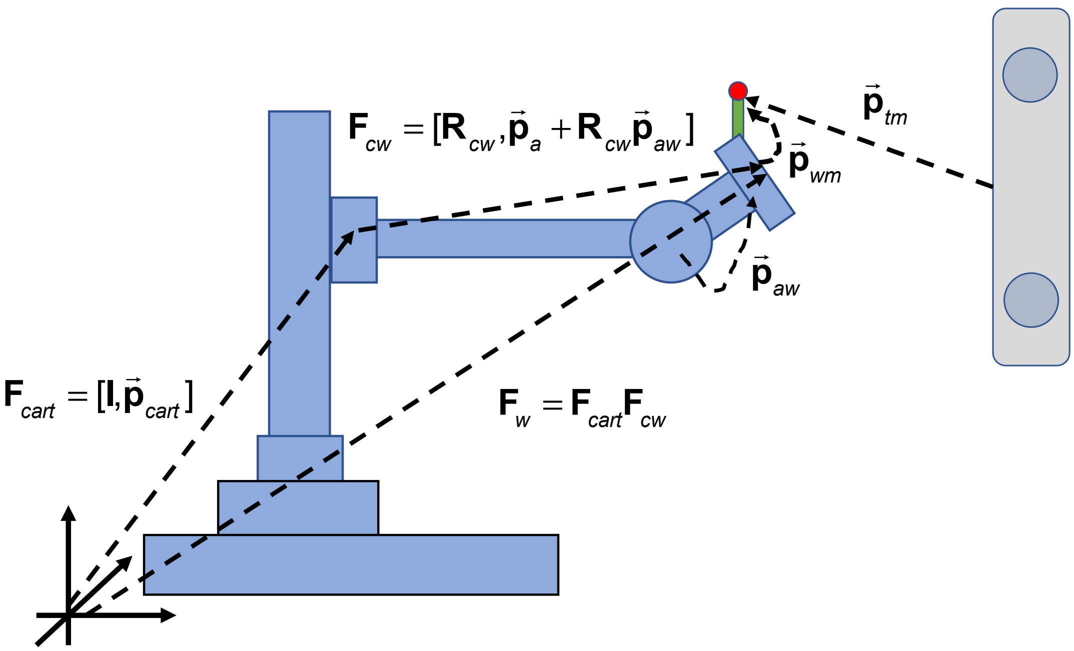
Homework Assignment 3 – 600.455/655 Fall 2024

Name	Name
Email	Email
Other contact information (optional)	Other contact information (optional)
Signature (required) I/We have followed the rules in completing this assignment ______________________________	Signature (required) I/We have followed the rules in completing this assignment ______________________________

1. Remember that this is a graded homework assignment. It is the functional equivalent of a take-home exam.
2. You are to work <u>**alone**</u> or in <u>**teams of two**</u> and are not to discuss the problems with anyone other than the TAs or the instructor.
3. **IMPORTANT NOTE:** If you work in teams of two, you are <u>**not**</u> to split up the questions and each answer a subset individually. You are to work <u>**together**</u>. I encourage teaming on these problems because I believe that it encourages learning, not as a way to reduce the required work for students taking the course. By signing this sheet you are asserting that each of you has contributed equally to each answer and can individually explain the answer as well as if you had answered the question alone. I view this as a question of trust and ethics.
4. It is otherwise open book, notes, and web. But you should cite any references you consult.
5. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
6. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
7. Submit the assignment on GradeScope as a neat and legible PDF file. We will not insist on typesetting your answers, but we must be able to read them. We will not go to extraordinary lengths to decipher what you write. If the graders cannot make out an answer, the score will be 0.
8. Sign and hand in this page as the first sheet of your assignment. If you work with a partner, then you both should sign the sheet, but you should only submit one PDF file for both of you, using the GradeScope teaming feature. Indicate clearly who it is from.

This assignment has more than 100 points, but the most that will be applied to your grade is 100

Scenario: Robot Calibration



This is a robot calibration problem. Consider the robotic system shown above. The robot consists of a cartesian base carrying an arm, at the end of which is a 3-axis spherical wrist carrying a tool attachment plate. The cartesian base consists of three orthogonal linear stages. As shown in the figure, the nominal kinematics of the system are given by

$$\mathbf{F}_{cart} = \mathbf{F}_{cart}(\vec{\mathbf{s}}) = \mathbf{F}_{cart}(s_x, s_y, s_z) = [\mathbf{I}, \vec{\mathbf{p}}_{cart}] = [\mathbf{I}, s_x\vec{\mathbf{x}} + s_y\vec{\mathbf{y}} + s_z\mathbf{z}]$$

$$0 \le s_i \le L \ \text{ for } i \in \{x, y, z\}$$

$$\mathbf{R}_{cw} = \mathbf{R}_{cw}(\vec{\theta}) = Rot(\vec{\mathbf{x}}, \theta_1)Rot(\vec{\mathbf{y}}, \theta_2)Rot(\vec{\mathbf{x}}, \theta_3)$$

$$\mathbf{F}_{cw} = \mathbf{F}_{cw}(\vec{\theta}) = [\mathbf{R}_{cw}(\vec{\theta}), \vec{\mathbf{p}}_a + \mathbf{R}_{cw}(\vec{\theta})\vec{\mathbf{p}}_{aw}]$$

$$|\theta_x| \le \pi \qquad |\theta_y| \le 3\pi/4 \qquad |\theta_z| \le \pi$$

$$\mathbf{F}_w = \mathbf{F}(\vec{\mathbf{s}}, \vec{\theta}) = \mathbf{F}_{cart}(\vec{\mathbf{s}})\mathbf{F}_{cw}(\vec{\theta})$$

The linear stages are very straight, but they are not mounted exactly perpendicular to each other, and there is some inaccuracy in the displacement produced by the actuators, so that

$$\vec{\mathbf{p}}^*_{cart}(\vec{\mathbf{s}}) = \left(s_x + \Delta s_x\right)\vec{\mathbf{x}} + \left(s_y + \Delta s_y\right)Rot(\vec{\mathbf{z}}, \phi_{xy})\vec{\mathbf{y}} + \left(s_z + \Delta s_z\right)Rot(\vec{\mathbf{z}}, \phi_{xy})Rot(\vec{\mathbf{y}}, \phi_{yz})\vec{\mathbf{z}} = \vec{\mathbf{s}} + \vec{\varepsilon}_{cart}(\vec{\mathbf{s}})$$

The spherical wrist also has small manufacturing errors, so that

$$\begin{aligned}\mathbf{R}^*_{cw} &= Rot(\Delta R_1\vec{\mathbf{x}},\vec{\theta}_x+\Delta\theta_1)Rot(\Delta R_2\vec{\mathbf{y}},\vec{\theta}_2+\Delta\theta_y)Rot(\Delta R_3\vec{\mathbf{x}},\vec{\theta}_3+\Delta\theta_z)\\ &= Rot(\vec{\mathbf{a}}_1,\theta_1+\Delta\theta_1)Rot((\vec{\mathbf{a}}_2,\vec{\theta}_2+\Delta\theta_2)Rot(\vec{\mathbf{a}}_3,\vec{\theta}_3+\Delta\theta_3)\end{aligned}$$

We also know that the nonlinear displacement and rotation angle errors are characterized by

$$\left|\nabla\vec{\varepsilon}_{cart}(\vec{\mathbf{s}})\right| \le \mu \qquad \frac{d\Delta\theta_i(\theta_i)}{d\theta_i} = \nu_i$$

Fortunately, all joints have index marks so that $\Delta s_i(0) = \Delta\theta(0) = 0$.

To perform our calibration, we have an optical tracking system and an optical marker that can be mounted to the end-effector tooling plate of the robot. For the purposes of this calibration problem, you may assume that the optical tracker and robot are both rigidly mounted to an optical bench, so that there is no motion of the optical tracker relative to the robot base. The optical marker is mounted at an unknown position $\vec{\mathbf{p}}_{wm}$ relative to the end effector, but its position $\vec{\mathbf{p}}_{tm}(\mathbf{F}_w)$ relative to the optical tracker can be measured for any pose $\mathbf{F}_w$ of the robot. The tracker error is characterized by $\vec{\mathbf{p}}^*_{tm} = \vec{\mathbf{p}}_{tm} + \vec{\varepsilon}_{tm}$, where $\left\|\vec{\varepsilon}_{tm}\right\| \le \delta$.

Question 1 (initial calibration of the cartesian stage)

A. (10 points) Suppose that you have two nominal values $\mathbf{F}_w(\vec{\mathbf{s}}_1,\vec{\theta})$ and $\mathbf{F}_w(\vec{\mathbf{s}}_2,\vec{\theta})$ with corresponding estimated true values

$$\mathbf{F}_w(\vec{\mathbf{s}}_1^*,\vec{\theta}) = \Delta\mathbf{F}_w(\vec{\mathbf{s}}_1,\vec{\theta})\mathbf{F}_w(\vec{\mathbf{s}}_1,\vec{\theta}) \approx [\mathbf{I}+sk(\vec{\alpha}_1),\vec{\varepsilon}_1]\bullet\mathbf{F}_w(\vec{\mathbf{s}}_1,\vec{\theta})$$
$$\mathbf{F}_w(\vec{\mathbf{s}}_2^*,\vec{\theta}) = \Delta\mathbf{F}_w(\vec{\mathbf{s}}_2,\vec{\theta})\mathbf{F}_w(\vec{\mathbf{s}}_2,\vec{\theta}) \approx [\mathbf{I}+sk(\vec{\alpha}_2),\vec{\varepsilon}_2]\bullet\mathbf{F}_w(\vec{\mathbf{s}}_2,\vec{\theta})$$

Consider a point at nominal location $\mathbf{F}_w(\vec{\mathbf{s}}_\lambda,\vec{\theta})$ with $\vec{\mathbf{s}}_\lambda = (1-\lambda)\vec{\mathbf{s}}_1 + \lambda\vec{\mathbf{s}}_2$. Provide a bound on the value of $\left\|\vec{\varepsilon}_\lambda\right\|$ where $\mathbf{F}_w(\vec{\mathbf{s}}_\lambda^*,\vec{\theta}) \approx [\mathbf{I}+sk(\vec{\alpha}_\lambda),\vec{\varepsilon}_\lambda]\bullet\mathbf{F}_w(\vec{\mathbf{s}}_\lambda,\vec{\theta})$.

B. (10 points) What is the largest value of $\Delta\vec{\mathbf{s}} = \left\|\vec{\mathbf{s}}_2 - \vec{\mathbf{s}}_1\right\|$ for which $\left\|\vec{\varepsilon}_\lambda\right\| \le 2\delta$

C. (10 points) Describe a calibration method for determining the value of $\Delta s_A(s)$ to an accuracy $\left|\Delta s_A(s)\right| \le 2\delta$ based on measuring $\vec{\mathbf{p}}_{tm}$ for multiple values of s. You should include a detailed workflow for taking the necessary data and formulas for computing $\Delta s_A(s)$. **Note:** Here, "A" refers to one of the linear stages, and *s* represents a commanded displacement for that stage. Essentially, we are asking you to create a calibration function to correct for the displacement error. **Hint:** I am just using a form of linear interpolation for my suggested answer. You will make it easier on the graders if you do likewise.

D. (10 points) Describe to compute a commanded position stage s_{cmd} that will place the stage at a desired position s_{des}. Here, you are welcome to use the data that you gathered you gathered in Question 1.C and/or you may assume that you have successfully found a reliable and efficient method to compute $\Delta s_A(s)$

E. (15 points) Describe a method for determining ϕ_{xy} and ϕ_{yz}, including the step-by-step workflow for gathering data and the formulas for computing ϕ_{xy} based on your data. **Note:** Here, you should assume that you have successfully calibrated the individual stages so that we know how to move the stages to the desired displacement, I.e. you have a function $\vec{\mathbf{s}}_{cmd}(\vec{\mathbf{s}}_{des})$ that will move the stages to the desired displacements. For notational convenience, we will use the notation $\vec{\mathbf{p}}_{tm}(\vec{\mathbf{s}}_{des})$ to be the value read by the tracker when the stages are moved to $\vec{\mathbf{s}}_{cmd}(\vec{\mathbf{s}}_{des})$.

F. (10 pts) Describe a method for computing the transformation $\mathbf{F}_{RT} = [\mathbf{R}_{RT},\vec{\mathbf{p}}_{RT}]$ from robot coordinates to tracker coordinates.

Question 2 (calibration of the spherical wrist)

A. (15 pts) Describe a method for determining the actual values of the rotation axes $\left\{\vec{\mathbf{a}}_X,\vec{\mathbf{a}}_y,\vec{\mathbf{a}}_Z\right\}$ including the step-by-step workflow for gathering data and the formulas for computing $\left\{\vec{\mathbf{a}}_X,\vec{\mathbf{a}}_y,\vec{\mathbf{a}}_Z\right\}$ based on your data. In answering this question, you should consider the effect of $\mathbf{F}_{wm} = [\mathbf{R}_{wm},\vec{\mathbf{p}}_{wm}]$ on your method.

B. (15 pts) For each axis *A*, provide an estimate on how accurately you can determine $\vec{\mathbf{a}}_A$. I.e., can you provide a bound on the error $\gamma_A = \sin^{-1}\left\|\vec{\mathbf{a}}_A^* \times \vec{\mathbf{a}}_A\right\| = \sin^{-1}\left\|\Delta\vec{\mathbf{a}}_A \times \vec{\mathbf{a}}_A\right\| \approx \left\|\Delta\vec{\mathbf{a}}_A \times \vec{\mathbf{a}}_A\right\| the$

C. (15 pts) As described in the scenario, the rotational axis encoders are subject to scale distortions, so that $\theta_A^* = \nu_A \theta_A$ Describe a calibration method for determining the actual values of ν_k, including the step-by-step workflow for gathering data and the formulas for computing ν_A based on your data.