



NSF Engineering Research Center for
Computer Integrated Surgical Systems and
Technology



LABORATORY FOR
**Computational
Sensing + Robotics**
THE JOHNS HOPKINS UNIVERSITY

Registration – Part 1

600.455/655 Computer Integrated Surgery



100 YEARS
JOHNS HOPKINS ENGINEERING

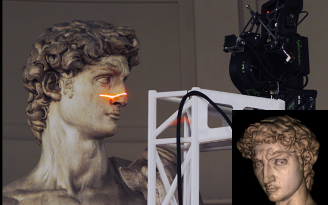
**WHITING
SCHOOL OF
ENGINEERING**
THE JOHNS HOPKINS UNIVERSITY

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John C. Malone Professor of Computer Science,
with joint appointments in Mechanical Engineering, Radiology & Surgery
Director, Laboratory for Computational Sensing and Robotics
The Johns Hopkins University
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Why is Registration Important?



Digitize important cultural artifacts




Medical interventions



Archeology

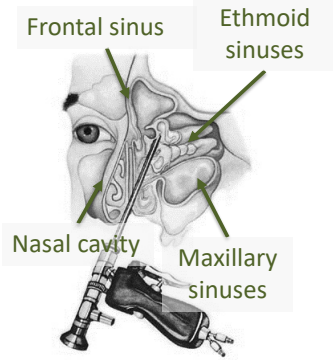
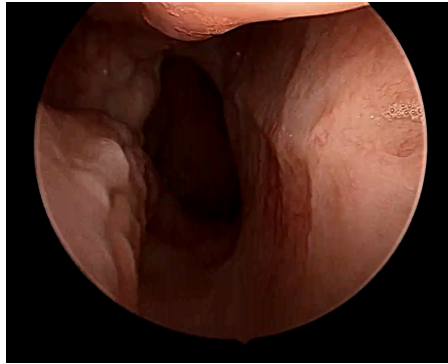
And many more applications...

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Why is Registration Important?



Typical Example: Sinus Endoscopy. The surgeon can only see video from the endoscope. But crucial data is in the CT about structures that cannot be seen.

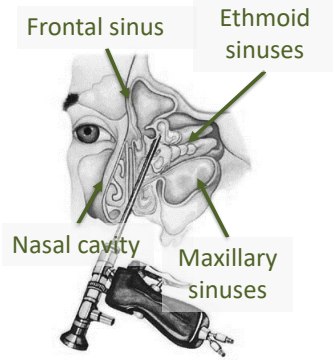
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Why is Registration Important?



Typical Example: Sinus Endoscopy. After registration, the computer can create video overlays, help guide a robot, or provide other assistance.

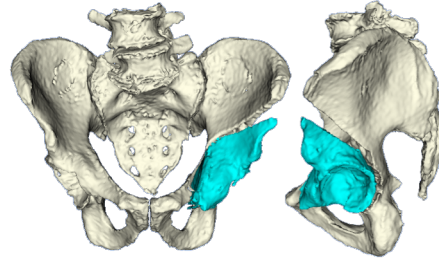
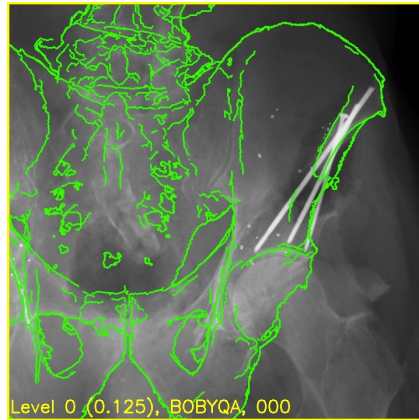
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Why is Registration Important?



Typical Example: Osteotomies. Surgeon needs to know the position and orientation of bone fragment relative to pelvis, based on x-ray images.

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What needs registering?

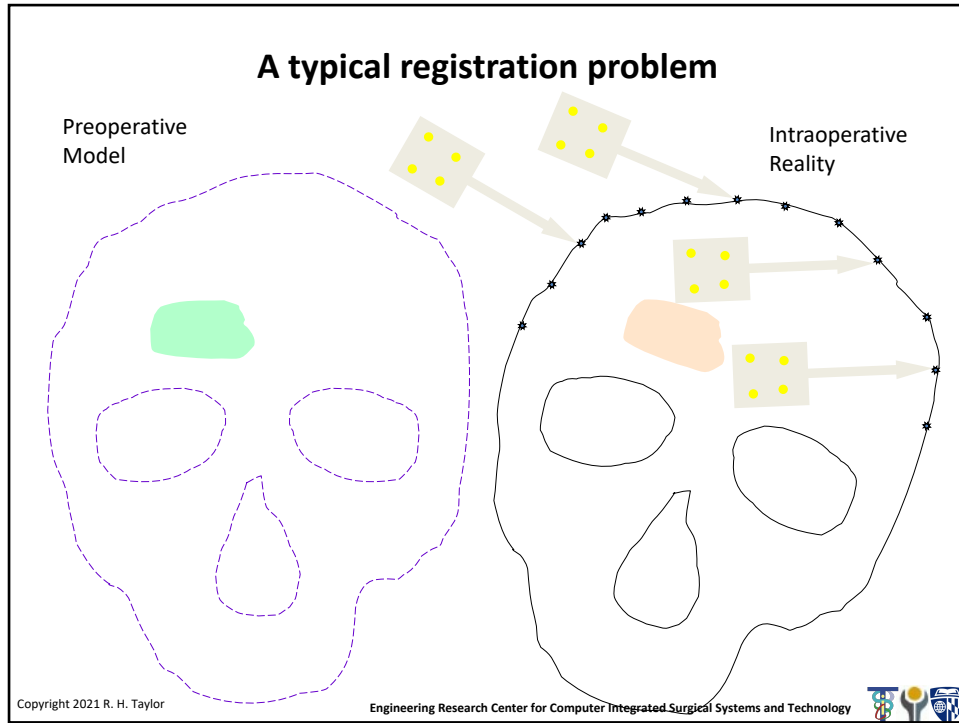
- **Preoperative Data**
 - 2D & 3D medical images
 - Models
 - Preoperative positions
- **Intraoperative Data**
 - 2D & 3D medical images
 - Models
 - Intraoperative positioning information
- **The Patient**

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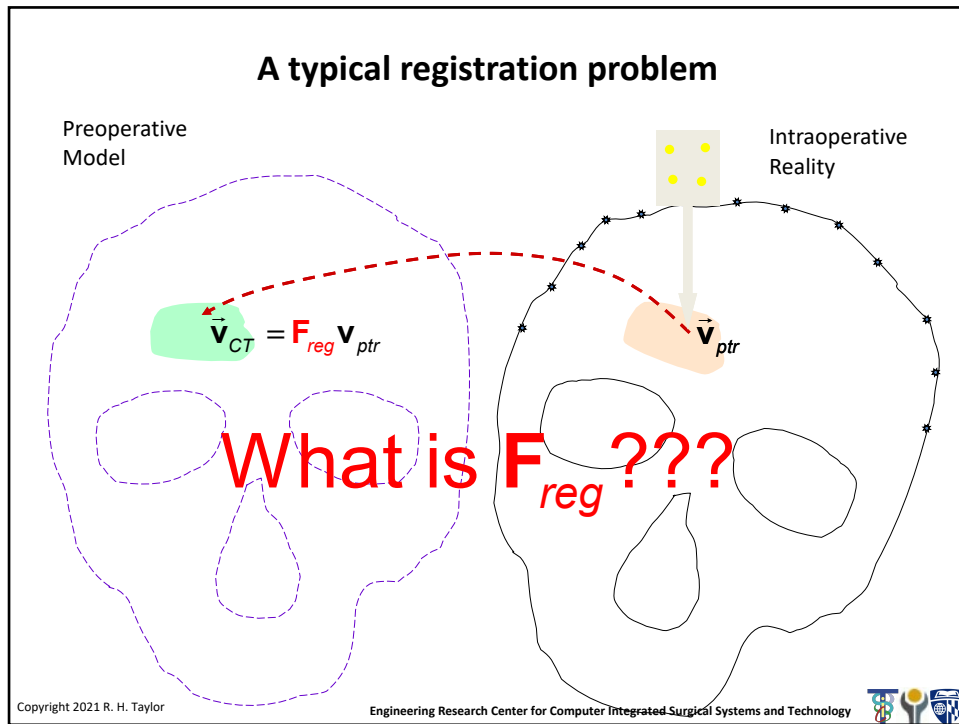
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Taxonomy of methods

- Feature-based
- Intensity-based

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Framework for feature-based methods

- Definition of coordinate system relations
- Segmentation of reference features
- Definition of disparity function between features
- Optimization of disparity function

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Definitions

Overall Goal: Given two coordinate systems,

Ref_A & Ref_B

and coordinates

x_A & x_B

associated with corresponding features in the two coordinate systems, the general goal is to determine a transformation function T that transforms one set of coordinates into the other:

$$x_A = T(x_B)$$

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Definitions

- **Rigid Transformation:** Essentially, our old friends 2D & 3D coordinate transformations:

$$T(x) = R \cdot x + p$$

The key assumption is that deformations may be neglected.

- **Similarity Transformation:** Essentially, rigid+scale change. Preserves angles and shape, but not size

$$T(x) = sR \cdot x + p$$

- **Elastic Transformation:** Cases where must take more general deformations into account. Many different flavors, depending on what is being deformed

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Uses of Rigid Transformations

- Register (approximately) multiple image data sets
- Transfer coordinates from preoperative data to reality (especially in orthopaedics & neurosurgery)
- Initialize non-rigid transformations

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Uses of Elastic Transformations

- Register different patients to common data base (e.g., for statistical analysis)
- Overlay atlas information onto patient data
- Study time-varying deformations
- Assist segmentation

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Typical Features

- Point fiducials
- Point anatomical landmarks
- Ridge curves
- Contours
- Surfaces
- Line fiducials

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Distance Functions

Given two (possibly distributed) features F_i and F_j , need to define a distance metric distance (F_i, F_j) between them. Some choices include:

- Minimum distance between points
- Maximum of minimum distances
- Area between line features
- Volume between surface features
- Area between point and line
- etc.

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Distance Functions Between Feature Sets

Let $\mathcal{F}_A = \{\dots F_{Ai} \dots\}$ and $\mathcal{F}_B = \{\dots F_{Bi} \dots\}$ be corresponding sets of features in \mathbf{Ref}_A and \mathbf{Ref}_B , respectively. We need to define an appropriate disparity function $D(\mathcal{F}_A, \mathcal{F}_B)$ between feature sets. Some typical choices include:

$$D = \sum_i w_i [\text{distance}(F_{Ai}, \mathbf{T}(F_{Bi}))]^2$$

$$D = \max_i \text{distance}(F_{Ai}, \mathbf{T}(F_{Bi}))$$

$$D = \text{median}_i \text{distance}(F_{Ai}, \mathbf{T}(F_{Bi}))$$

$$D = \text{Cardinality}\{i | \text{distance}(F_{Ai}, \mathbf{T}(F_{Bi})) > \text{threshold}\}$$

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Optimization

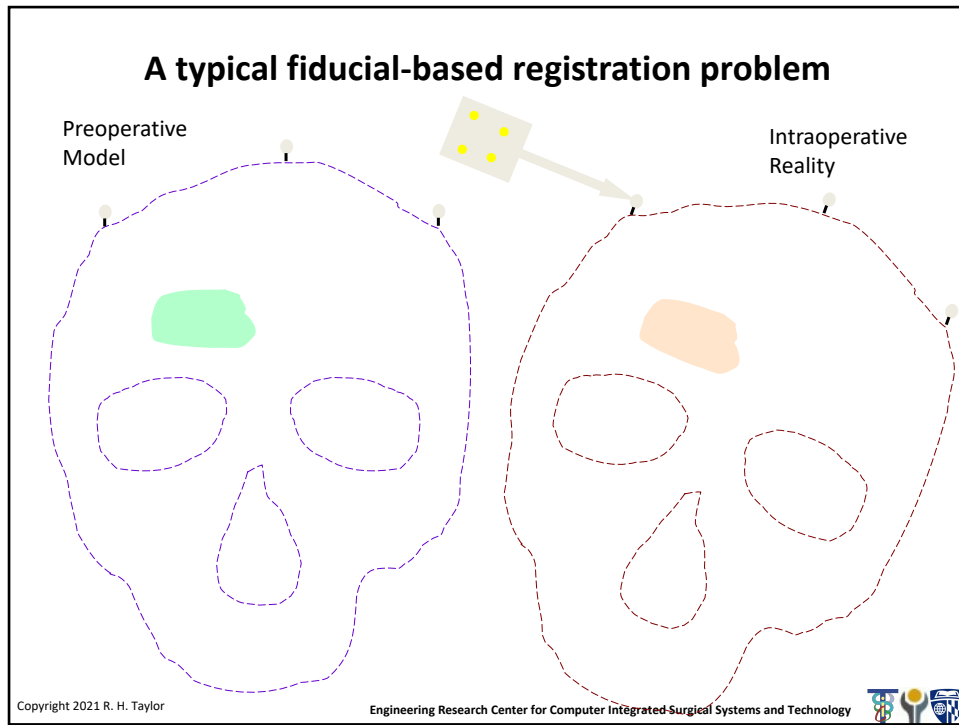
- Global vs Local
- Numerical vs Direct Solution
- Local Minima

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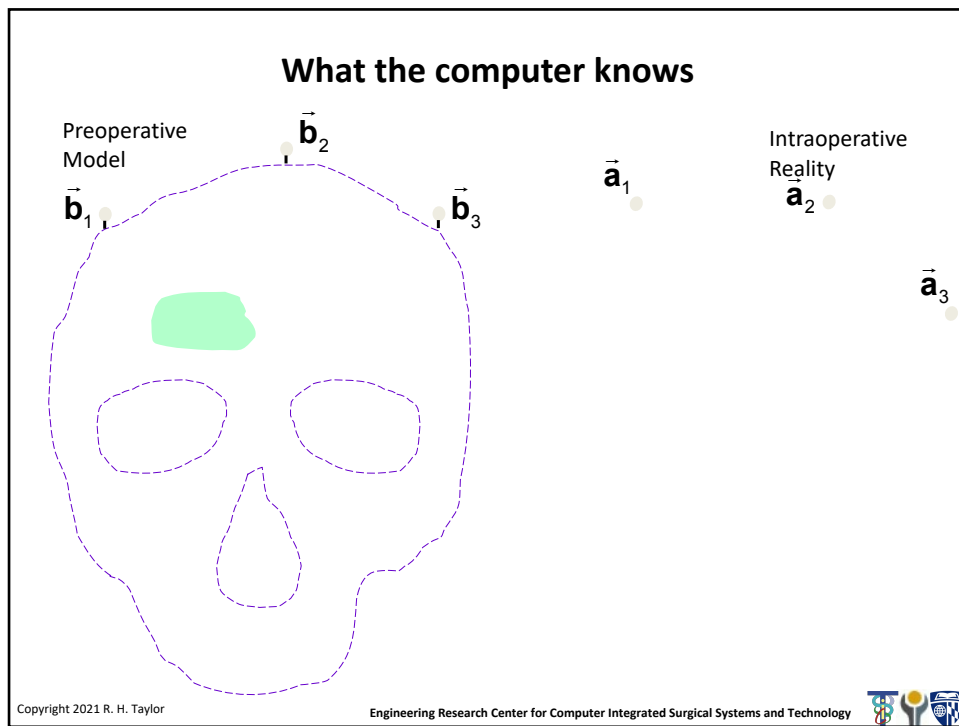
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Identify corresponding points

Preoperative Model

Intraoperative Reality

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Find best rigid transformation!

Preoperative Model

Intraoperative Reality

$$\min_{F_{reg}} \sum_i w_i D(F_{reg} \vec{a}_i, \vec{b}_i)$$

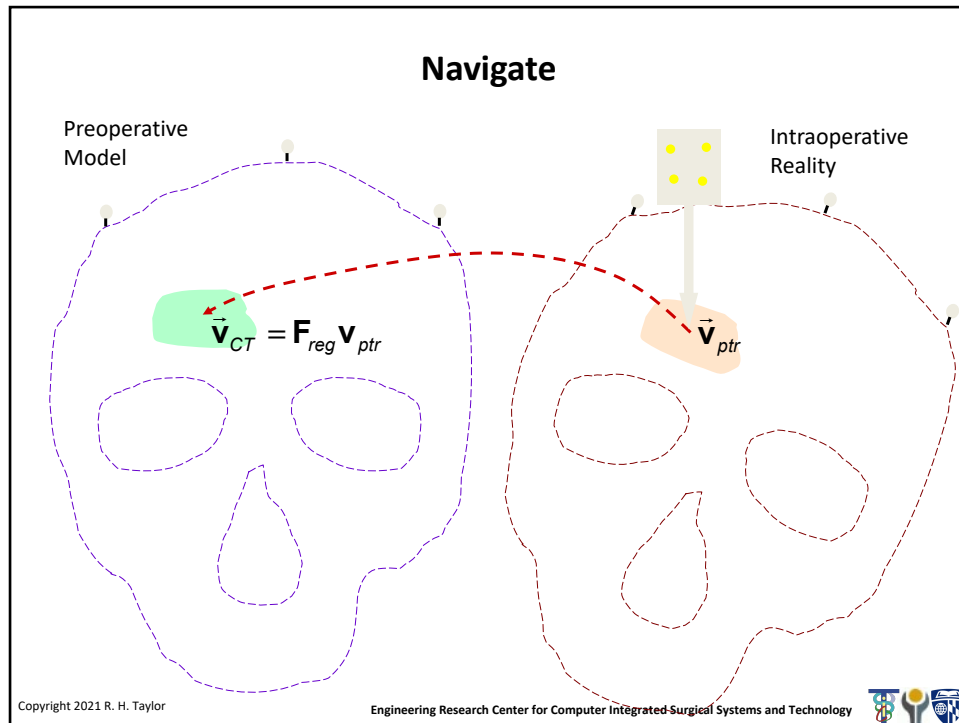
e.g.,

$$\min_{F_{reg}} \sum_i w_i \|F_{reg} \vec{a}_i - \vec{b}_i\|^2$$

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Sampled 3D data to surface models

Outline:

- Select large number of sample points
- Determine distance function $d_S(\mathbf{f}, \mathcal{F})$ for a point \mathbf{f} to a surface feature \mathcal{F} .
- Use d_S to develop disparity function D .

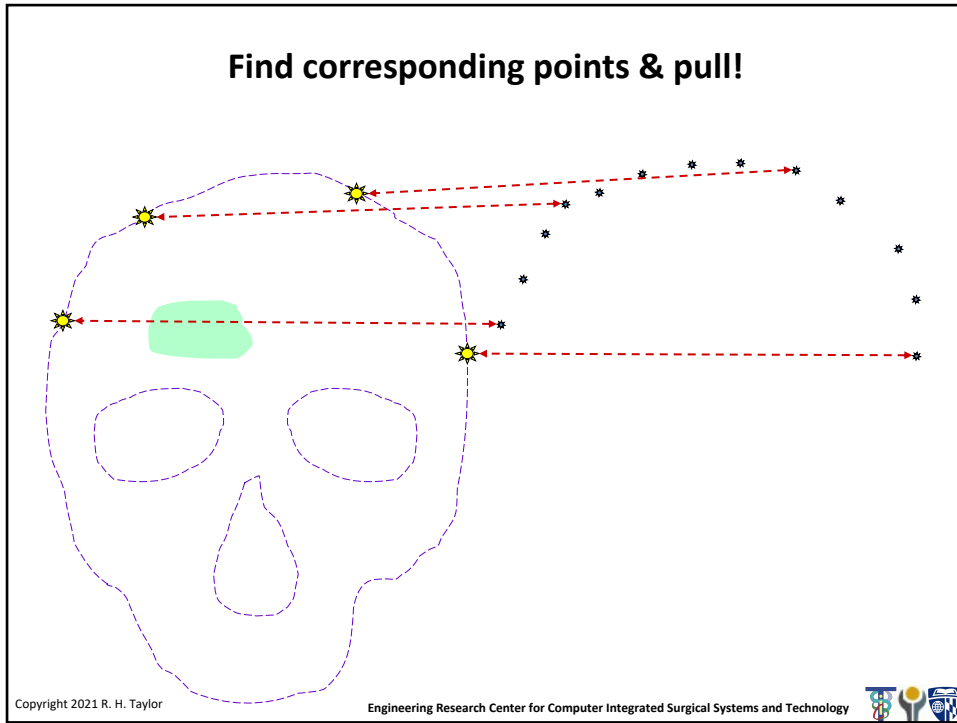
Examples

- Head-in-hat algorithm [Levin et al., 1988; Pelizzari et al., 1989]
- Distance maps [e.g., Lavallee et al.]
- Iterative closest point [Besl and McKay, 1992]

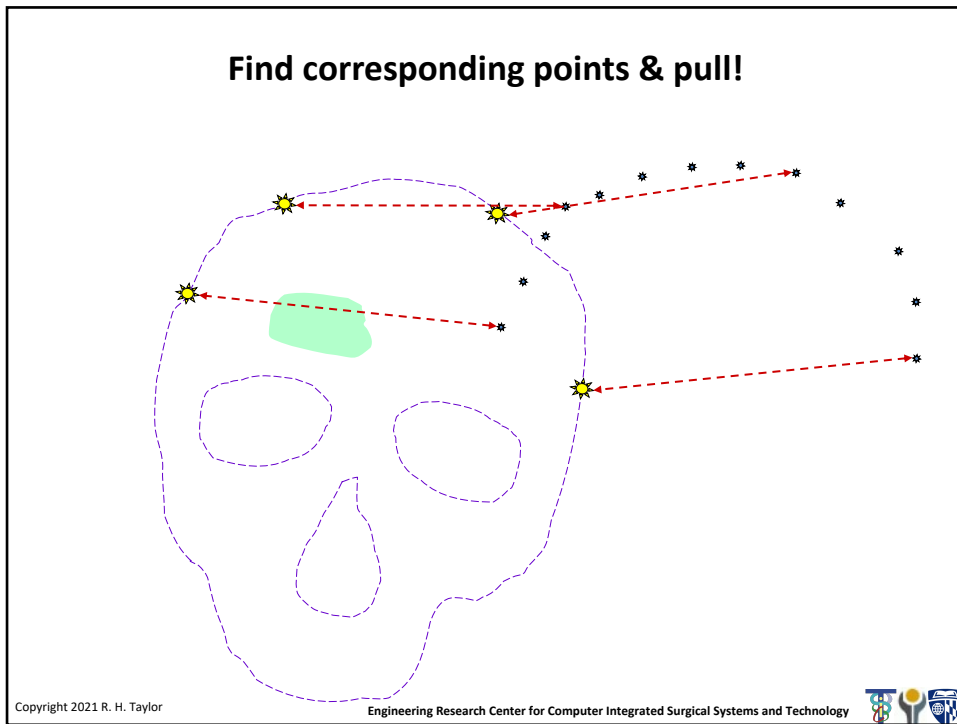
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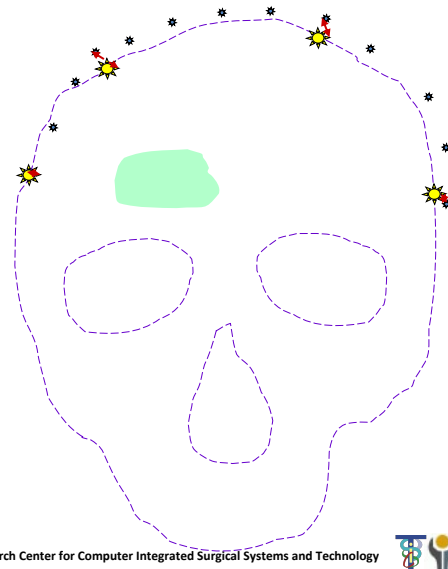
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Find corresponding points & pull!

Iterate this until converge

Find new point pairs every iteration

Key challenge is finding point pairs efficiently.



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Head in Hat Algorithm

- Levin et al, 1988; Pelizzari et al, 1989
- Originally used for Pet-to-MRI/CT registration
- Given $\mathbf{f}_i \in \mathcal{F}_A$, and a surface model \mathcal{F}_B , computes a rigid transformation \mathbf{T} that minimizes

$$D = \sum_i [d_S(\mathcal{F}_B, \mathbf{T} \cdot \mathbf{f}_i)]^2$$

where d_S is defined below, given a good initial guess for \mathbf{T} .

- Optimization uses standard numerical method (steepest gradient descent [Powell]) to find six parameters (3 rotations, 3 translations) defining \mathbf{T} .

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Iterative Closest Point

- Besl and McKay, 1992
- Start with an initial guess, \mathbf{T}_0 , for \mathbf{T} .
- At iteration k
 1. For each sampled point $\mathbf{f}_i \in \mathcal{F}_A$, find the point $\mathbf{v}_i \in \mathcal{F}_B$ that is closest to $\mathbf{T}_k \cdot \mathbf{f}_i$.
 2. Then compute \mathbf{T}_{k+1} as the transformation that minimizes

$$D_{k+1} = \sum_i \|\mathbf{v}_i - \mathbf{T}_{k+1} \cdot \mathbf{f}_i\|^2$$

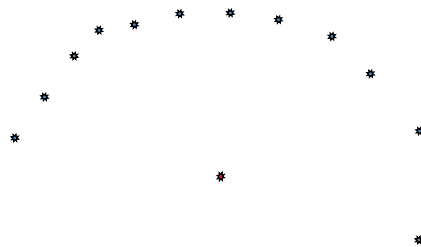
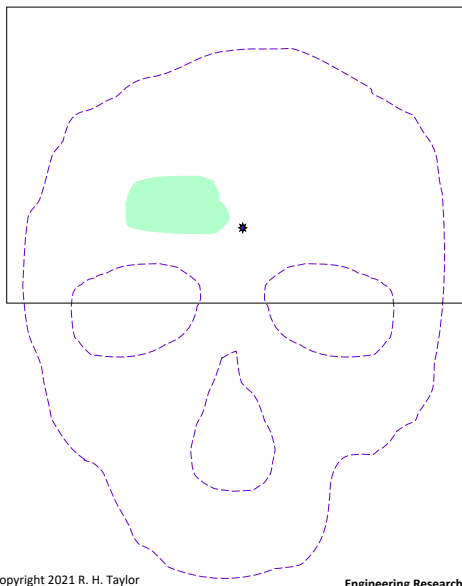
- Physical Analogy

Cop



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Iterative Closest Point: step 0

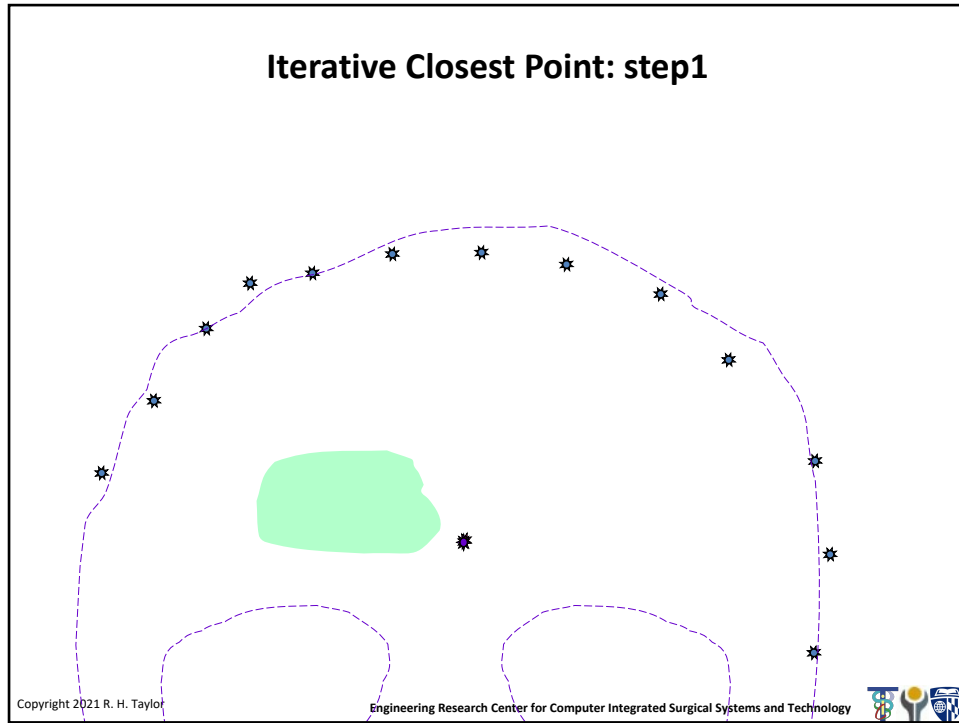


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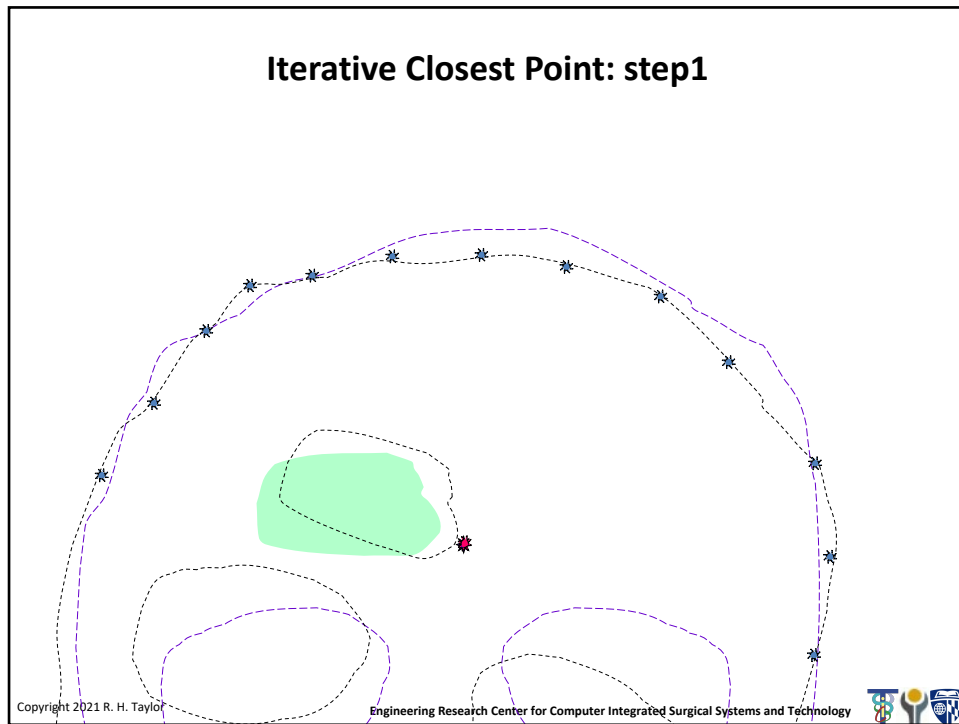
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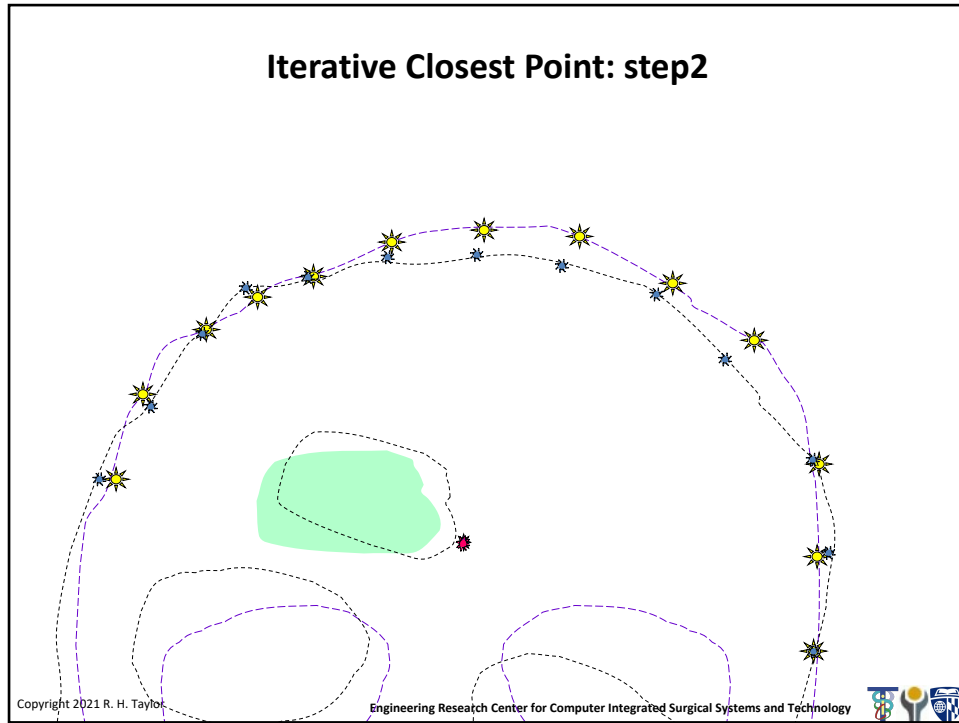
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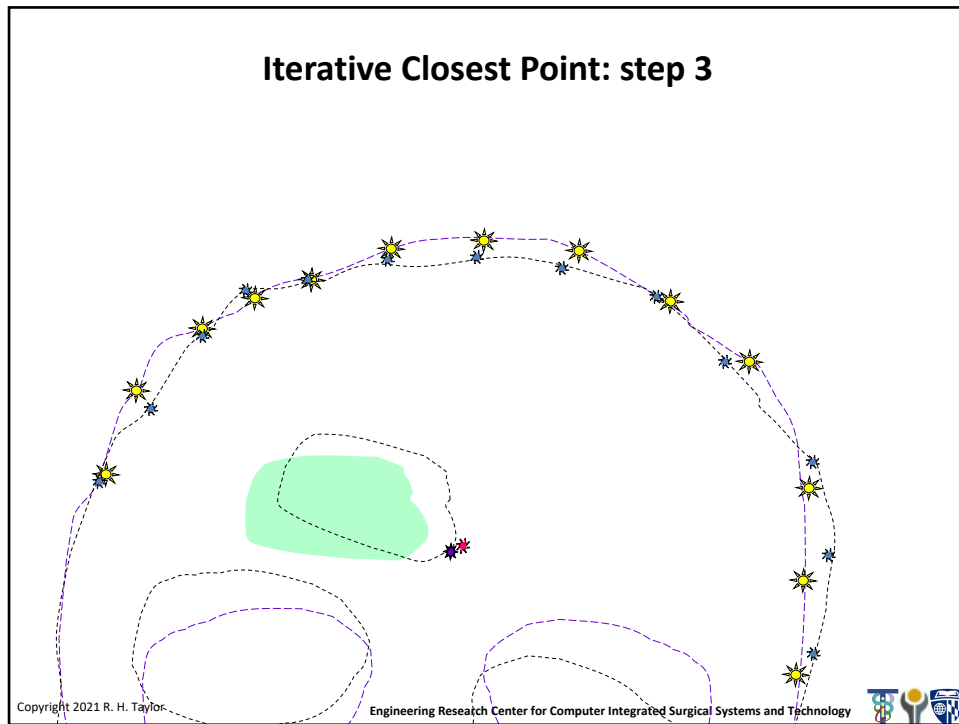
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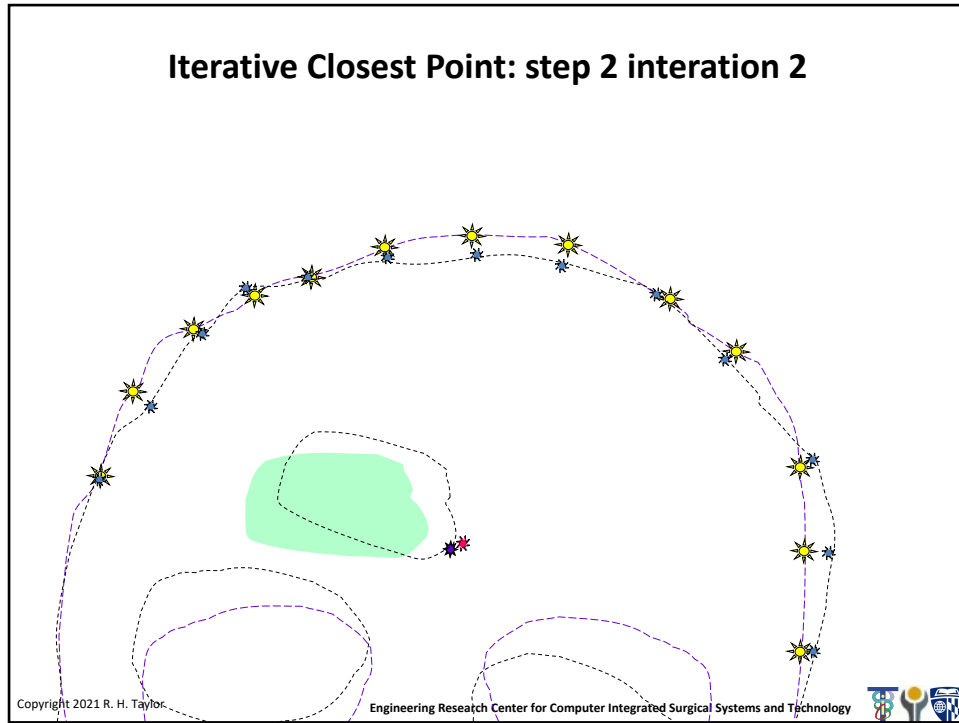
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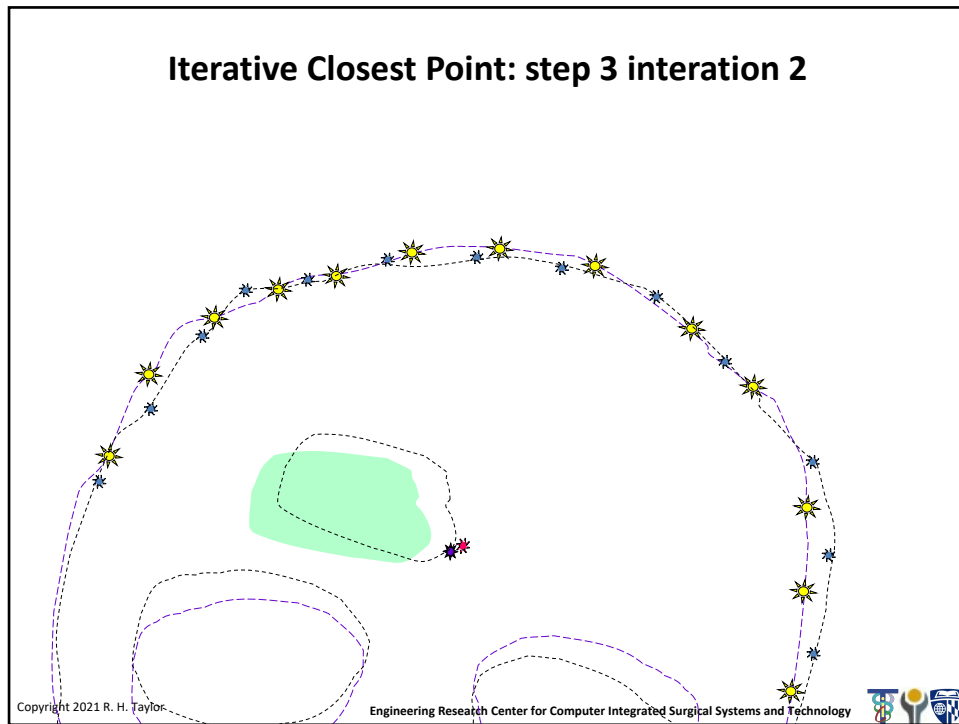
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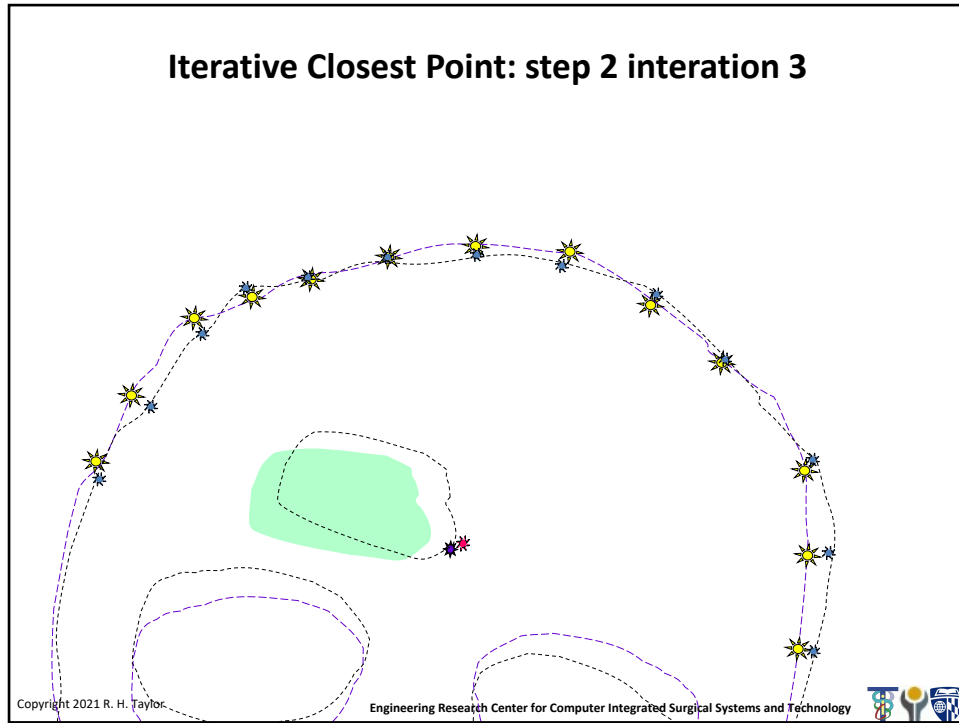
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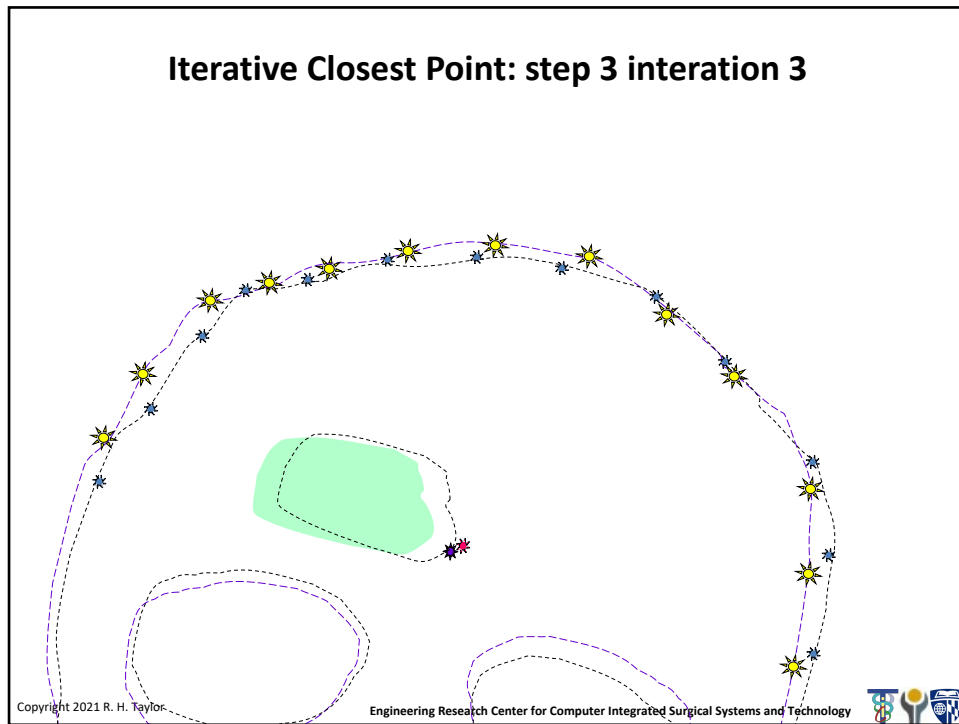
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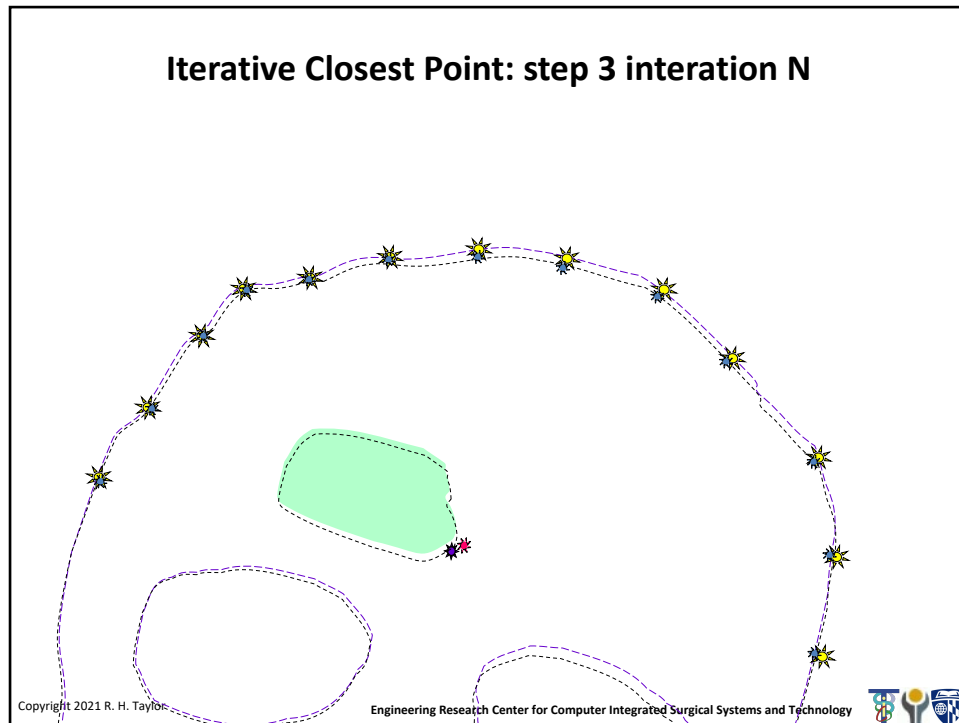
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Iterative Closest Point: Discussion

- Minimization step can be fast
- Crucially requires fast finding of nearest points
- Local minima still an issue
- Data overlap still an issue

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Digression: Finding Point Pairs

The diagram illustrates the process of finding point pairs for registration. On the left, a purple dashed outline of a skull is shown with a green shaded area on the forehead. Several yellow star-shaped points are marked on the skull's contour. On the right, a scatter plot of black star-shaped points is shown. Dashed red arrows connect the points on the skull to their corresponding points in the scatter plot, demonstrating the matching process. A red button with the text "Click here" is located in the bottom right corner of the diagram area.

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Robust Pose Estimation ...

- Basic idea is to identify outliers and give them little or no weight.

The diagram shows a cluster of blue star-shaped points on the left and a single yellow star-shaped point on the right. Dotted lines connect the points in the cluster to the outlier. A red dashed oval encircles the outlier, and the word "Outliers" is written in red below it.

Outliers

R. Kumar and A. R. Hanson, "Robust methods for estimating pose and a sensitivity analysis," *Comput. Vision, Graphics, Image Processing-1U*, vol. 60, no. 3, pp. 313-342, 1994.

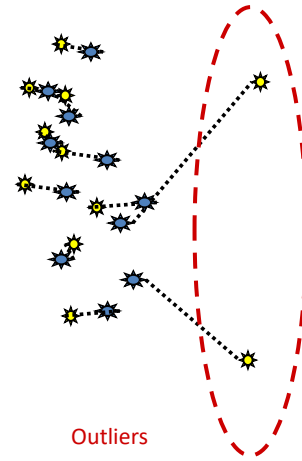
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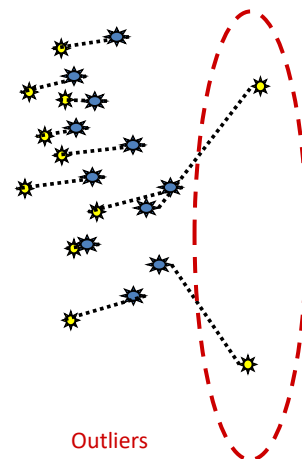
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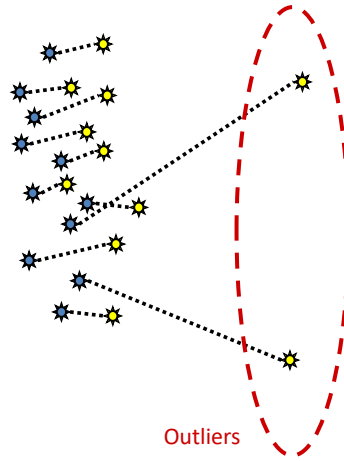
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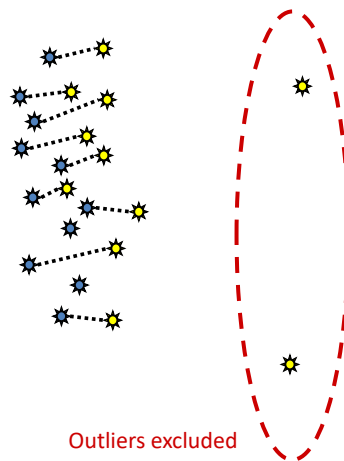
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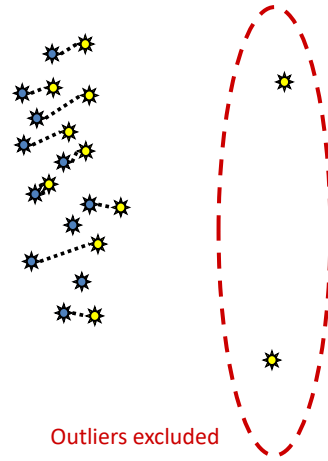
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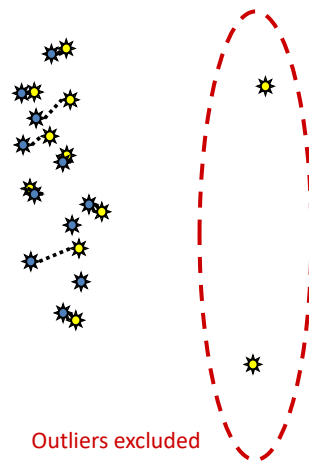
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Outline of a practical ICP code

Given

1. Surface model M consisting of triangles $\{m_i\}$
2. Set of points $Q = \{\bar{\mathbf{q}}_1, \dots, \bar{\mathbf{q}}_N\}$ known to be on M .
3. Initial guess \mathbf{F}_0 for transformation \mathbf{F}_0 such that the points $\mathbf{F}_0 \cdot \bar{\mathbf{q}}_k$ lie on M .
4. Initial threshold η_0 for match closeness

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Outline of a practical ICP code

Temporary variables

n	Iteration number
$\mathbf{F}_n = [\mathbf{R}, \bar{\mathbf{p}}]$	Current estimate of transformation
η_n	Current match distance threshold
$C = \{\dots, \bar{\mathbf{c}}_k, \dots\}$	Closest points on M to Q
$D = \{\dots, d_k, \dots\}$	Distances $d_k = \ \bar{\mathbf{c}}_k - \mathbf{F}_n \cdot \bar{\mathbf{q}}_k\ $
$I = \{\dots, i_k, \dots\}$	Indices of triangles m_{i_k} corresp. to $\bar{\mathbf{c}}_k$
$A = \{\dots, \bar{\mathbf{a}}_k, \dots\}$	Subset of Q with valid matches
$B = \{\dots, \bar{\mathbf{b}}_k, \dots\}$	Points on M corresponding to A
$E = \{\dots, \bar{\mathbf{e}}_k, \dots\}$	Residual errors $\bar{\mathbf{b}}_k - \mathbf{F}_n \cdot \bar{\mathbf{a}}_k$
$[\sigma_n, (\varepsilon_{\max})_n, \bar{\varepsilon}_n]$	$= \left[\frac{\sum_k \bar{\mathbf{e}}_k \cdot \bar{\mathbf{e}}_k}{\text{NumElts}(E)}; \max_k \sqrt{\bar{\mathbf{e}}_k \cdot \bar{\mathbf{e}}_k}; \frac{\sum_k \sqrt{\bar{\mathbf{e}}_k \cdot \bar{\mathbf{e}}_k}}{\text{NumElts}(E)} \right]$

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Outline of a practical ICP code

Step 0: (initialization)

Input surface model M and points Q .

Build an appropriate data structure (e.g., octree, kD tree) T
to facilitate finding the closest point matching search.

$n \leftarrow 0; \quad \eta_n \leftarrow \text{large number}$

$I \leftarrow \{\dots, 1, \dots\}$

$C \leftarrow \{\dots, \text{point on } m_1, \dots\}$

$D \leftarrow \{\dots, \|\bar{\mathbf{c}}_k - \mathbf{F}_0 \cdot \bar{\mathbf{q}}_k\|, \dots\}$

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Outline of a practical ICP code

Step 1: (matching)

$A \leftarrow \emptyset; \quad B \leftarrow \emptyset$

For $k \leftarrow 1$ step 1 to N do

begin

$bnd_k = \|\mathbf{F}_n \cdot \bar{\mathbf{q}}_k - \bar{\mathbf{c}}_k\|$

$[\bar{\mathbf{c}}_k, i, d_k] \leftarrow \text{FindClosestPoint}(\mathbf{F}_n \cdot \bar{\mathbf{q}}_k, \bar{\mathbf{c}}_k, i_k, bnd_k, T);$

// Note: develop first with simple

// search. Later make more

// sophisticated, using T

if ($d_k < \eta_n$) then { put $\bar{\mathbf{q}}_k$ into A ; put $\bar{\mathbf{c}}_k$ into B ; };

// See also subsequent notes

end

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Outline of a practical ICP code

Step 1: (matching)

$A \leftarrow \emptyset; B \leftarrow \emptyset$

For $k \leftarrow 1$ step 1 to N do

begin

$$bnd_k = \|\mathbf{F}_n \cdot \vec{\mathbf{q}}_k - \vec{\mathbf{c}}_k\|$$

$[\vec{\mathbf{c}}_k, i, d_k] \leftarrow \text{Find}$

Note: If using a tree search, you can use
// previous match to get a reasonable initial

// bound. E.g.,

$$// \quad bnd_k = \|\vec{\mathbf{c}}_k - \mathbf{F}_n \cdot \vec{\mathbf{q}}_k\|$$

if ($d_k < \eta_n$) then

and then pass that to the tree search.

// Alternatively, you can find the closest point

end

on triangle i_k and use that to get an initial
bound bnd_k for the search

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Outline of a practical ICP code

Step 2: (transformation update)

$n \leftarrow n + 1$

$\mathbf{F}_n \leftarrow \text{FindBestRigidTransformation}(A, B)$

$$\sigma_n \leftarrow \frac{\sqrt{\sum_k \vec{\mathbf{e}}_k \cdot \vec{\mathbf{e}}_k}}{\text{NumElts}(\mathbf{E})}; \quad (\epsilon_{\max})_n \leftarrow \max_k \sqrt{\vec{\mathbf{e}}_k \cdot \vec{\mathbf{e}}_k}; \quad \bar{\epsilon}_n \leftarrow \frac{\sum_k \sqrt{\vec{\mathbf{e}}_k \cdot \vec{\mathbf{e}}_k}}{\text{NumElts}(\mathbf{E})}$$

Step 3: (adjustment)

Compute η_n from $\{\eta_0, \dots, \eta_{n-1}\}$ // see notes next page

// May also update \mathbf{F}_n from $\{\mathbf{F}_0, \dots, \mathbf{F}_n\}$ (see Besl & McKay)

Step 4: (iteration)

if TerminationTest($\{\sigma_0, \dots, \sigma_n\}, \{(\epsilon_{\max})_0, \dots, (\epsilon_{\max})_n\}, \{\bar{\epsilon}_0, \dots, \bar{\epsilon}_n\}$)

then stop. Otherwise, go back to step 1 // see notes

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Outline of practical ICP code

Threshold η_n update

The threshold η_n can be used to restrict the influence of clearly wrong matches on the computation of \mathbf{F}_n . Generally, it should start at a fairly large value and then decrease after a few iterations. One not unreasonable value might be something like $3\bar{\epsilon}_n$. If the number of valid matches begins to fall significantly, one can increase it adaptively. Too tight a bound may encourage false minima

Also, if the mesh is incomplete, it may be advantageous to exclude any matches with triangles at the edge of the mesh.

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Outline of practical ICP code

Termination test

There are no hard and fast rules for deciding when to terminate the procedure. One criterion might be to stop when $\sigma_n, \bar{\epsilon}_n$ and/or $(\epsilon_{\max})_n$ are less than desired thresholds and $\gamma \leq \frac{\bar{\epsilon}_n}{\bar{\epsilon}_{n-1}} \leq 1$ for some value γ (e.g., $\gamma \cong .95$) for several iterations.

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Short further note: ICP related methods

- There is an extensive literature on methods based on ideas similar to ICP. Surveys and tutorials describing some of them may be found at
 - http://www.cs.princeton.edu/~smr/papers/fasticp/fasticp_paper.pdf
 - http://www.mrpt.org/Iterative_Closest_Point_%28ICP%29_and_other_matching_algorithms
- There are also a number of methods that incorporate a probabilistic framework. One example is the “Generalized-ICP” method of Segal, Haehnel, and Thrun
 - Aleksandr V. Segal, Dirk Haehnel, and Sebastian Thrun, “Generalized-ICP”, in *Robotics: Science and Systems*, 2009.
 - http://www.robots.ox.ac.uk/~avsegal/resources/papers/Generalized_ICP.pdf

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Typical Generalized ICP Algorithm

Outline below is based mostly on from paper by A. Segal, D. Haehnel, and S. Thrun, “Generalized-ICP”, in *Robotics: Science and Systems*, 2009.

```

n ← 0; initialize  $F_0$ , threshold value  $\eta_0$ , distribution parameters  $\Phi$ 
Step 1: (matching)
A ← ∅; B ← ∅
For k ← 1 step 1 to N do
  begin
    [ $\vec{c}_k, i_k, d_k$ ] ← FindClosestPoint( $F_n \cdot \vec{q}_k, \vec{c}_k, i_k, T$ );
    if ( $d_k < \eta_n$ ) then { put  $\vec{q}_k$  into A; put  $\vec{c}_k$  into B; };
    \ alternative: test if  $prob(\vec{q}_k \sim \vec{c}_k) > \eta_n$ 
  end
Step 2: (transformation update)
n ← n + 1
 $F_n \leftarrow \underset{F}{\operatorname{argmax}} \operatorname{prob}(F \cdot A \sim B; \Phi) = \underset{F}{\operatorname{argmax}} \prod_i \operatorname{prob}(F \cdot \vec{a}_i \sim \vec{b}_i; \Phi)$ 
=  $\underset{F}{\operatorname{argmin}} \sum_i -\log \operatorname{prob}(F \cdot \vec{a}_i \sim \vec{b}_i; \Phi)$ 
Step 3: (adjustment)
  update threshold  $\eta_n$  and distribution parameters  $\Phi$ 
Step 4: (iteration)
  if TerminationTest(...) then stop. Otherwise, go back to step 1 // see notes

```

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Related concept: Estimation with Uncertainty

Suppose you know something about the uncertainty of the sample data at each point pair (e.g., from sensor noise and/or model error). I.e.,

$$\vec{\mathbf{a}}_k \in \mathbf{A}_k; \vec{\mathbf{b}}_k \in \mathbf{B}_k; \text{cov}(\mathbf{A}_k, \mathbf{B}_k) = \mathbf{C}_k = \mathbf{Q}_k \Lambda_k \mathbf{Q}_k^T$$

Then an appropriate distance metric is the Mahalabonis distance

$$D(\vec{\mathbf{a}}_k, \vec{\mathbf{b}}_k) = (\vec{\mathbf{a}}_k - \vec{\mathbf{b}}_k)^T \mathbf{C}_k^{-1} (\vec{\mathbf{a}}_k - \vec{\mathbf{b}}_k) = \vec{\mathbf{d}}_k^T \Lambda_k^{-1} \vec{\mathbf{d}}_k$$

where

$$\vec{\mathbf{d}}_k = \mathbf{Q}_k^T (\vec{\mathbf{a}}_k - \vec{\mathbf{b}}_k)$$

This approach is readily extended to the case where the samples are not independent.

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Distance Maps

- Many authors
- Somewhat related to ICP and also to level sets
- Basic idea is to precompute the distance to the surface for a dense sampling of the volume.
- Then use the gradient of the distance map to compute an incremental motion that reduces the sum of the distances of all the moving points to the surface.
- Then iterate

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Distance Maps

There are a number of very fast algorithms for computing the Euclidean Distance Transform (distance to surface of each point in an image at each point in a 3D volume grid). One example is:

J. C. Torelli, R. Fabbri, G. Travieso, and O. Bruno, "A High Performance 3D Exact Euclidean Distance Transform Algorithm for Distributed Computing", *International Journal of Pattern Recognition and Artificial Intelligence*, vol. 24- 6, pp. 897-915, 2010.

But a web search will disclose many others, together with open source code

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Distance Maps

Given

a current registration transformation \mathbf{F}

Euclidean distance map $d(\vec{\mathbf{p}})$

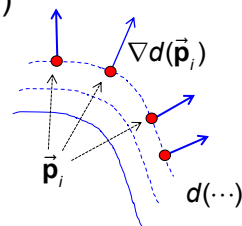
For each sample point $\vec{\mathbf{f}}_i$ compute $\vec{\mathbf{p}}_i = \mathbf{F} \cdot \vec{\mathbf{f}}_i$

Compute a small motion $\Delta\mathbf{F}$

$$\Delta\mathbf{F} = \operatorname{argmin}_{\Delta\mathbf{F}} \sum_i (\Delta\mathbf{F} \cdot \vec{\mathbf{p}}_i - \vec{\mathbf{p}}_i) \cdot \nabla d(\vec{\mathbf{p}}_i)$$

Update $\mathbf{F} \leftarrow \Delta\mathbf{F} \cdot \mathbf{F}$

Iterate



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Distance Maps

Given

a current registration transformation \mathbf{F}
 Euclidean distance map $d(\vec{\mathbf{p}})$

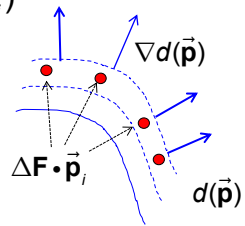
For each sample point $\vec{\mathbf{f}}_i$ compute $\vec{\mathbf{p}}_i = \mathbf{F} \cdot \vec{\mathbf{f}}_i$

Compute a small motion $\Delta\mathbf{F}$

$$\Delta\mathbf{F} = \operatorname{argmin}_{\Delta\mathbf{F}} \sum_i (\Delta\mathbf{F} \cdot \vec{\mathbf{p}}_i - \vec{\mathbf{p}}_i) \cdot \nabla d(\vec{\mathbf{p}}_i)$$

Update $\mathbf{F} \leftarrow \Delta\mathbf{F} \cdot \mathbf{F}$

Iterate



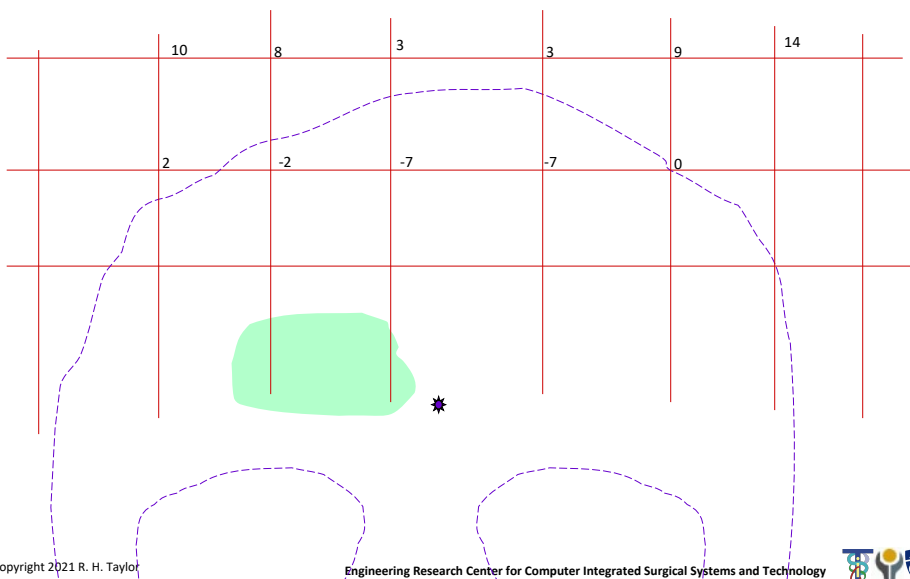
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Distance Maps: step 0

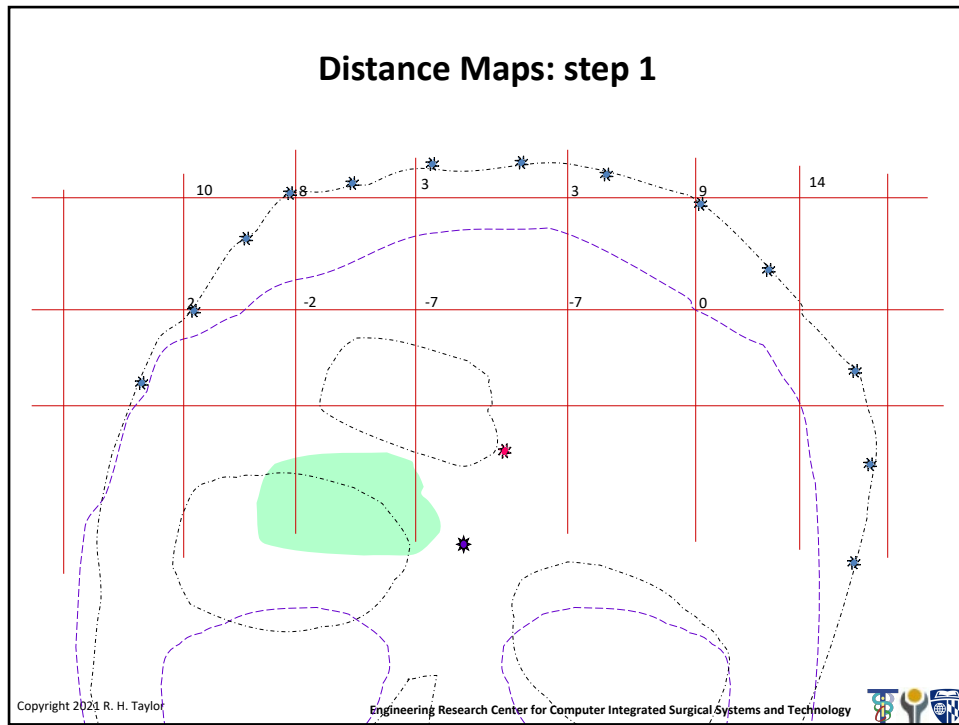


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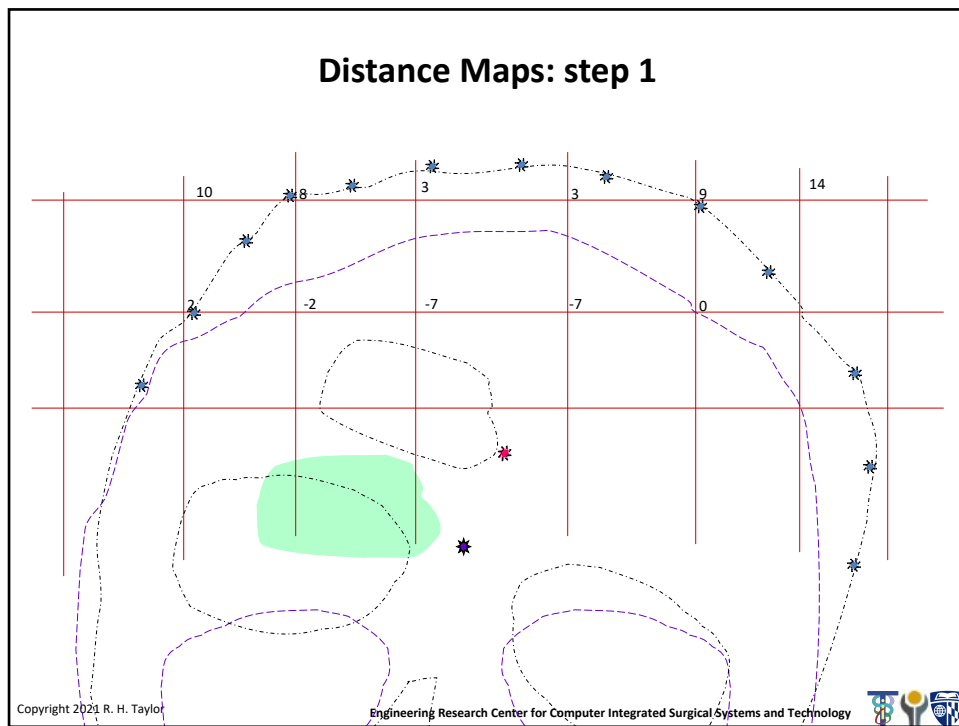
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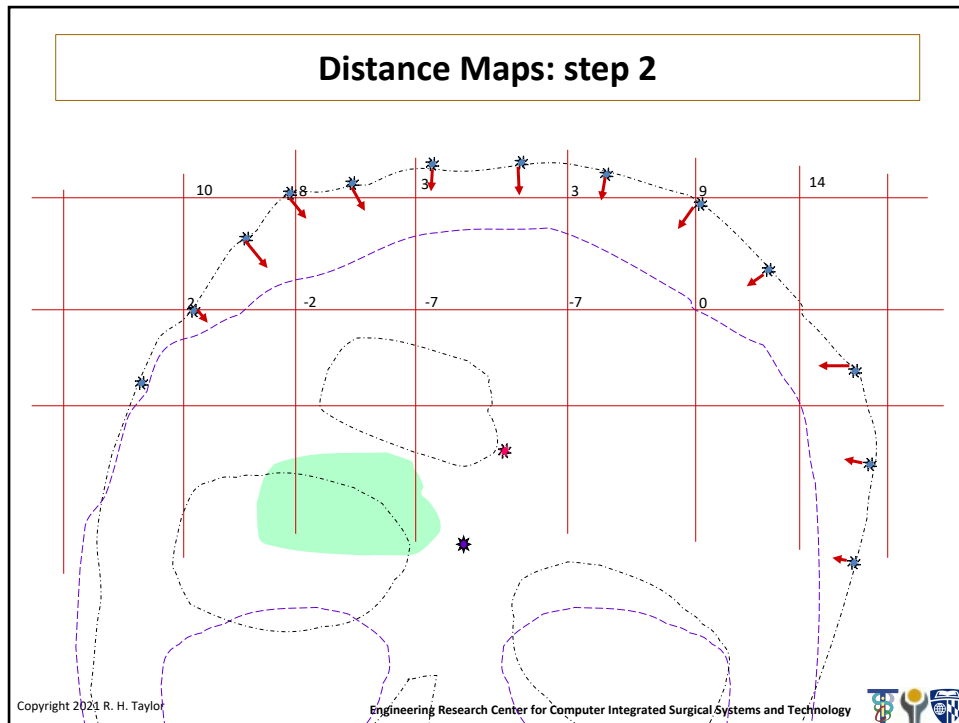
86



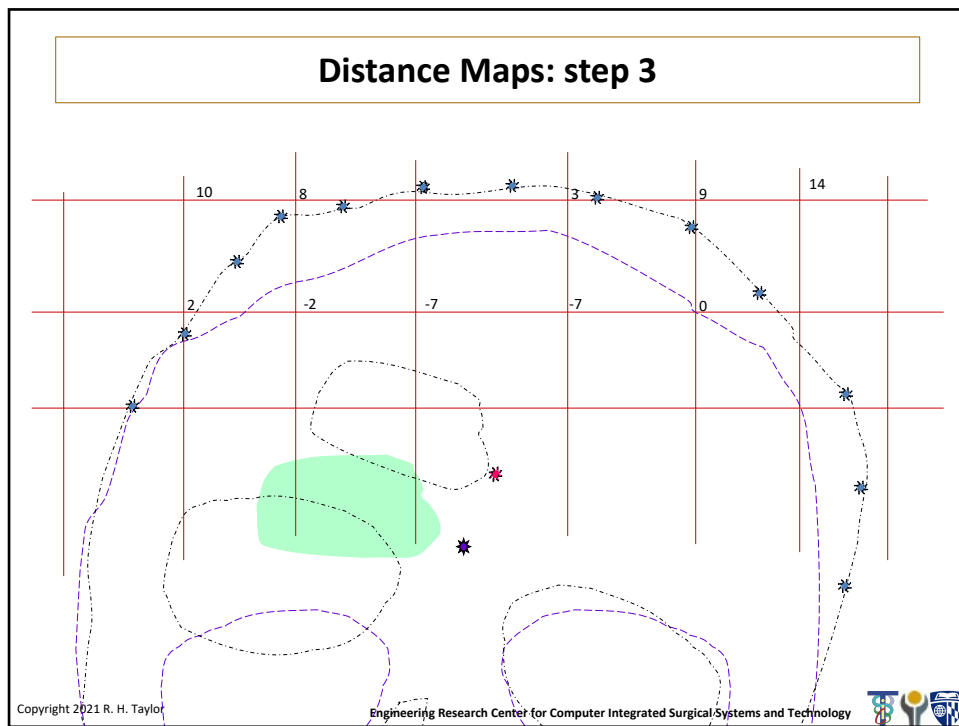
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