Medical Robots, Constrained Robot Motion Control, and "Virtual Fixtures"

(Part 2)

Russell H. Taylor
601.455/655

Disclosures & Acknowledgments

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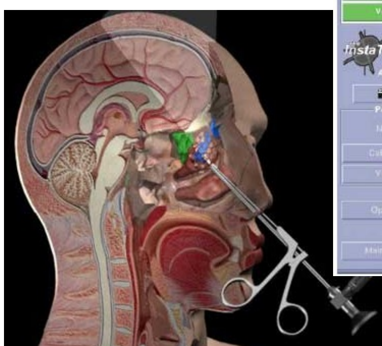
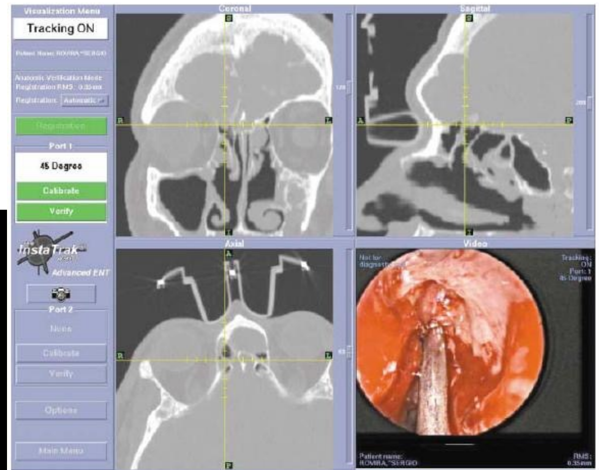
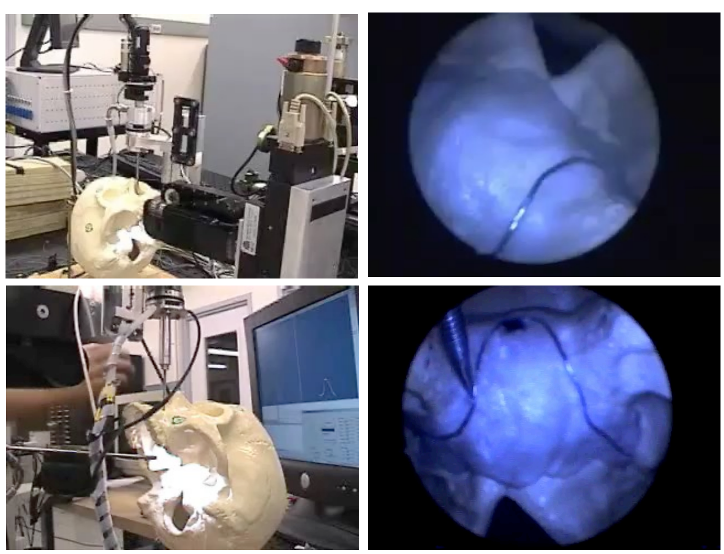
Typical application domain: endoscopic sinus surgery





Kennedy, D.W., W.E. Bolger , S. J. Zinreich , J. Zinreich, *Diseases of the Sinuses: Diagnosis and Management*. 2001.

Sample task: steady hand path tracing



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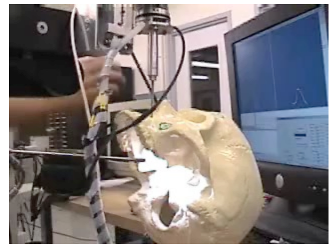
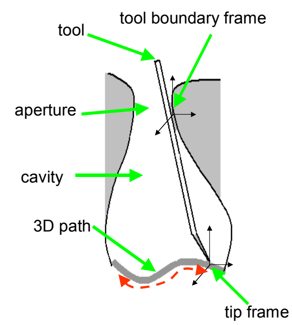
Goal: robotically-assisted sinus surgery

- **Difficulties with conventional approach**
  - Complicated geometry
  - Safety-critical structures
  - Limited work space
  - Awkward tools
- **Our approach**
  - Cooperatively controlled "Steady hand" robot
  - Registered to CT models
  - "Virtual fixtures" automatically derived from models





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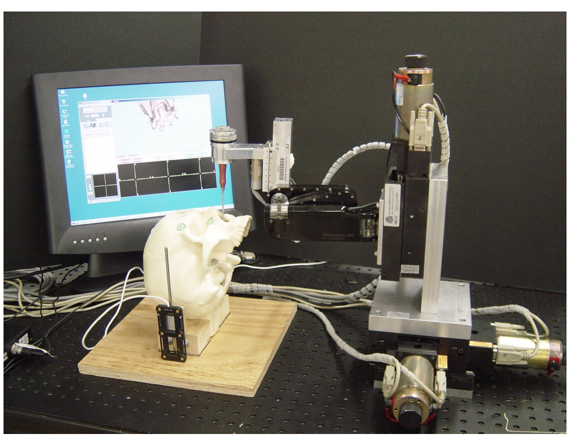
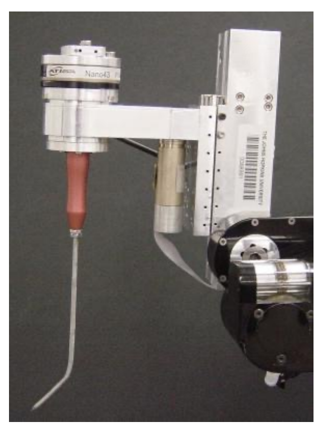
Experiment Setup





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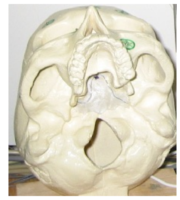
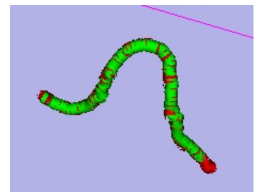
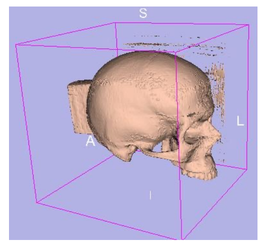
Experimental setup



- Plastic Skull Phantom
  - Target path defined by embedded wire
  - Radioopaque fiducials implanted on skull for registration
- Computer model
  - Extracted from CT scan using standard software (Slicer)
- 3D tracking of tools, etc. using Northern Digital Optotrak®
- Co-register model, robot, and optical tracker using standard techniques





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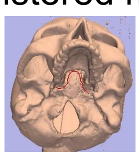
Virtual Fixture Online Implementation

Registered model



Path

Constraint generation

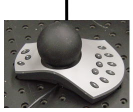
Tool tip guidance virtual fixture

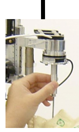
$$\min \left\| W \cdot \left(J_{tip} \cdot \Delta q - \Delta P_{des} \right) \right\|^2$$

Subject to

$$G \cdot \Delta q \geq g$$

Robot interface





State

M. Li; R. Taylor; ICRA 2005

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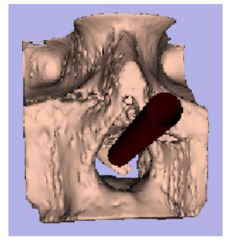
Boundary Constraints Generation



- Anatomy – triangulated surface models
  - Patient-specific model of nose & sinus derived from CT
  - High complexity: 182,000 triangles & 99,000 vertices
- Tool shaft -- cylinder
- The boundary constraint generation requires us to find close-point pairs between boundary surface model & tool shaft

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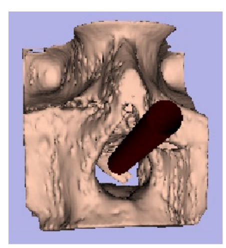
Boundary Constraints Generation



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  - High complexity: 182,000 triangles & 99,000 vertices
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- The boundary constraint generation requires us to find close-point pairs between boundary surface model & tool shaft
- **Problem: How can we generate the right constraints in real time???**

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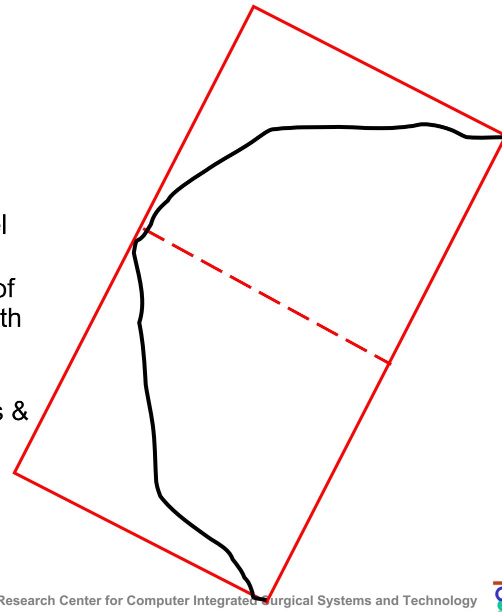
Our solution: efficient search method using covariance tree representation of model

Covariance trees:
- Williams & Taylor, 1998; other authors
- Variation of k-d trees
- Basic idea:
  - Hierarchically split 3D model into sub-volumes
  - Realign coordinate system of each sub-volume to align with moments of inertia
- Produces bounding boxes that closely approximate boundaries & fast searches



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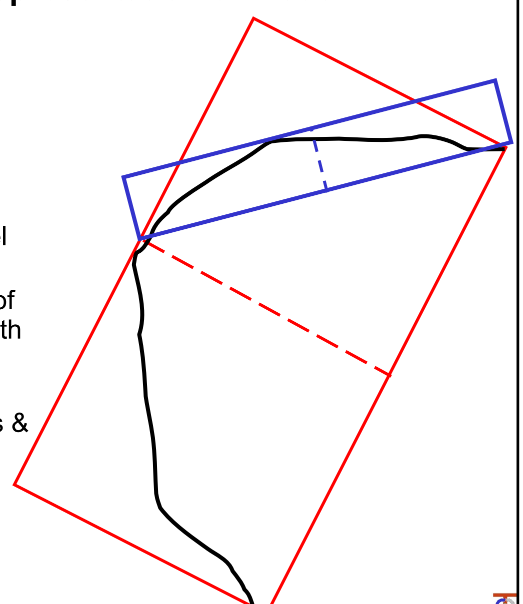

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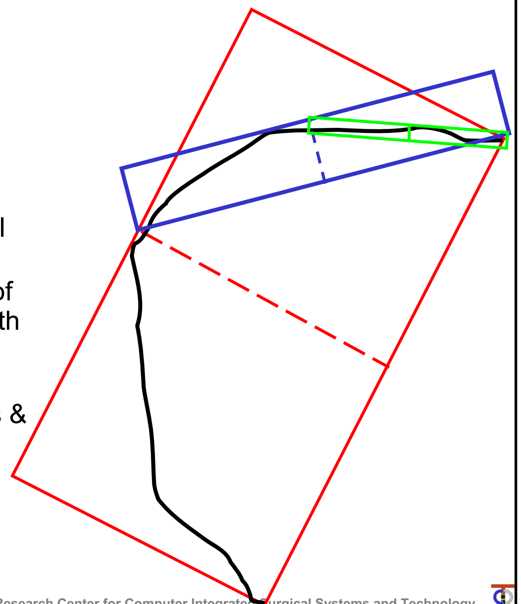

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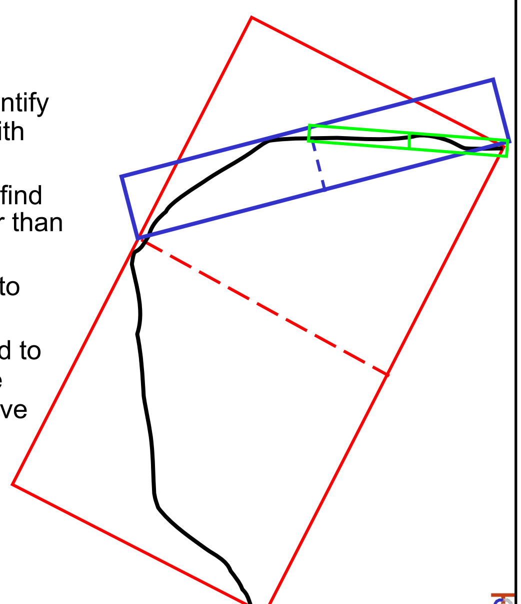
One difference from ICP problem

One difference from ICP problem:

- Here we in principle need to identify all anatomy that can interfere with tool shaft
- Consequently modify search to find all triangle edges that are closer than some threshold to tool shaft
- Further modify to prune search to eliminate redundant constraints
- **NOTE:** Generally, you only need to consider surfaces that are close enough so that the tool may move there in one time step.



Control Implementation

- Formulate constrained least squares problem
- Constraints & objective function include terms for desired tip motion, joint limits, boundary constraints

$$\zeta = \min_{\Delta q} \left\| \begin{bmatrix} W_{tip} & & \\ & W_k & \\ & & W_{joints} \end{bmatrix} \cdot \left(\begin{bmatrix} J_{tip}(q) \\ J_k(q) \\ I \end{bmatrix} \Delta q - \begin{bmatrix} \Delta P_{tip-des} \\ 0 \\ 0 \end{bmatrix} \right) \right\|$$

$$\text{subject to} \begin{bmatrix} H_{tip} & & \\ & H_k & \\ & & H_{joints} \end{bmatrix} \cdot \begin{bmatrix} J_{tip}(q) \\ J_k(q) \\ I \end{bmatrix} (\Delta q) \geq \begin{bmatrix} h_{tip} \\ h_k \\ h_{joints} \end{bmatrix}$$

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Control Implementation

- Tip frame $\Delta P_{tip} = J_{tip}(q) \cdot \Delta q$

$\left\| \Delta P_{tip} - \Delta P_{tip-des} \right\|$

$\Delta P_{tip_d}^{T} \cdot \Delta P_{tip} \geq THD$

min $\zeta_{tip} = \left\| W_{tip} \cdot \left(J_{tip}(q) \Delta q - \Delta P_{tip-des} \right) \right\|$

subject to $H_{tip-des} J_{tip}(q) \Delta q \geq h_{tip}$

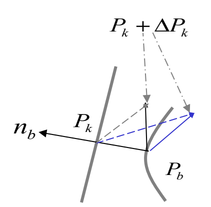
- Boundary constraint $\Delta P_k = J_k(q) \cdot \Delta q$

$\left\| W_k \cdot \Delta P_k \right\|$

$n_b^{T} \cdot (P_k + \Delta P_k - P_b) \geq d$

min $\zeta_k = \left\| W_k J_k(q) \Delta q \right\|$

subject to $H_k J_k(q) \Delta q \geq h_k$



- Joints limitation

$\left\| W_{joint} \cdot \Delta q \right\|$

$q_{\min} - q \leq \Delta q \leq q_{\max} - q$

min $\zeta_{joints} = \left\| W_{joints} \Delta q \right\|$

subject to $H_{joints} \Delta q \geq h_{joints}$

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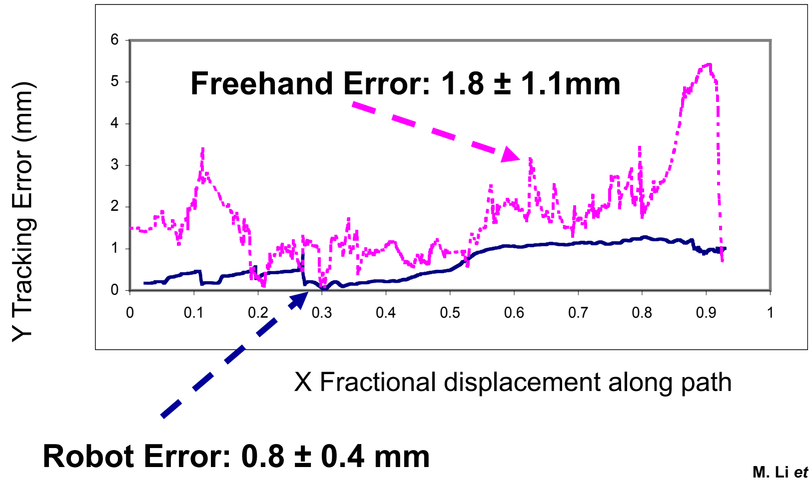
Results: Robot *vs* Freehand



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Results: Robot *vs* Freehand

Trial#	Free hand		Robot Guidance	
	Average Error (mm)	Average Time (s)	Average Error (mm)	Average Time (s)
1	1.785	26.354	0.736	18.972
2	1.632	29.358	0.757	15.275
3	1.796	27.372	0.765	16.29
4	2.061	25.436	0.779	19.439
5	2.119	24.533	0.777	16.209
avg	1.819	26.611	0.763	17.237
std	1.126	1.863	0.395	1.848

Approx 1.5:1 improvement in time!

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Combine constraints

Single Frame

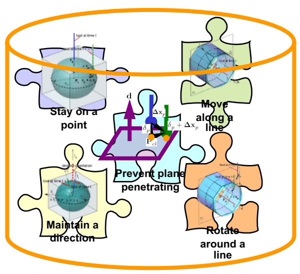
Translational part

$$\begin{bmatrix} A_p \\ A_r \end{bmatrix} J(q) \cdot \Delta q \le \begin{bmatrix} b_p \\ b_r \end{bmatrix}$$

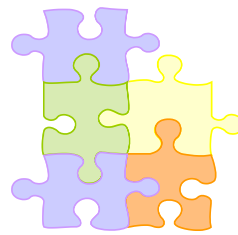
Rotational part

Multiple Frame

$$\begin{bmatrix} A_1, 0 \\ \ddots \\ 0, A_n \end{bmatrix} \begin{bmatrix} J_1(q) \\ \vdots \\ J_n(q) \end{bmatrix} \Delta q \le \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$



Select one or more



Customized virtual fixtures

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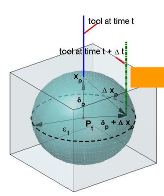
5 Basic Geometric Constraints
(Virtual fixture library)



Stay on a point

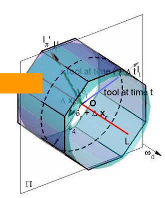
Optimization

$$\arg\min_{\Delta\vec{q}} C(\vec{x}(\vec{q}+\Delta\vec{q}), \vec{s}, \vec{x}^d)$$

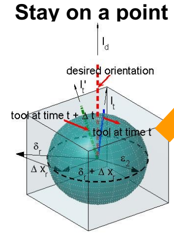
$$\text{s.t.} \quad A(\vec{x}(\vec{q}+\Delta\vec{q}), \vec{s}) \le \vec{b},$$

$$\vec{s}_{up} \ge s \ge \vec{s}_{low} \ge 0,$$

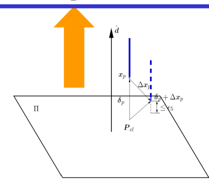
$$\Delta\vec{q}_{up} \ge \Delta\vec{q} \ge \Delta\vec{q}_{low}$$



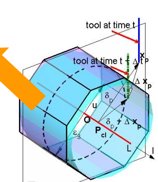
Move along a line



Maintain a direction



Prevent plane penetrating



Rotate around a line

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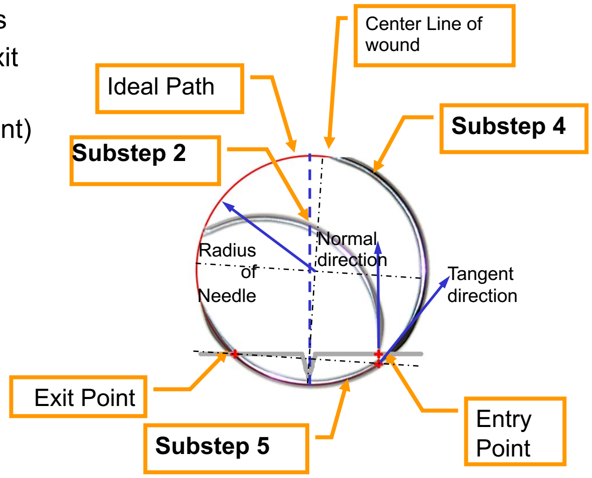
Example: Suturing

The suturing task involves
- Select entry and exit points
- Align (Move & Orient) Needle
- Bite: Pass Needle
- Loop
- Knot



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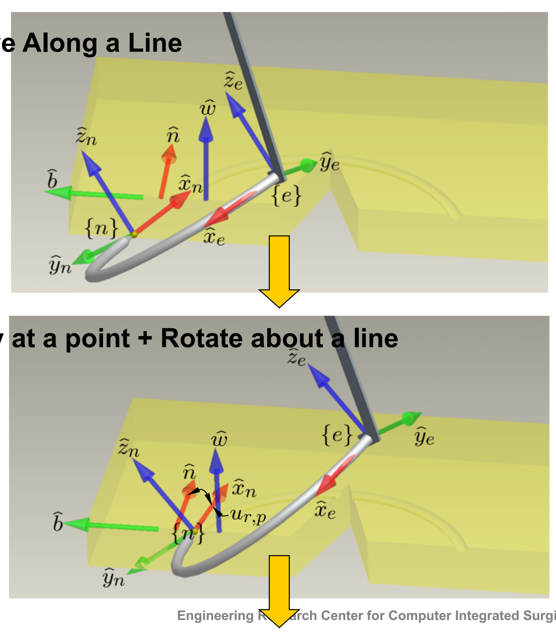
Suturing: Align Step

0. Move Along a Line

1. Stay at a point + Rotate about a line



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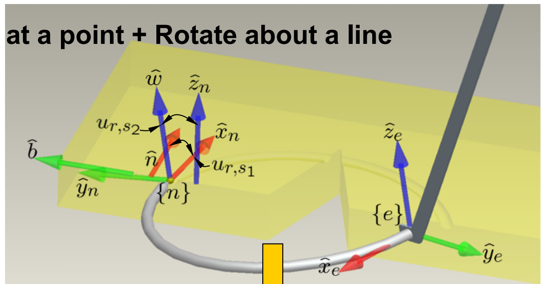
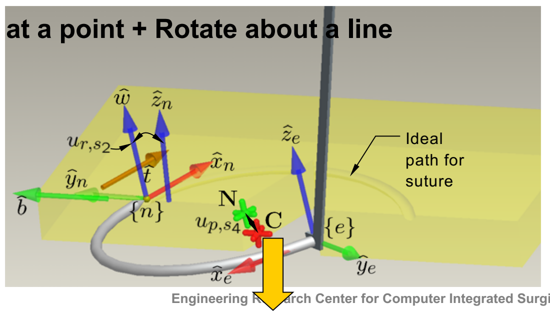
Suturing: Align Step

2. Stay at a point + Rotate about a line



3. Puncture

4. Stay at a point + Rotate about a line



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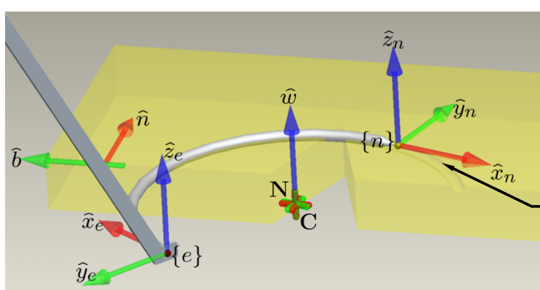
Suturing: Bite Step



Ideal path for suture

- Ideal trajectory is a circle with radius equal to needle radius.
- Needle plane is parallel to entry and exit points and surface normal.

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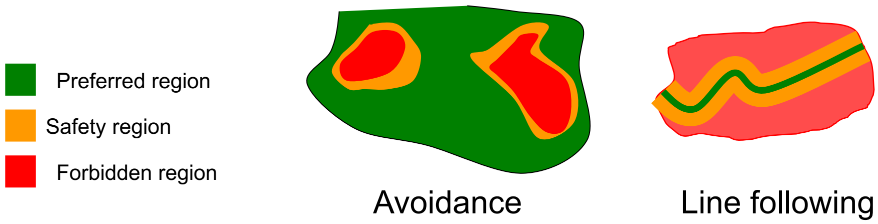
Hard and soft constraints



- Constraints on the task can be “hard” or “soft”
- The relative sizes depend on the procedure, ranging from micros to tenths of millimeter.
- Soft constraints allow the controller to accommodate uncertainties inherent in surgical procedures.

Thanks: **A. Kapoor**

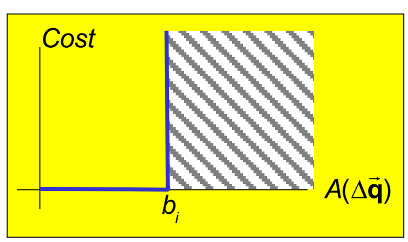
“Soft” constraint implementation

Suppose that we have a problem of the form

$$\Delta\vec{\mathbf{q}}_{des} = \arg\min \left\| E(\Delta\vec{\mathbf{q}}) \right\|^2$$

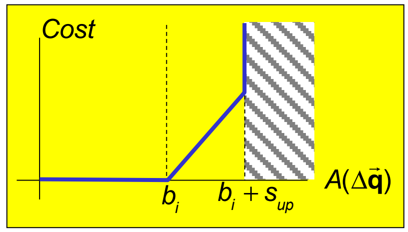
subject to a constraint of the form

$$A_i(\Delta\vec{\mathbf{q}}) \le b_i$$



"Soft" constraint implementation

But suppose we want to make the barrier "soft". I.e., allow the robot to go beyond the barrier at increasing cost until it hits a harder barrier later



Add an explicit slack s_i and add a penalty term to the objective function

$$\Delta\vec{\mathbf{q}}_{des} = \arg\min \left\|E(\Delta\vec{\mathbf{q}})\right\|^2 + \eta_i s_i^2$$

subject to a constraint of the form

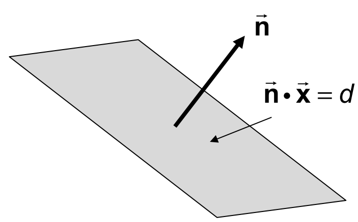
$$A_i(\Delta\vec{\mathbf{q}}) - s_i \leq b_i$$

$$0 \leq s_i \leq s_{up,i}$$

This process can be repeated several times to produce progressively steeper costs

Example

- Stay below a plane
- Move freely until get within 2 mm
- Increasing resistance as get close



$$\Delta\vec{\mathbf{q}}_{cmd} = \arg\min \ \eta_s s^2 + \sum_k \eta_k \left\|\cdots\text{other objective function terms}\right\|$$

such that

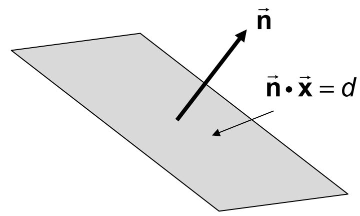
$$\vec{\mathbf{n}} \bullet (\vec{\mathbf{x}} + \Delta\vec{\mathbf{x}}) - s \leq d - 2$$

$$\Delta\vec{\mathbf{x}} = \mathbf{J}_{\vec{\mathbf{x}}}(\vec{\mathbf{q}})\Delta\vec{\mathbf{q}}$$

$$0 \leq s \leq 2$$

Example

- Stay below a plane
- Move freely until get within 2 mm
- Increasing resistance as get close



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such that

$$\vec{\mathbf{n}} \bullet \Delta\vec{\mathbf{x}} - s \le d - 2 - \vec{\mathbf{n}} \bullet \vec{\mathbf{x}}$$
$$\Delta\vec{\mathbf{x}} = \mathbf{J}_{\vec{\mathbf{x}}}(\vec{\mathbf{q}})\Delta\vec{\mathbf{q}}$$
$$0 \le s \le 2$$

Nonlinear Optimization

- One problem with linearized least squares is the proliferation of constraints to approximate the real constraints
- Consequently, it is worth considering alternatives that can handle more general formulas "directly"

$$\Delta\vec{\mathbf{q}}_{des} = \arg\min_{\Delta\vec{\mathbf{q}}} C(\Delta\vec{\mathbf{x}}, \Delta\vec{\mathbf{q}}, \vec{\mathbf{s}})$$

subject to

$$\Delta\vec{\mathbf{x}} = \mathbf{J}\Delta\vec{\mathbf{q}}$$
$$\mathbf{A}(\Delta\vec{\mathbf{x}}, \Delta\vec{\mathbf{q}}, \vec{\mathbf{s}}) \le \vec{\mathbf{b}}$$

Example: Stay near a point

Target Position: $\vec{x}_0$

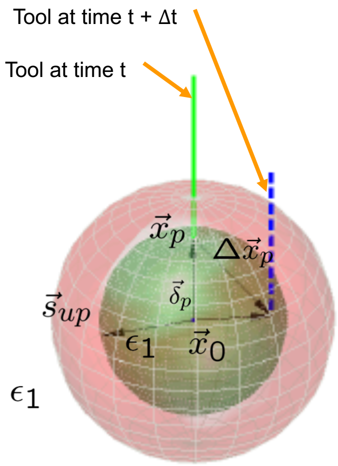
After incremental motion

$\vec{x}_p + \Delta\vec{x}_p$ close to $\vec{x}_0$

We want…

$$A(\vec{x}, s) = \|\vec{\delta}_p + \Delta\vec{x}_p\|^2 - s \le \epsilon_1$$

where $\vec{\delta}_p = \vec{x}_p - \vec{x}_0$



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Using Linear Constrained Quadratic Optimization

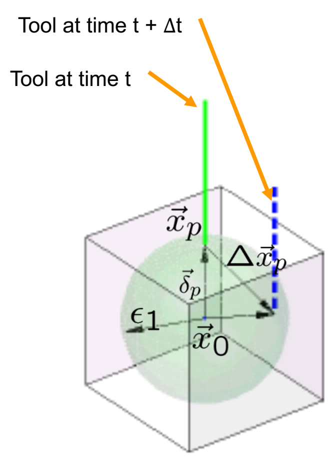
Matrix representation

$$A \cdot \Delta\vec{x} - s \le b$$

Use Constrained Least Squares to solve

$$\underset{\Delta\vec{q}}{\arg\min} \quad \|\Delta\vec{x} - \Delta\vec{x}^d\|^2$$

$$s.t \quad A \cdot \Delta\vec{x} - s \le b$$



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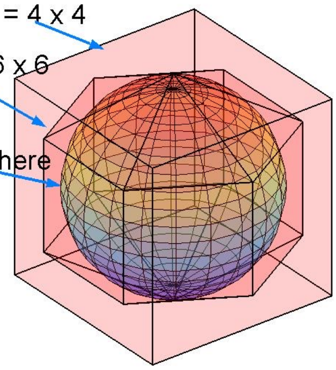
Linear approximation for constraints

- n x m increase
  - Polyhedron approaches the inscribed sphere
  - Linearized conditions are a better approximation
  - More constraints require more time to solve the optimization problem
- Symmetrical polyhedron
  - nxm = 4x4
- Bounded polyhedron
  - nxm = 3x3



A. Kapoor, *et al.*

Using Non-Linear Constrained Optimization

- Use Sequential Quadratic Program* method
- SQP solves the following problem iteratively

$$\mathbf{d}^{(k)} = \arg\min_{\mathbf{d}^{(k)}} \quad \nabla C\big(\mathbf{x}(\mathbf{q}+\Delta\mathbf{q}^{(k)}), \mathbf{s}^{(k)}, \mathbf{x}^d\big)^T \mathbf{d}^{(k)} + \frac{1}{2}\mathbf{d}^{(k)T}\mathbf{B}^{(k)}\mathbf{d}^{(k)}$$

$$\text{s.t.} \quad \nabla A_j\big(\mathbf{x}(\mathbf{q}+\Delta\mathbf{q}^{(k)}), \mathbf{s}^{(k)}\big)^T \mathbf{d}^{(k)} \le b_j; \quad j \in \mathcal{A}_k$$

- Start with a solution $[\Delta q^k, s^k]^t$
- Descent direction along with step size determine next solution $[\Delta q^{k+1}, s^{k+1}]^t$

*P. Spellucci, *Math. Prog.*, '98

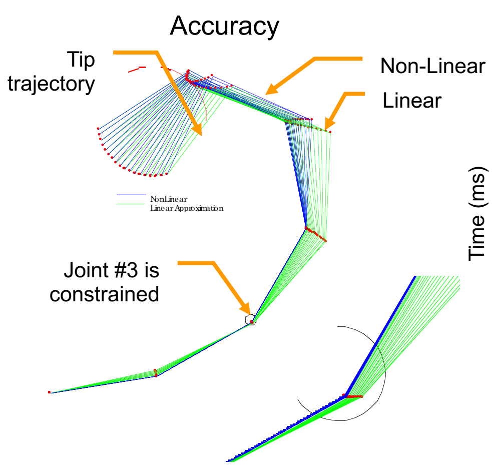
A. Kapoor, *et al.*

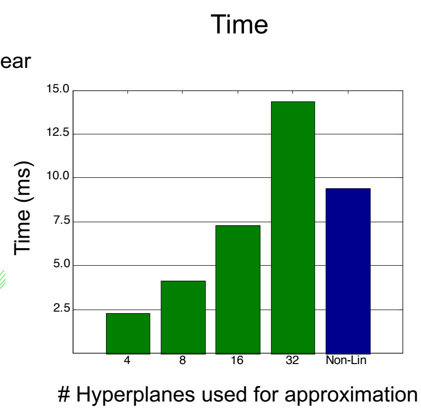
Remarks: Non-Linear Constraints

- Current incremental motion can be used as starting guess for next motion
- Worst case number of constraints n times m, n = # variables, m = # nonlinear constraints
- Analytical gradient increases speed

A. Kapoor, *et al.*

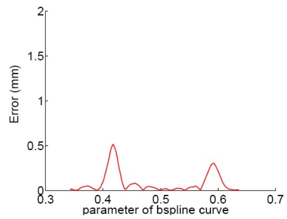
Linear v. Non-Linear Constraints



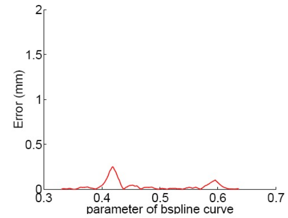


A. Kapoor, *et al.*

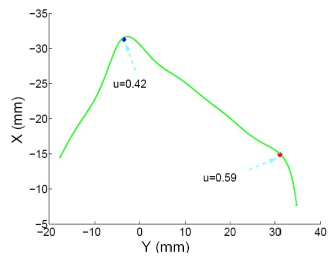
Effect of increasing control-loop time



Interval: 150ms



Interval: 40ms



- Large error at sharp turning
- Small interval reduces error

Ming Li et al., IROS '05

Longer Straight Line Motions

- In many cases, one wants to command a fairly long "straight line" motion from some initial pose to a final goal pose.
- This can be done fairly straightforwardly as follows:

$\mathbf{F}_0 =$ initial pose; $\mathbf{F}_G =$ goal pose;

$\dot{\theta}_{max} =$ max angular velocity; $v_{max} =$ max linear speed

Define $\mathbf{F}_{G0} = [\mathbf{R}_{G0}, \vec{\mathbf{p}}_{G0}]$ such that $\mathbf{F}_G \mathbf{F}_{G0} = \mathbf{F}_0$

Compute axis-angle representation for $\mathbf{R}_{G0} = Rot(\vec{\mathbf{n}}_{G0}, \theta_{G0})$

Compute $T_{move} = \max(\theta_{G0} / \dot{\theta}_{max}, \|\vec{\mathbf{p}}_{G0}\| / v_{max})$; $T_{left} = T_{move}$

while $T_{left} > 0$ do

  Wait for next time interval

  Perform housekeeping; input state ($\vec{\mathbf{q}}$, $\dot{\vec{\mathbf{q}}}$, forces, etc.)

  $T_{left} \leftarrow \max(T_{left} - \Delta T, 0); \lambda \leftarrow T_{left} / T_{max}$; $\mathbf{F}_T \leftarrow \mathbf{F}_G \bullet [\mathbf{R}(\vec{\mathbf{n}}_{G0}, \lambda\theta_{G0}), \lambda\vec{\mathbf{p}}_{G0}]$

  Set up optimization function to minimize $\left\| \mathbf{F}_T^{-1} \mathbf{F}(\vec{\mathbf{q}} + \Delta\vec{\mathbf{q}}) \right\|^2$

  Output velocity goal $\Delta\vec{\mathbf{q}} / \Delta T$

end

Longer Straight-Line Motions

To minimize $\left\| \mathbf{F}_T^{-1}\mathbf{F}(\vec{\mathbf{q}}+\Delta\vec{\mathbf{q}}) \right\|^2$ we actually want to try to make $\mathbf{F}_T \approx \mathbf{F}(\vec{\mathbf{q}}+\Delta\vec{\mathbf{q}}) = \mathbf{F}(\vec{\mathbf{q}})\Delta\mathbf{F}(\vec{\xi})$

in a least-squares sense, where $\vec{\xi} = \mathbf{J}_{kins}(\vec{\mathbf{q}})\Delta\vec{\mathbf{q}}$ and $\vec{\xi} = \left[\vec{\alpha}^T, \vec{\varepsilon}^T\right]^{T^{-1}}$

$$\Delta\mathbf{R} \approx \mathbf{R}(\vec{\mathbf{q}})^{-1}\mathbf{R}_T = Rot(\vec{\mathbf{n}}_{RT}, \vec{\theta}_{RT})$$
$$\vec{\mathbf{p}}_T \approx \mathbf{R}(\vec{\mathbf{q}})\vec{\varepsilon} + \mathbf{p}(\vec{\mathbf{q}})$$

This gives us the following minimization

$$\Delta\vec{\mathbf{q}} = \underset{\Delta\vec{\mathbf{q}}}{\operatorname{argmin}} \quad \nu_\alpha \left\| \vec{\alpha} - \vec{\theta}_{RT} \right\|^2 + \nu_\varepsilon \left\| \mathbf{R}(\vec{\mathbf{q}})\vec{\varepsilon} + \mathbf{p}(\vec{\mathbf{q}}) - \vec{\mathbf{p}}_T \right\|^2$$

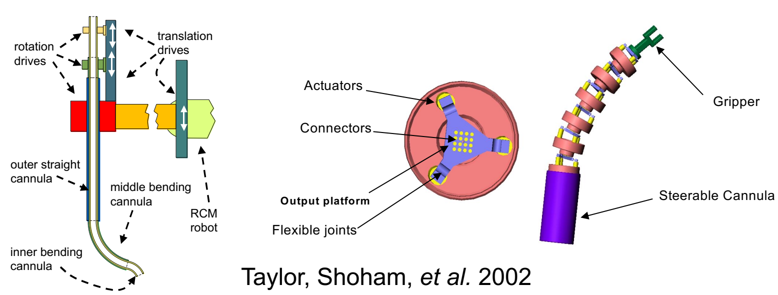
subject to

$\vec{\alpha} = \mathbf{J}_{kins}^{\alpha}(\vec{\mathbf{q}})\Delta\vec{\mathbf{q}} \quad \vec{\varepsilon} = \mathbf{J}_{kins}^{\varepsilon}(\vec{\mathbf{q}})\Delta\vec{\mathbf{q}}$

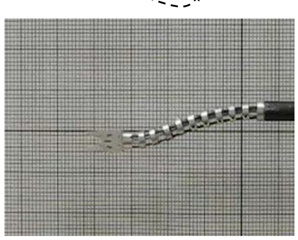
... other constraints such as joint limits, virtual fixture constraints, etc.

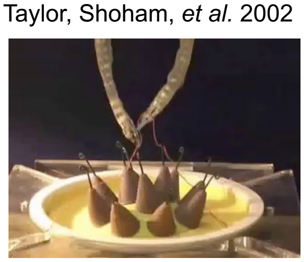
where ν_α and ν_ε can be used to control the relative importance of orientation and translation

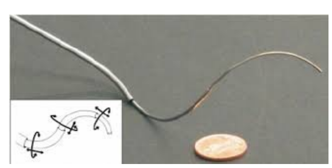
High Dexterity in a Small Package



Taylor, Shoham, *et al.* 2002



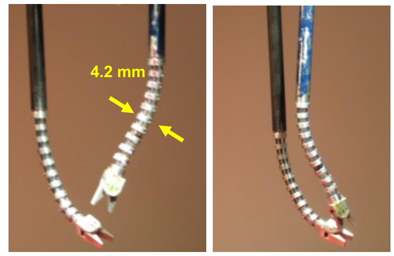


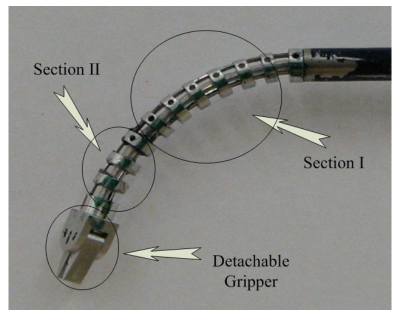


Webster *et al., 2006*

Simaan, Taylor *et al.* 2004, 2007

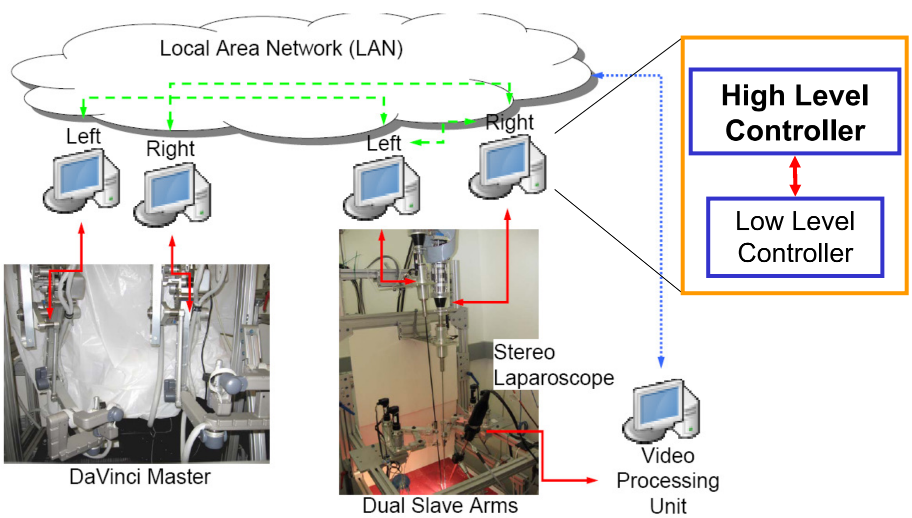
Scalable Robot for Dexterous Surgery in Small Spaces (aka Snake Like Robot)





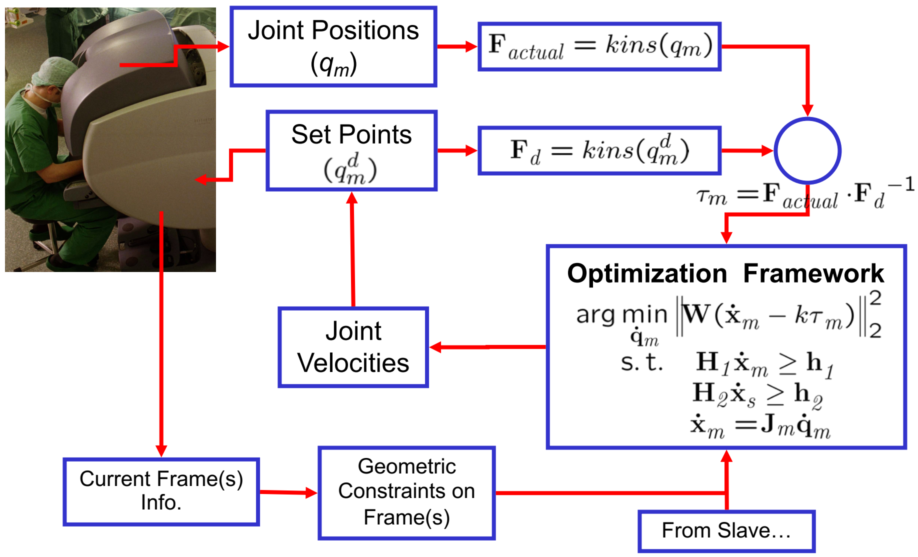
Team: A. Kapoor, Kai Xu, Wei Wei, N. Simaan, P. Kazanzides, R. H. Taylor
Collaborator: P. Flint, MD

Snake Like Robot
System Architecture

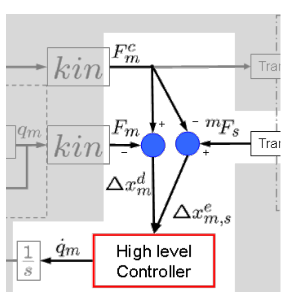


Master Tool Manipulator (MTM)

High Level Constrained Control



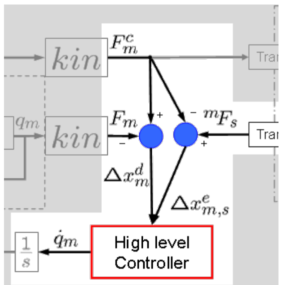
Master Side High-Level Controller



$$\min_{\Delta \vec{q}_m} \left\| W_{m,t} \left(\Delta x_m - K_a \left[p_m^e; \theta_m \omega_m \right] \right) \right\|$$
$$+ \left\| W_{m,s} \left(\Delta x_m - K_f \left[p_{m,s}^e; \theta_{m,s} \omega_{m,s} \right] \right) \right\|$$
$$+ \left\| W_{m,j} \Delta q_m \right\|$$

- **Objectives**:
  - Minimize error between desired motion and actual motion
  - Oppose motion that increases master-slave tracking error
  - Minimize the extraneous motion of the joints, and
  - Avoid large incremental joint motions that could occur near singularities

Master Side High-Level Controller



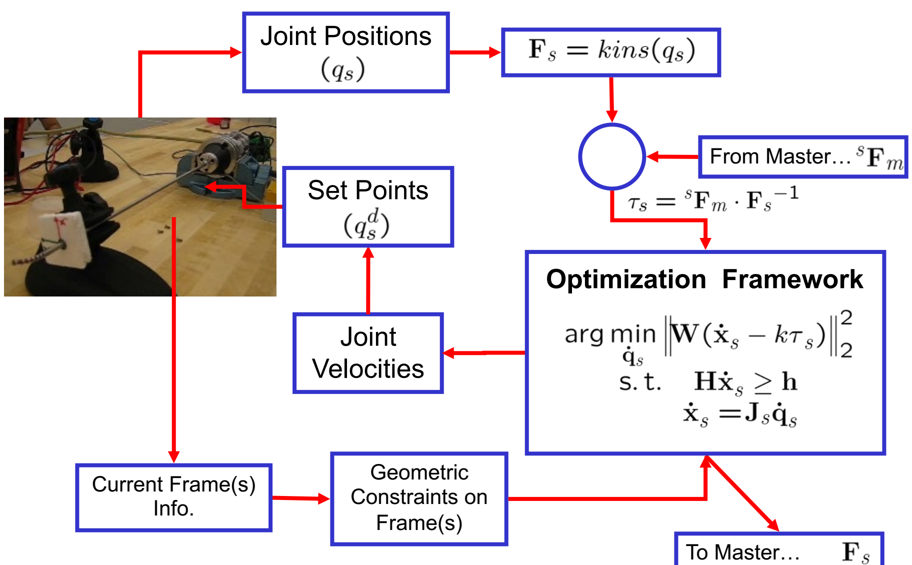
$$H_m \Delta q_m \geq h_m$$

that is $$\begin{bmatrix} I \\ -I \\ I \\ -I \end{bmatrix} \Delta q_m \geq \begin{bmatrix} q_{m,L} - q_m \\ q_m - q_{m,U} \\ \dot{q}_{m,U} \cdot \Delta t \\ \dot{q}_{m,U} \cdot \Delta t \end{bmatrix}$$

- **Constraints**:
  - General form: $H_{m,j}+\Delta q_m \geq h_{m,j}$
  - Not allow motion outside joint range
  - Not allow motion that exceeds joint velocity limits
  - Additional constraints can be added from the VF Library

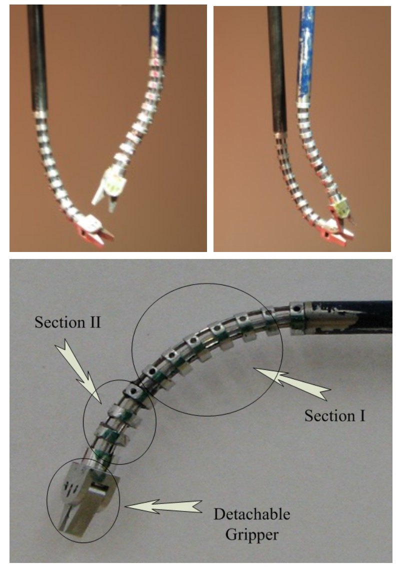
Patient-Side Manipulator (PSM)

High Level Constrained Control

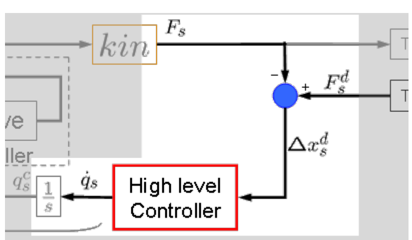


Patient-Side Snakes

- Actual snake section bends are a fairly complicated function of the linear displacements of the individual tubes and wires in the bending parts. But these displacements can be computed from the desired bending angles.
- Therefore, create pseudo-"joints" q_{sec1} and q_{sec2} corresponding to the bending angles in the two bend sections.
- Solve the optimization problem for q_{sec1} and q_{sec2} and the other joint angles of the slave robot. Then compute linear displacements from q_{sec1} and q_{sec2}. This also involves some calculations for redundancy resolution that can be done with a similar optimization method or can be done analytically.



Patient-Side High-Level Controller



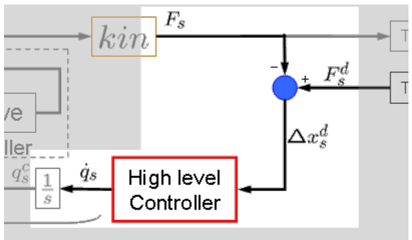
$$\min_{\Delta \vec{q}_s} \left\| W_{s,t} \left(\Delta x_s - K_a \left[p_s^e; \theta_s \omega_s \right] \right) \right\|$$

$$+ \left\| W_{s,j}(q) \Delta q_s \right\|$$

$$+ \left\| W_{s,} s \right\|$$

- **Objectives**:
  - Minimize error between desired motion and actual motion
  - Minimize the extraneous motion of the joints, and
  - Avoid large incremental joint motions that could occur near singularities

Patient-Side High-Level Controller



such that $\begin{bmatrix} I \\ -I \\ I \\ -I \end{bmatrix} \Delta q_s \geq \begin{bmatrix} q_{s,L} - q_s \\ q_s - q_{s,U} \\ \dot{q}_{s,U} \cdot \Delta t \\ \dot{q}_{s,U} \cdot \Delta t \end{bmatrix}$

and $\|\vec{d}\| + \Delta x_b \cdot \hat{d} + \vec{v} \cdot \hat{d} + s \geq d_{safe}$

$0 \leq s \leq s_{lim}$

- **Constraints**:
  - Not allow motion outside joint range
  - Not allow motion that exceeds joint velocity limits
  - **Collision avoidance between manipulators**
  - More constraints can be added from the VF Library