Homework Assignment 6 – 601.455/655 Fall 2022 (Circle One)

Instructions and Score Sheet (hand in with answers)

Name	Name
Email	Email
Other contact information (optional)	Other contact information (optional)
Signature (required) I/We have followed the rules in completing this assignment ______________________________	Signature (required) I/We have followed the rules in completing this assignment ______________________________

Remember that this is a graded homework assignment. It is the functional equivalent of a take-home exam.

1. You are to work **alone** or **in teams of two** and are **not to discuss the problems with anyone** other than the TAs or the instructor.
2. It is otherwise open book, notes, and web. But you should cite any references you consult.
3. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
4. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
5. Sign and hand in the score sheet as the first sheet of your assignment.

NOTE: This assignment has a total of 110 points. However, at most 100 points will count toward your course grade.

Scenario – Essentially the same as for HW # 5

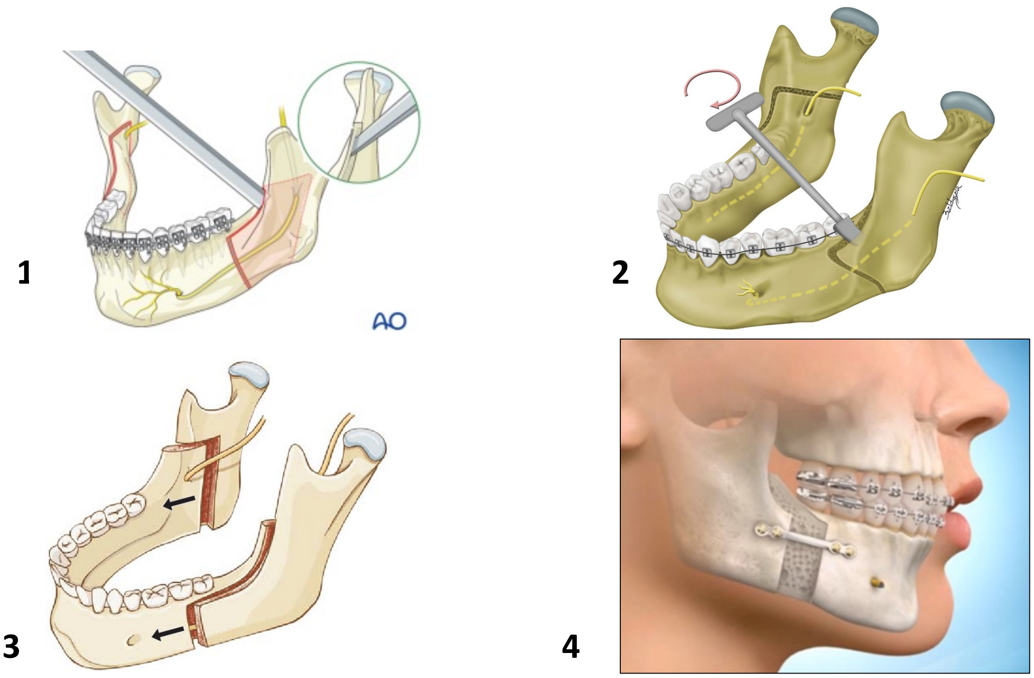

Figure 1: Mandibular Osteotomy

Consider the mandibular osteotomy procedure illustrated in Figure 1. Planar cuts are made in the mandible (i.e., the lower jaw) to split it into a front and back section. The back section is then translated forward and resecured to the back section by screws and metal plates. This procedure requires high precision and great care in order not to damage delicate nerves within the jaw. The cuts need to be planar and parallel in order to facilitate sliding the jaw forward.

For this assignment, we will assume that a CT-based presurgical plan has been made. The planning information for one of the cuts is shown in Figure 2. The planning system has defined a mandibular coordinate system $\mathbf{F}_{mandible}$ associated with the mandible. The cut is to be made in the XY plane of a cut coordinate system $\mathbf{F}_{mc}$ defined relative to $\mathbf{F}_{mandible}$. The cutting tool will be a drill-like rotary cutter, in which both the side of the cutter will be used to make the cut. The planning system has specified a path $\vec{\mathbf{p}}_{cut}(\vec{\zeta},\lambda)$ in XY plane of the cut coordinate system that the tip of the cutter is to follow, where $\vec{\zeta}$ are parameters describing the path and λ represents a displacement along the path. The cutter has a cylindrical shape with a radius r_{drill} and a length L_{drill}.

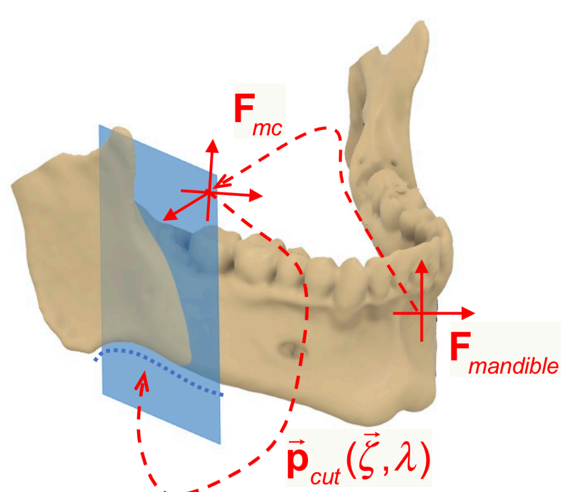

Figure 2: Planned Cut

During the cut, the direction $\vec{\mathbf{d}}_c$ of the cutter shaft must stay within a specified angle $\theta_{\max}$ of the desired shaft direction $\vec{\mathbf{d}}_{des}$. (Here, $\vec{\mathbf{d}}_c$ and $\vec{\mathbf{d}}_{des}$ are unit vectors). For safety reasons, all parts of the cutter must stay within a defined distance ρ_Z of the XY plane of $\mathbf{F}_{mc}$. Also, the tip of the cutter must stay within a projected distance ρ_{XY} of the nominal path $\vec{\mathbf{p}}_{cut}(\vec{\zeta},\lambda)$. I.e., the projected position of the tip onto the XY plane of $\mathbf{F}_{mc}$ must be within ρ_{XY} of the nominal path $\vec{\mathbf{p}}_{cut}(\vec{\zeta},\lambda)$.

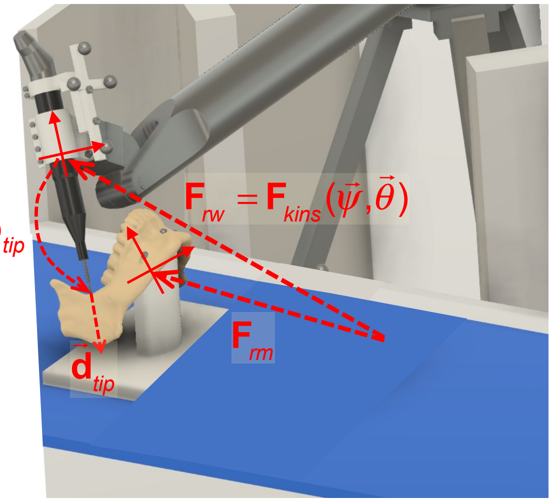

Figure 3: Robotic Cutting

We wish to use the hand-over-hand surgical robot shown in Figure 3 to assist in performing the cuts. Here, the position of the robot's tool holder relative to its base is $\mathbf{F}_{rw}(\vec{\theta}) = \mathbf{F}_{kins}(\vec{\psi},\vec{\theta})$, where $\vec{\psi}$ are structural parameters associated with the robot design, and $\vec{\theta}$ are the positions of the joints of the robot. The robot controller is also able to compute the Jacobean

$$\mathbf{J}_{kins}(\vec{\theta}) = \begin{bmatrix} \mathbf{J}_{\vec{\alpha}}(\vec{\theta}) \\ \mathbf{J}_{\vec{\varepsilon}}(\vec{\theta}) \end{bmatrix}$$ such that

$$\mathbf{F}_{rw}(\vec{\theta} + \Delta\vec{\theta}) \approx \left[\mathbf{I} + sk(\vec{\alpha}), \vec{\varepsilon}\right] \bullet \mathbf{F}_{rw}(\vec{\theta})$$

where

$$\begin{bmatrix} \vec{\alpha}_{rw} \\ \vec{\varepsilon}_{rw} \end{bmatrix} = \vec{\eta}_{rw} = \mathbf{J}_{kins}(\vec{\theta})\Delta\vec{\theta}$$

For convenience, we will adopt the notation $\Delta\mathbf{F}(\vec{\eta}) = \Delta\mathbf{F}(\vec{\alpha},\vec{\varepsilon}) = [\Delta\mathbf{R}(\vec{\alpha}),\vec{\varepsilon}] \approx [\mathbf{I} + sk(\vec{\alpha}),\vec{\varepsilon}]$ where

$$\vec{\eta} = \begin{bmatrix} \vec{\alpha} \\ \vec{\varepsilon} \end{bmatrix}.$$

The position of the tip of the cutter is located at $\vec{\mathbf{p}}_{tip}$ relative to the tool holder, i.e., $\mathbf{F}_{rw} \bullet \vec{\mathbf{p}}_{tip}$ relative to the robot. Similarly, the direction of the cutter shaft relative to the tool holder is $\vec{\mathbf{n}}_{tip}$. As shown in the figure, $\vec{\mathbf{n}}_{tip}$ points "away" from the drill body, i.e., in the direction from the drill body toward the tip.

A registration process needs to be performed to determine the transformation $\mathbf{F}_{rm}$ between robot and mandible coordinates.

New material for the assignment

The goal of this assignment is to fill in some of the things that may be needed for a complete clinical application.

As you may have noticed from a change in Figure 3, an optical tracking system is available that can be used to track optical markers attached to the drill. In fact, a supply of marker spheres is available, and it will be possible to fabricate additional marker bodies that can be attached to the patient's anatomy or to other objects in the surgical field.

You also have a portable x-ray c-arm available. Further, you can assume that this c-arm has been calibrated so that every point in the x-ray image corresponds to a known line in space (in the coordinate system of the x-ray detector) between the x-ray source and a point on the x-ray detector.

You have the ability to fabricate simple radiolucent apparatus for holding the patient and supporting multiple optical marker spheres. The spheres are assumed to be sufficiently radio-opaque so that they appear as circles in the x-ray images. Software is available to locate the centers of these circles in the images.

The c-arm is assumed to be motorized and capable of reorienting itself in multiple poses relative to the surgical scene.

The drill is also assumed to be equipped with sensors that can detect touch and forces between the tool tip and surrounding objects. The computer can determine the magnitude of these forces and their direction relative to the tool holder coordinate system.

Question 1 – Registration

(40 points) Describe a method for determining the registration transformation $\mathbf{F}_{rm}$ between the robot and the mandible. If you wish, you can simplify matters that the coordinate system $\mathbf{F}_{mandible}$ is the same as the CT coordinate system and that good CT segmentation software is available.

Describe the equipment you will use; enumerate the step-by-step workflow; and describe the measurements taken; and provide sufficient computational detail so that it is clear how you will perform the registration. It is not necessary to enumerate all the steps of a known algorithm, so long as the inputs, outputs, and other conditions for using it are clear.

Note that there is not one single "right" answer for this question, and there are many possibilities, given the equipment available. In addition to explaining how you will perform the registration, you should provide a brief rationale for your choice.

Note, also, that you do need to be sure that you do not leave any missing-links in your proposed registration chain. E.g., if you use the x-ray system, you will need to find a way to relate x-ray to robot coordinates.

Question 2 – Calibration

(40 points) So far, we have assumed that the robot is perfectly accurate. This is unrealistic, although the robot is highly precise and repeatable. I.e. it can move to a spot, move away, and return with very high accuracy to the same spot. Also, relative motions over short distances can be performed to high accuracy. I.e., If we command the robot to move from $\mathbf{F}_1$ to $\mathbf{F}_2$ with the actual positions being $\Delta\mathbf{F}_1\mathbf{F}_1$ and $\Delta\mathbf{F}_2\mathbf{F}_2$ then the rate at which the $\Delta\mathbf{F}$'s change is fairly slow, so that $\left\|\vec{\varepsilon}_1 - \vec{\varepsilon}_2\right\| \le \mu \left\|\vec{\mathbf{p}}_1 - \vec{\mathbf{p}}_2\right\|$ and $\left\|\vec{\alpha}_1 - \vec{\alpha}_2\right\| \le \nu \left\|\mathbf{R}_1^{-1}\mathbf{R}_2\right\|$ where the magnitude of a rotation is the magnitude of the angle of rotation.

Your job is to perform a geometric calibration of the robot using the optical tracking system. Your goal is to find a function $\Delta\mathbf{F}_{cal}(\vec{\theta})$ such that the actual pose $\mathbf{F}^*(\vec{\theta})$ of the robot's tool holder for joint positions $\vec{\theta}$ is $\mathbf{F}^*(\vec{\theta}) = \Delta\mathbf{F}_{cal}(\vec{\theta})\mathbf{F}_{kins}(\vec{\theta})$.

In doing this, you can make several simplifying assumptions:

- The transformation between the robot's tool holder and the optical markers on the drill is known. In fact, you can assume that the marker body has been configured so that the coordinate transformation $\mathbf{F}_{cw}$ between the tracking camera and the marker body is the same as that between the tracking camera and the tool holder.
- The robot's design consists of two modules: 1) a wrist mechanism that is capable of 3 DOF rotations about a single rotation center; and 2) a translation mechanism that moves the wrist in the X, Y, and Z directions. The nominal kinematics of the robot are thus $\mathbf{F}_{kins}(\vec{\theta}_{wrist}, \vec{\theta}_{xyz}) = [\mathbf{R}_{wrist}(\vec{\theta}_{wrist}), \vec{\mathbf{p}}(\vec{\theta}_{xyz}) + \mathbf{R}_{wrist}(\vec{\theta}_{wrist})\vec{\mathbf{p}}_{wrist}]$.
- For the purposes of this exercise, you can assume that $\vec{\mathbf{p}}_{wrist}$ is known exactly. Also, you can assume that $\mathbf{F}_{kins}(\vec{\mathbf{0}}, \vec{\mathbf{0}}) = \mathbf{I}$.
- However, the cartesian component is not quite ideal, in the sense that it may introduce some small rotational errors. I.e., $\Delta\mathbf{F}_{kins}(\vec{\mathbf{0}}, \vec{\theta}_{xyz}) = [\Delta\mathbf{R}_{xyz}(\vec{\theta}_{xyz}), \Delta\vec{\mathbf{p}}_{xyz}(\vec{\theta}_{xyz})] \approx [\mathbf{I} + sk(\vec{\alpha}_{xyz}), \vec{\varepsilon}_{xyz}]$

Describe the equipment you will use; enumerate the step-by-step workflow; and describe the measurements taken; and provide sufficient computational detail so that it is clear how you will perform the registration. It is not necessary to enumerate all the steps of a known algorithm, so long as the inputs, outputs, and other conditions for using it are clear.

Hint: Start by calibrating the cartesian stage of the robot to find a way to compute $\Delta\mathbf{F}_{kins}(\vec{\mathbf{0}}, \vec{\theta}_{xyz})$ by moving only the $\vec{\theta}_{xyz}$ joints. Then return the robot to $\mathbf{F}(\vec{\mathbf{0}}, \vec{\mathbf{0}})$ and move the $\vec{\theta}_{wrist}$ joints to determine $\Delta\mathbf{F}_{kins}(\vec{\theta}_{wrist}, \vec{\mathbf{0}})$, which will only have a $\Delta\mathbf{R}$ component. Then $\Delta\mathbf{F}_{kins}(\vec{\theta}_{wrist}, \vec{\theta}_{xyz}) = \Delta\mathbf{F}_{kins}(\vec{\theta}_{wrist}, \vec{\mathbf{0}})\Delta\mathbf{F}_{kins}(\vec{\mathbf{0}}, \vec{\theta}_{xyz})$.

Another hint: You will likely need some markers on the robot's base to account for possible camera motion.

Question 3 – Clinical Use

(30 Points) Assuming that you have a well-calibrated robot and a robust registration method, describe the steps that you must take to use this system clinically. Some considerations include testing, sterility, cleaning, training, etc. Here, we are looking for a fairly short, well organized, and easy-to-follow essay, perhaps with some bulletized points.