Homework Assignment 4 – 601.455/655 Fall 2022 (Circle One)

Instructions and Score Sheet (hand in with answers)

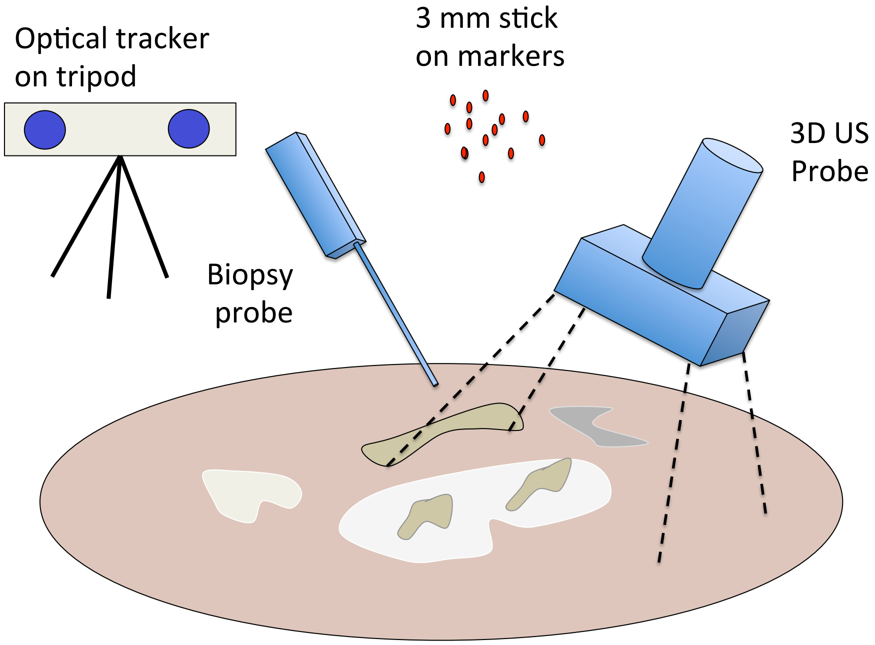
Name	Name
Email	Email
Other contact information (optional)	Other contact information (optional)
Signature (required) I/We have followed the rules in completing this assignment ____________________________	Signature (required) I/We have followed the rules in completing this assignment ____________________________

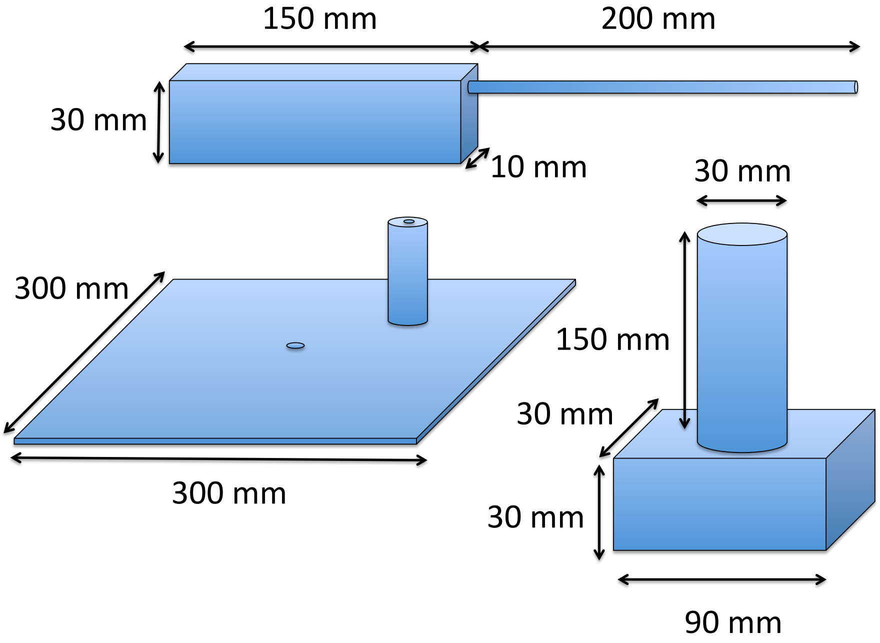
Remember that this is a graded homework assignment. It is the functional equivalent of a take-home exam.

1. You are to work **alone** or **in teams of two** and are **not to discuss the problems with anyone** other than the TAs or the instructor.
2. It is otherwise open book, notes, and web. But you should cite any references you consult.
3. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
4. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
5. Sign and hand in the score sheet as the first sheet of your assignment.

NOTE: This assignment has a total of 110 points. However, at most 100 points will count toward your course grade.

Scenario: Image Guided Surgical Biopsy System





The goal of this exercise is design of an image guided biopsy system. You have been given several components to work from. These include:

- **A biopsy probe**. This probe has a rectangular handle approximately 30x10x150 mm. A thin, stiff shaft approximately 200 mm long extends from the end of this handle, approximately 5 mm from one of the thin sides, and is the portion that may be inserted into the patient. The actual tissue-sampling portions are at the end of this shaft.

- **An ultrasound phantom with a corresponding CT scan** in which some anatomic structures visible in CT and ultrasound have been identified. Segmented models of these structures are available for use by the ultrasound-to-CT registration software. The phantom has a roughly ovoid cross section, and is 200 mm thick and 400 mm wide. A CT image of the phantom has been taken, and surface triangular mesh models S_n are available for anatomic structures within the phantom. Each mesh consists of a list of triangles $\{\cdots,T_j^{(n)}=[\vec{\mathbf{p}}_j,\vec{\mathbf{q}}_j,\vec{\mathbf{r}}_j],\cdots\}$ whose vertices $[\vec{\mathbf{p}}_j,\vec{\mathbf{q}}_j,\vec{\mathbf{r}}_j]$ are expressed in CT coordinates.
- **A 3D ultrasound system**. The probe for the ultrasound system has a cylindrical handle approximately 150 mm long and 30 mm in diameter. The probe is roughly "T" shaped, and the actual 3D US image sensor is contained in a rectangular parallelepiped 30x30x90 mm in size. For the purposes of this problem you may make the (unrealistic) assumption that the ultrasound images are large enough to cover a substantial fraction of the phantom and (in particular) of the anatomic structures S_n.
- **Ultrasound Image Segmentation Software:** The ultrasound system has excellent image segmentation software. For a given ultrasound image k, the software can find a cloud of surface points $\{\cdots,\vec{\mathbf{s}}_i^{(k)},\cdots\}$ on anatomical structures visible within the image. Unfortunately, the software may return a few "outlier" points that are not actually on any anatomic structure, and it is unable to tell you which structure any of the returned points are located.
- **An optical tracking device** mounted on a tripod but that can also be removed from the tripod and moved freely in space. The optical tracker is capable of tracking small markers to an absolute accuracy of $\delta = 0.25\ mm$ relative to the optical tracker. However, the tripod on which the tracker camera is mounted is subject to small random room vibrations or other displacements $\Delta\mathbf{F}_C \approx [\mathbf{I}+sk(\vec{\alpha}_C),\vec{\varepsilon}_C]$ relative to the floor for each measurement, so that $\Delta\mathbf{F}_C[t_i]\neq\Delta\mathbf{F}_C[t_j]$ for $t_i \neq t_j$.
- **A collection of 3 mm stick-on markers** capable of being tracked by the optical tracker. These are very adhesive and can be attached to any object in the room. Note that when markers are attached to something, only the *approximate* position (within about 3 mm) of the marker relative to the object can be controlled during the placement process.
- **A 300x300 mm metal plate.** This plate has a small dimple in the middle. Also, rigidly attached to the plate is a tube with approximate length 150 mm. The inner diameter of this tube closely matches the outer diameter of the shaft of the biopsy probe.

Question 1

One requirement for the biopsy system is to determine the position of the tip of the biopsy probe and also the direction of the shaft relative to some tracking base coordinate system $\mathbf{F}_T$. If the system is capable of determining the pose $\mathbf{F}_{TB}(t)$ of a coordinate system associated with the biopsy probe at time t, then the problem reduces to finding the position $\vec{\mathbf{p}}_{tip}$ of the biopsy tip and the direction $\vec{\mathbf{n}}_{shaft}$ relative to the biopsy probe coordinate system.

A. **(15 points)** Describe a workflow and algorithm outline for determining $\vec{\mathbf{p}}_{tip}$ and $\vec{\mathbf{n}}_{shaft}$. Note that this will probably involve using some of the stick-on markers. You should indicate where you plan to place them and also should include information about workflow, sensing, and formulation. Again, you do not have to recite internal details of any point-cloud-to-point-cloud registration algorithms. But make clear how they are being used. Your answer will probably include a few sketches, as well as step-by-step workflow, clear descriptions of the measurements to be taken, and formulas. **Hint:** There are some pretty clear suggestions in the lecture notes and programming assignment about how to do this. Note that your answer should include a method for establishing a tracking coordinate system $\mathbf{F}_T$ that is insensitive to the jiggling motion $\Delta\mathbf{F}_C(t)$ of the tracking camera.

B. **(10 points)** Assuming that you have determined $\vec{\mathbf{p}}_{tip}$ and $\vec{\mathbf{n}}_{shaft}$ accurately, how accurately will you be able to determine $\mathbf{F}_{TB}(t)\vec{\mathbf{p}}_{tip}$ and $\mathbf{R}_{TB}(t)\vec{\mathbf{n}}_{shaft}$ (i.e., what can you say about the position of the biopsy tool tip relative to the tracker and the angle between the computed value of $\mathbf{R}_{TB}(t)\vec{\mathbf{n}}_{shaft}$ and the actual vector?) Note that this might depend somewhat on where you put the markers onto the biopsy probe. Your answer should start with an <u>*approximate*</u> estimate of bounds on $\vec{\alpha}_{TB}$ and $\vec{\varepsilon}_{TB}$ where $\Delta\mathbf{F}^*_{TB} = \mathbf{F}_{TB}\Delta\mathbf{F}_{TB}$ and $\Delta\mathbf{F}_{TB} \approx [\mathbf{I} + sk(\vec{\alpha}_{TB}), \vec{\varepsilon}_{TB}]$. You should first express your answer in terms of δ and the placement locations of any tracking markers and then give a numerical answer based on $\delta = 0.25\ mm$.

C. **(15 points)** Suppose you suspect that the optical measurement system has a systematic error in its "depth" direction. I.e., if the reported value of a marker is at position $\vec{\mathbf{m}} = [m_x, m_y, m_z]$ relative to the actual position will be at position $\vec{\mathbf{m}}^* = [m_x, m_y, \phi m_z] + \vec{\varepsilon}$, where $\left\|\vec{\varepsilon}\right\| \le \delta$. Describe a method to determine the value of ϕ, using only the apparatus you have available. Include a description of the apparatus you will use, the complete workflow, what measurements you will take, and the mathematical calculations that you will use. **Hint:** You will not need to fabricate any new apparatus beyond what you needed for Question 1A.

Question 2

Our image-guided interventional system will also require that we be able to determine the relative transformation $\mathbf{F}_{TI}(t)$ between the tracker and the 3D ultrasound image. Again, this will probably require that you define a coordinate system using markers stuck to the ultrasound probe that will enable you to compute a transformation $\mathbf{F}_{TU}(t)$ between the tracker and the ultrasound probe body and then do some sort of calibration process to determine a transformation $\mathbf{F}_{UI}$ between the probe body and the actual 3D image, so that a point $\vec{\mathbf{c}}(t)$ in an ultrasound image taken at time *t* is located at $\mathbf{F}_{TU}(t)\mathbf{F}_{UI}\vec{\mathbf{c}}(t)$ relative to the tracker.

A. **(15 points)** Describe how you will determine $\mathbf{F}_{TU}(t)$ and provide an estimate of how accurately you can determine $\Delta\mathbf{F}_{TU}(t) \approx [\mathbf{I} + sk(\vec{\alpha}_{TU}), \vec{\varepsilon}_{TU}]$. Your answer will doubtless include some markers, and you should explain where you would place them on the probe and some analysis to justify your accuracy estimates. A sketch or two may be useful. Again, first give an answer in terms of δ and the approximate placement of any markers on the ultrasound probe and then a numerical answer assuming $\delta = 0.25\ mm$.

B. **(25 points)** Describe a workflow and algorithm outline for determining $\mathbf{F}_{UI}$ and give a formula for computing $\mathbf{F}_{TI}(t)$, following the same general guidelines as those for Question 1A. **Hint:** you may want to review the lecture notes on calibration for some ideas about how to go about this.

C. **(20 points)** Assume that your answer to Question 2A has enabled you to compute $\mathbf{F}_{UI}$ to very high accuracy. Suppose also that you are able to locate a biopsy target on an actual patient at a location $\vec{\mathbf{c}}(t)$ relative to a 3D US image when your system reports a value of $\mathbf{F}_{TU}(t)$. Describe a workflow and human-machine interfaces that will enable you to perform the biopsy (i.e., to insert the biopsy device into the patient along a straight line to the target and place the tip onto the target.)

D. **(10 points)** Estimate how accurately you can place the biopsy tool onto the target. Here, you should assume that you have an accurate estimate for the value of $\vec{\mathbf{p}}_{tip}$ but that you will still need to account for the error in $\mathbf{F}_{TB}(t)\vec{\mathbf{p}}_{tip}$, as well as the error $\Delta\mathbf{F}_{TU}(t) \approx [\mathbf{I} + sk(\vec{\alpha}_{TU}), \vec{\varepsilon}_{TU}]$.