{F}_G(t)$ gives the position and orientation of the pointer body at time t with respect to some tracker coordinate system, then $\mathbf{F}_G(t) \bullet \vec{t}_G$ gives the coordinates of the pointer tip with respect to the same tracker coordinate system.

   b. For each "frame" k of pivot data, compute a transformation $\mathbf{F}_G[k]$ such that
   $$\vec{G}_j = \mathbf{F}_G[k] \bullet \mathbf{g}_j.$$

   c. Now use the method discussed in class to solve the system

$$\vdots$$
$$\vec{P}_{\text{dimple}} = \mathbf{F}_G[k] \bullet \vec{\mathbf{t}}_G$$
$$\vdots$$

6. Apply the optical tracking data to perform a pivot calibration of the optical tracking probe. The suggested method is the same as above <u>except</u> that you should first use your value for $\mathbf{F}_D$ to transform the optical tracker beacon positions into EM tracker coordinates. Note that the optical tracker may not be in exactly the same position and orientation with respect to the EM tracker base for each observation frame of optical tracker data, so this is an important step.

Data file formats

- "NAME-CALBODY.TXT" – input file which describes the calibration object

LINE	Data	Description
1	$N_D, N_A, N_C,$ NAME-CALBODY.TXT	Number of optical markers on EM base, number of optical markers on calibration object, number EM markers on calibration object. Followed by ascii string giving file name
Next N_D records	$d_{x,i}, d_{y,i}, d_{z,i}$	Coordinates of $\vec{\mathbf{d}}_i$
Next N_A records	$a_{x,i}, a_{y,i}, a_{z,i}$	Coordinates of $\vec{\mathbf{a}}_i$
Next N_C records	$c_{x,i}, c_{y,i}, c_{z,i}$	Coordinates of $\vec{\mathbf{c}}_i$

- "NAME-CALREADINGS.TXT" – input file which provides the values read by the sensor

LINE	Data	Description
1	$N_D, N_A, N_C, N_{\text{frames}},$ NAME-CALREADINGS.TXT	Number of optical markers on EM base, number of optical markers on calibration object, number EM markers on calibration object, number of "data frames" of data, file name
Frame 1	$D_{x,i}, D_{y,i}, D_{z,i}$	Coordinates of $\vec{D}_i$
	…	
	$A_{x,i}, A_{y,i}, A_{z,i}$	Coordinates of $\vec{A}_i$
	…	

	$C_{x,i}, C_{y,i}, C_{z,i}$	Coordinates of $\vec{C}_i$
…	…	…
Frame N_{frames}	$D_{x,i}, D_{y,i}, D_{z,i}$	Coordinates of $\vec{D}_i$
	…	
…	$A_{x,i}, A_{y,i}, A_{z,i}$	Coordinates of $\vec{A}_i$
	…	
…	$C_{x,i}, C_{y,i}, C_{z,i}$	Coordinates of $\vec{C}_i$

- "NAME-EMPIVOT.TXT" – input file which provides the values read by the sensor

LINE	Data	Description
1	$N_G, N_{\text{frames}},$NAME-EMPIVOT.TXT	Number of EM markers on probe, number of "data frames" of data, file name
Frame 1	$G_{x,1}, G_{y,1}, G_{z,1}$	Coordinates of $\vec{G}_1$
…	…	…
…	$G_{x,N_G}, G_{y,N_G}, G_{z,N_G}$	Coordinates of $\vec{G}_{N_G}$
…	…	…
Frame N_{frames}	$G_{x,1}, G_{y,1}, G_{z,1}$	Coordinates of $\vec{G}_1$
…	…	…
…	$G_{x,N_G}, G_{y,N_G}, G_{z,N_G}$	Coordinates of $\vec{G}_{N_G}$

- "NAME-OPTPIVOT.TXT" – input file which provides the values read by the sensor

LINE	Data	Description
1	$N_D, N_H, N_{\text{frames}},$NAME-OPTPIVOT.TXT	Number of optical markers on EM base, number of optical markers on probe, number of "data frames" of data, file name
Frame 1	$D_{x,1}, D_{y,1}, D_{z,1}$	Coordinates of $\vec{D}_1$
	$D_{x,N_D}, D_{y,N_D}, D_{z,N_D}$	Coordinates of $\vec{D}_{N_D}$

	$H_{x,1}, H_{y,1}, H_{z,1}$	Coordinates of $\vec{H}_1$
…	…	…
…	$H_{x,N_H}, H_{y,N_H}, H_{z,N_H}$	Coordinates of $\vec{H}_{N_H}$
…	…	…
Frame N_{frames}	$D_{x,1}, D_{y,1}, D_{z,1}$	Coordinates of $\vec{D}_1$
…		
…	$D_{x,N_D}, D_{y,N_D}, D_{z,N_D}$	Coordinates of $\vec{D}_{N_D}$
	$H_{x,1}, H_{y,1}, H_{z,1}$	Coordinates of $\vec{H}_1$
	…	…
	$H_{x,N_H}, H_{y,N_H}, H_{z,N_H}$	Coordinates of $\vec{H}_{N_H}$

- "NAME-OUTPUT-1.TXT" – output file for problem 1

LINE	Data	Description
1	$N_C, N_{\text{frames}},$NAME-OUTPUT1.TXT	Number of EM markers on cal object, number of "data frames" of data, file name
2	P_x, P_y, P_z	Estimated post position with EM probe pivot calibration
3	P_x, P_y, P_z	Estimated post position with optical probe pivot calibration
Frame 1	$C_{x,1}, C_{y,1}, C_{z,1}$	Coordinates of $\vec{C}_1^{(\text{expected})}$
…	…	…
…		Coordinates of $\vec{C}_{N_C}^{(\text{expected})}$
…	…	…
Frame N_{frames}	$C_{x,1}, C_{y,1}, C_{z,1}$	Coordinates of $\vec{C}_1^{(\text{expected})}$
…	…	…
…	$C_{x,N_C}, C_{y,N_C}, C_{z,N_C}$	Coordinates of $\vec{C}_{N_C}^{(\text{expected})}$

Assignment 2: Distortion calibration & application

In this assignment, you will fit polynomials to model distortion and then use these polynomials to "de-warp" the EM tracker space. You will use the dewarped EM tracker space to repeat your pivot calibration. Then you will use the resulting calibration to compute registration to a CT coordinate system. Finally, given some pointer data frames, you will report corresponding CT coordinates.

The suggested procedure is as follows:

1. Input the body calibration data file and process it to determine the values of $\vec{C}_i^{(\text{expected})}[k]$ corresponding to each $\vec{C}_i[k]$ in each "frame" k of data.

2. Use the method described in class to produce a suitable distortion correction function.

3. Use this distortion correction function to repeat your "pivot calibration" for the EM probe.

4. Using the distortion correction and the improved pivot value, compute the locations $\vec{\mathbf{b}}_j$ of the fiducials points with respect to the EM tracker base coordinate system.

5. Compute the registration frame $\mathbf{F}_{reg}$.

6. Apply the distortion correction to all the subsequent values of $[\vec{G}_1, \cdots, \vec{G}_{N_G}]$, compute the pointer tip coordinates with respect to the tracker base, and apply $\mathbf{F}_{reg}$ to compute the tip location with respect to the CT image.

Data file formats

- The following files are the same format as in PA#1
    - "NAME-CALBODY.TXT" – input file which describes the calibration object
    - "NAME-CALREADINGS.TXT" – input file which provides the values read by the sensor
    - "NAME-EMPIVOT.TXT" – input file which provides the values read by the sensor
    - "NAME-OPTPIVOT.TXT" – input file which provides the values read by the sensor
    - "NAME-OUTPUT-1.TXT" – output file for problem 1 – Note: this will be provided for a couple of the "unknown" data sets. This is intended for the benefit of people who may have had some trouble with the first programming assignment. The only difference is that in this case, I will apply the dewarping computation to the individual points.
- "NAME-CT-FIDUCIALS.TXT" – input file that describes CT fiducial coordinates $\vec{\mathbf{b}}_j$

LINE	Data	Description
1	N_B, NAME-CT-FIDUCIALS.TXT	Number of CT foducials Followed by ascii string giving file name
2 to $N_B + 1$	$b_{x,i}, b_{y,i}, b_{z,i}$	Coordinates of $\vec{\mathbf{b}}_i$

- "NAME-EM-FIDUCIALS.TXT" – Input file that describes frames of data in which the probe is in contact with the corresponding CT fiducials

LINE	Data	Description
1	N_G, N_B,NAME-EM-FIDUCIALS.TXT	Number of EM markers on probe, number of "data frames" of data, file name
Frame 1	$G_{x,1}, G_{y,1}, G_{z,1}$	Coordinates of $\vec{G}_1$
…	…	…
…	$G_{x,N_G}, G_{y,N_G}, G_{z,N_G}$	Coordinates of $\vec{G}_{N_G}$
Frame 2	…	…
Frame N_B		Coordinates of $\vec{G}_1$
…	…	…
…	$G_{x,N_G}, G_{y,N_G}, G_{z,N_G}$	Coordinates of $\vec{G}_{N_G}$

- "NAME-EM-NAV.TXT" – Input file that describes frames of data defining test points. You are to find the corresponding positions of the probe tip with respect to CT coordinates

LINE	Data	Description
1	N_G, N_{frames},NAME-EMNAV.TXT	Number of EM markers on probe, number of "data frames" of data, file name
Frame 1	$G_{x,1}, G_{y,1}, G_{z,1}$	Coordinates of $\vec{G}_1$
…	…	…
…	$G_{x,N_G}, G_{y,N_G}, G_{z,N_G}$	Coordinates of $\vec{G}_{N_G}$
…	…	…
Frame N_{frames}	$G_{x,1}, G_{y,1}, G_{z,1}$	Coordinates of $\vec{G}_1$
…	…	…
…	$G_{x,N_G}, G_{y,N_G}, G_{z,N_G}$	Coordinates of $\vec{G}_{N_G}$

- "NAME-EM-OUTPUT2.TXT" – Output file that gives positions of probe tip in CT coordinates, corresponding to the frames of data in NAME-EM-NAV.TXT.

LINE	Data	Description
1	N_{frames},NAME-OUTPUT2.TXT	Number of "data frames" of data, file name

2 to $N_{\text{frames}}+1$	$v_{x,i}, v_{y,i}, v_{z,i}$	Coordinates of $\vec{v}_i$, the probe tip position in CT coordinates of the i'th frame of data in NAME-EM-NAV.TXT

Summary of problem data set creation

The data sets were generated with different amounts of simulated "EM distortion", "EM noise", and what I call "OT jiggle". The latter is simulated random motion of the optical tracking system about the simulated tripod on which it is supposedly mounted. For problem 2, I also have varying values of the registration transformation. The following table says which data sets have non-zero values for the different quantities.

Data set name	EM distortion	EM Noise	OT jiggle
pa1-debug-a	0	0	0
pa1-debug-b	0	X	0
pa1-debug-c	X	0	0
pa1-debug-d	0	0	X
pa1-debug-e	X	0	X
pa1-debug-f	X	X	X
pa1-debug-g	X	X	X
pa1-unknown-h	X	X	X
pa1-unknown-i	X	X	X
pa2-debug-a	0	0	0
pa2-debug-b	0	X	0
pa2-debug-c	X	0	0
pa2-debug-d	0	0	X
pa2-debug-e	X	X	X
pa2-debug-f	X	X	X
pa2-unknown-g	X	X	X
pa2-unknown-h	X	X	X