



NSF Engineering Research Center for
Computer Integrated Surgical Systems and
Technology



LABORATORY FOR
**Computational
Sensing + Robotics**
THE JOHNS HOPKINS UNIVERSITY

Registration – Part 2

600.455/655 Computer Integrated Surgery



**WHITING
SCHOOL OF
ENGINEERING**
THE JOHNS HOPKINS UNIVERSITY

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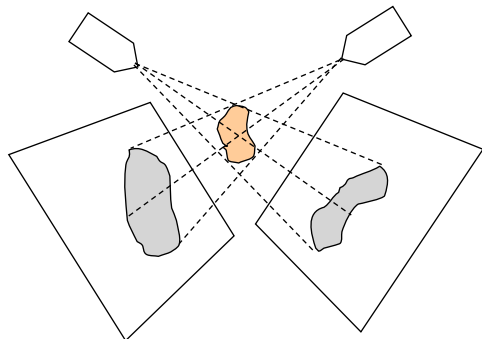
Feature-Based 2D-3D Registration

Given

- 3D surface model of an anatomic structure
- Multiple 2D x-ray projection images taken at known poses relative to some coordinate system C
- Initial estimate of the pose F of the anatomic object relative to the x-ray imaging coordinate system C


Goal

- Compute an accurate value for F

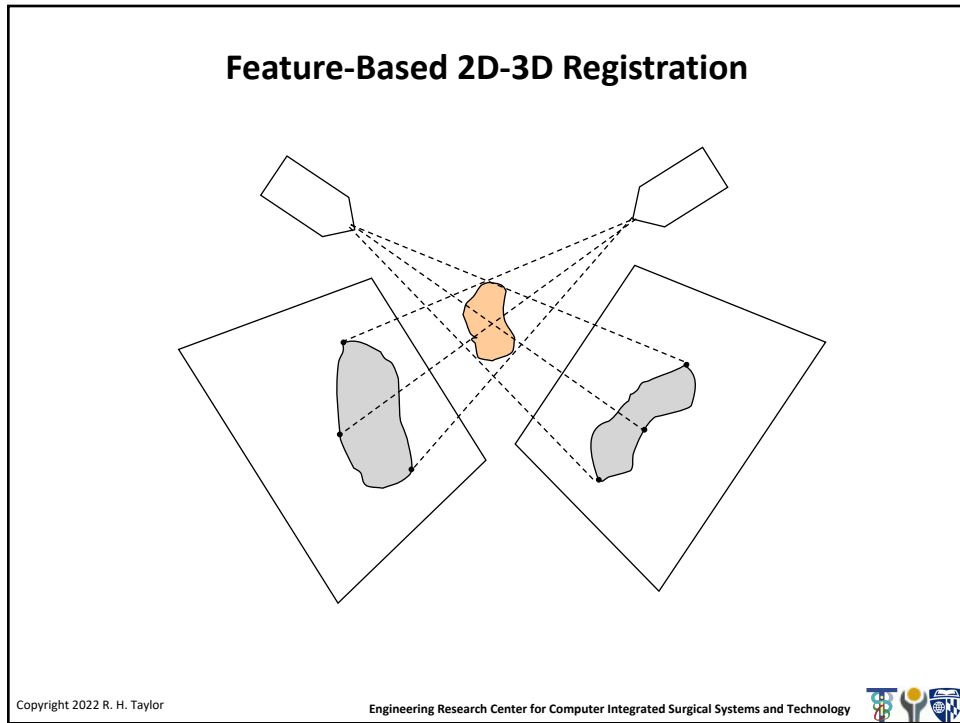


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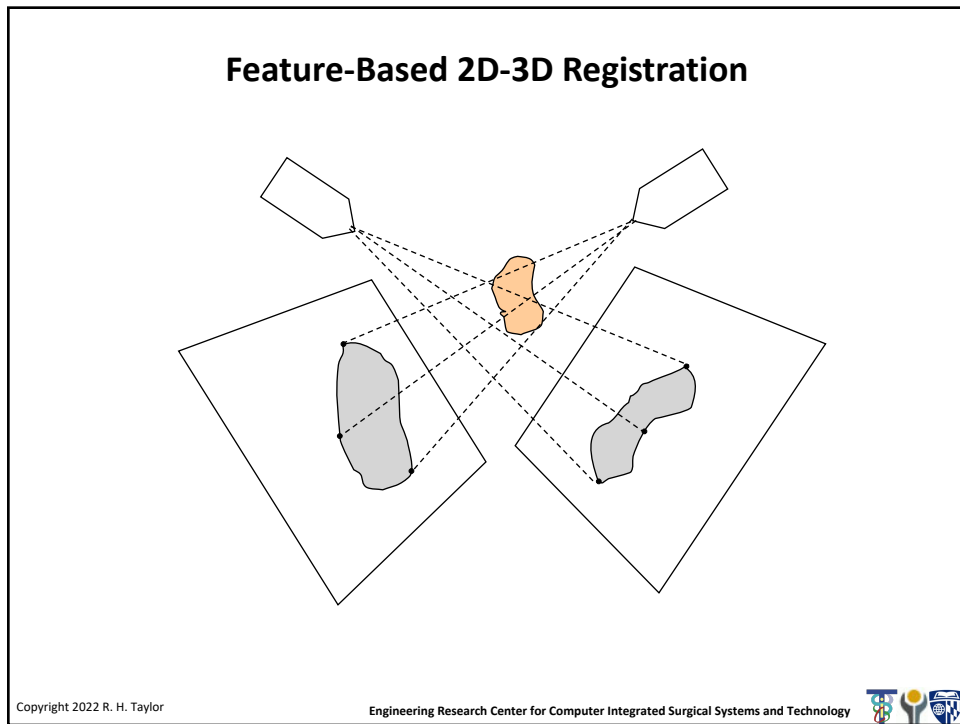
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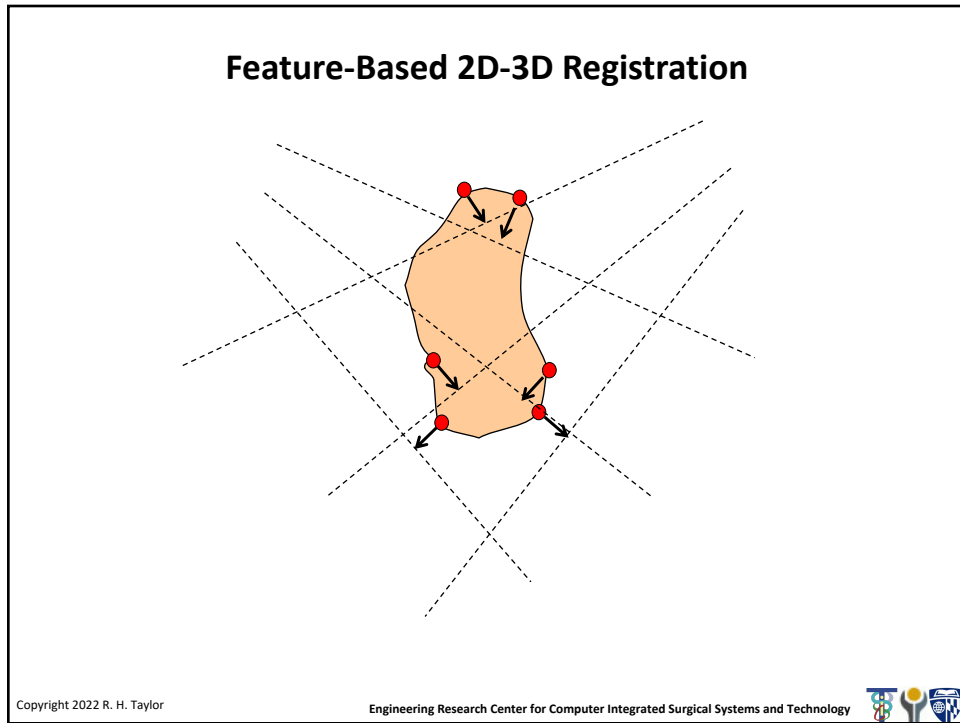
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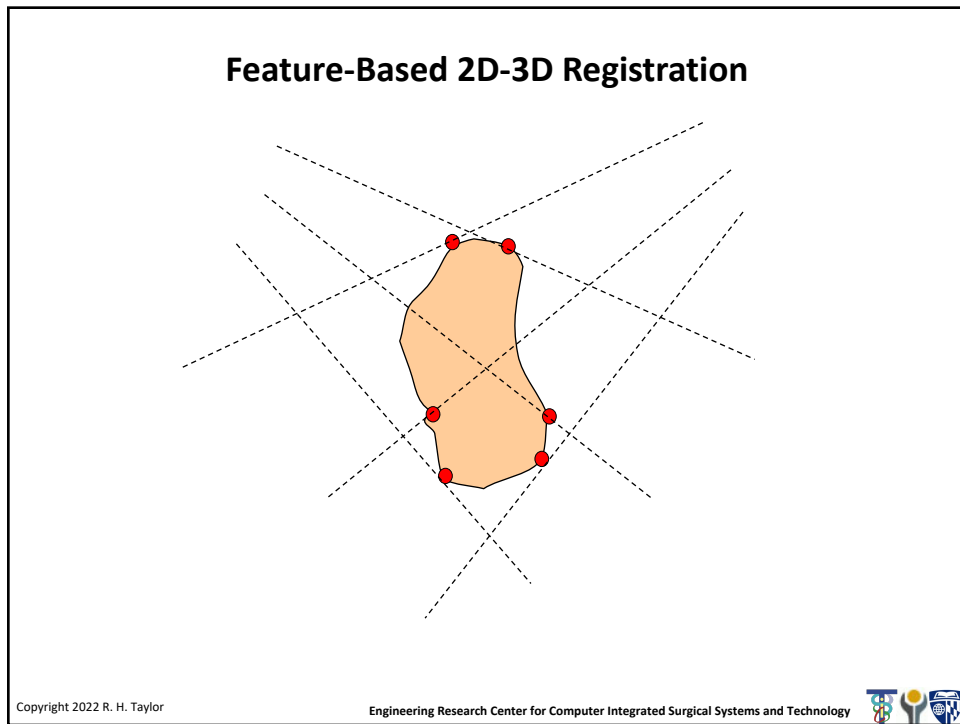
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Feature-Based 2D-3D Registration

The diagram illustrates feature-based registration. It shows two 2D images (left and right) and a 3D model (top center). A central orange feature is highlighted in the 3D model. Dashed lines project this feature onto the 2D images, where corresponding points are marked on the contours. The 2D images also show other grey features and their corresponding points.

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A contour-based 2D-3D method ...

Guezic et al., 1998

Step 0: Extract contours from x-ray images and compute corresponding lines between source and detector

The diagram shows the process of contour-based registration. On the left, a 3D model is shown with a central orange feature. In the middle, a circular x-ray image shows the projection of this feature. On the right, a set of parallel lines represents the corresponding lines between the source and detector.

A. Guéziec, P. Kazanzides, B. Williamson, and R. Taylor, "Anatomy-Based Registration of CT-Scan and Intraoperative X-Ray Images for Guiding a Surgical Robot," IEEE Transactions on Medical Imaging, vol. 17, pp. 715-728, 1998.

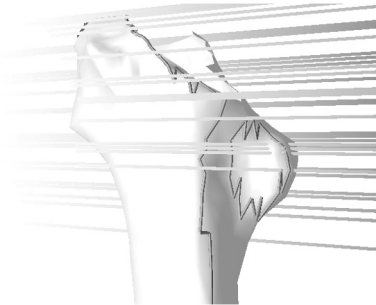
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A contour-based 2D-3D method ...

Gueziec *et al.*, 1998

- Step 1: Given the current estimate for $F = [R, t]$, compute the apparent projection contours of the model for each viewing direction.
- Step 2: For each x-ray path line L_i , identify the closest point p_i on an apparent projection contour. This will give a set of points on the body surface to be moved toward the corresponding x-ray lines



A. Guéziec, P. Kazanzides, B. Williamson, and R. Taylor, "Anatomy-Based Registration of CT-Scan and Intraoperative X-Ray Images for Guiding a Surgical Robot," IEEE Transactions on Medical Imaging, vol. 17, pp. 715-728, 1998.

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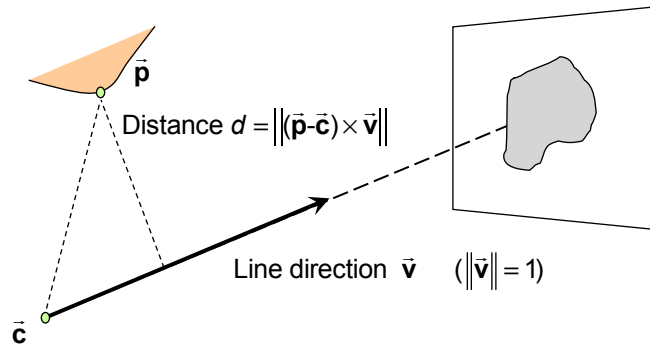
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A contour-based 2D-3D method ...

Gueziec *et al.*, 1998



Note: It is convenient to use the x-ray source position (i.e., the center of convergence for a bundle of x-ray projection lines) as the value for \vec{c} .

A. Guéziec, P. Kazanzides, B. Williamson, and R. Taylor, "Anatomy-Based Registration of CT-Scan and Intraoperative X-Ray Images for Guiding a Surgical Robot," IEEE Transactions on Medical Imaging, vol. 17, pp. 715-728, 1998.

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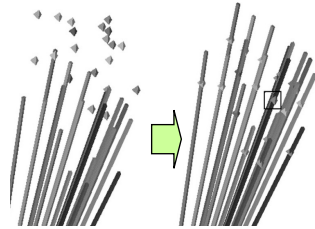


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A contour-based 2D-3D method ...

Guezic et al., 1998

Step 3: Solve an optimization problem to compute a value of F that minimizes the distance between the p_i and the L_i .



$$\min_{\mathbf{R}, \mathbf{t}} \sum_i d_i^2 = \min_{\mathbf{R}, \mathbf{t}} \sum_i \left\| \bar{\mathbf{v}}_i \times (\mathbf{c}_i - (\mathbf{R}\bar{\mathbf{p}}_i + \bar{\mathbf{t}})) \right\|^2$$

$$= \min_{\mathbf{R}, \mathbf{t}} \sum_i \left\| \text{skew}(\bar{\mathbf{v}}_i) \cdot (\mathbf{c}_i - (\mathbf{R}\bar{\mathbf{p}}_i + \bar{\mathbf{t}})) \right\|^2$$

Step 4: Iterate steps 1-3 until reach convergence

A. Guézic, P. Kazanzides, B. Williamson, and R. Taylor, "Anatomy-Based Registration of CT-Scan and Intraoperative X-Ray Images for Guiding a Surgical Robot," IEEE Transactions on Medical Imaging, vol. 17, pp. 715-728, 1998.

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Computational Note

Guezic uses the Cayley parameterization for rotations:

$$\mathbf{R}(\bar{\mathbf{u}}) = (\mathbf{I} - \text{skew}(\bar{\mathbf{u}}))(\mathbf{I} + \text{skew}(\bar{\mathbf{u}}))^{-1}$$

This leads to the approximation

$$\mathbf{R}(\bar{\mathbf{u}}) \approx \mathbf{I} + \text{skew}(2\bar{\mathbf{u}})$$

which is similar to our familiar $\mathbf{R}(\bar{\alpha}) \approx \mathbf{I} + \text{skew}(\bar{\alpha})$.

He also uses the notation $\mathbf{U} = \text{skew}(\bar{\mathbf{u}})$. So $\mathbf{R}(\bar{\mathbf{u}}) = (\mathbf{I} - \mathbf{U})(\mathbf{I} + \mathbf{U})^{-1}$

Similarly, we will see $\mathbf{V} = \text{skew}(\bar{\mathbf{v}})$, etc.

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A contour-based 2D-3D method ...

Guezic et al., 1998

Guezic compared three different methods for performing the minimization in Step 3:

- Levenberg Marquardt (LM) nonlinear minimization.
- Linearization and constrained minimization
- Use of a Robust M-Estimator

A. Guéziec, P. Kazanzides, B. Williamson, and R. Taylor, "Anatomy-Based Registration of CT-Scan and Intraoperative X-Ray Images for Guiding a Surgical Robot," IEEE Transactions on Medical Imaging, vol. 17, pp. 715-728, 1998.

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Levenberg-Marquardt ...

(Following development in Guezic et al., 1998)

Define $f_i(\vec{x}) = \|\mathbf{V}_i(\vec{c}_i - \mathbf{R}(\vec{u})\vec{p}_i - \vec{t})\|$ where $\vec{x}^t = [\vec{u}^t, \vec{t}^t]$, $\mathbf{V}_i = \text{skew}(\vec{v}_i)$

Our goal is to minimize

$$\varepsilon(\vec{x}) = \sum_i f_i(\vec{x})^2 = \sum_i \|\mathbf{V}_i(\vec{c}_i - \mathbf{R}(\vec{u})\vec{p}_i - \vec{t})\|^2$$

We note that $\varepsilon(\vec{x})$ is nonlinear. Levenberg-Marquardt is a widely used optimization method for problems of this type. However, it requires us to evaluate the partial derivatives $\partial f_i / \partial x_j$. Guezic worked these out symbolically for his problem

A. Guéziec, P. Kazanzides, B. Williamson, and R. Taylor, "Anatomy-Based Registration of CT-Scan and Intraoperative X-Ray Images for Guiding a Surgical Robot," IEEE Transactions on Medical Imaging, vol. 17, pp. 715-728, 1998.

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Levenberg-Marquardt ...

(Following development in Guezic et al., 1998)

Define $f_i(\bar{x}) = \|\mathbf{V}_i(\bar{\mathbf{c}}_i - \mathbf{R}(\bar{\mathbf{u}})\bar{\mathbf{p}}_i - \bar{\mathbf{t}})\|$ where $\bar{x}^t = [\bar{\mathbf{u}}^t, \bar{\mathbf{t}}^t]$, $\mathbf{V}_i = \text{skew}(\bar{\mathbf{v}}_i)$

$$\mathbf{J} = \begin{bmatrix} \dots & \frac{\partial f_i}{\partial \bar{x}} & \dots \end{bmatrix} = \begin{bmatrix} \dots & \frac{\partial f_i}{\partial \bar{\mathbf{u}}} & \dots \\ \dots & \frac{\partial f_i}{\partial \bar{\mathbf{t}}} & \dots \end{bmatrix}$$

$$\frac{\partial f_i}{\partial \bar{\mathbf{t}}} = \frac{\mathbf{V}_i^t \mathbf{V}_i (\mathbf{R}\bar{\mathbf{p}}_i - \mathbf{c} + \bar{\mathbf{t}})}{f_i}$$

$$\frac{\partial f_i}{\partial \bar{\mathbf{u}}} = \left(\frac{\partial \mathbf{R}\bar{\mathbf{p}}_i}{\partial \bar{\mathbf{u}}} \right)^t \frac{\mathbf{V}_i^t \mathbf{V}_i (\mathbf{R}\bar{\mathbf{p}}_i - \mathbf{c} + \bar{\mathbf{t}})}{f_i}$$

Details on this may be found
in reference [45] of
Guezic's paper

A. Guéziec, P. Kazanzides, B. Williamson, and R. Taylor, "Anatomy-Based Registration of CT-Scan and Intraoperative X-Ray Images for Guiding a Surgical Robot," IEEE Transactions on Medical Imaging, vol. 17, pp. 715-728, 1998.

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Levenberg-Marquardt ...

(Following development in Guezic et al., 1998)

Step 1: Pick $\lambda =$ a small number; pick initial guess for \bar{x}

Step 2: Evaluate $f_i(\bar{x})$ and \mathbf{J} and solve the least squares problem

$$\begin{bmatrix} \vdots \\ (\mathbf{J}^t \mathbf{J} + \lambda \mathbf{I}) \Delta \bar{x} - \mathbf{J}^t f_i \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ 0 \\ \vdots \end{bmatrix}$$

for $\Delta \bar{x}$.

Step 3: $\bar{x} \leftarrow \bar{x} + \Delta \bar{x}$; update λ .

Step 4: Evaluate termination condition. If not done, go back to step 2

Note: Usually λ starts small and grows larger. Consult standard references (e.g., Numerical Recipes) for more information.

A. Guéziec, P. Kazanzides, B. Williamson, and R. Taylor, "Anatomy-Based Registration of CT-Scan and Intraoperative X-Ray Images for Guiding a Surgical Robot," IEEE Transactions on Medical Imaging, vol. 17, pp. 715-728, 1998.

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Constrained Linearized Least Squares ...

(Following development in Guezic et al., 1998)

Step 0: Make an initial guess for \mathbf{R} and $\bar{\mathbf{t}}$

Step 1: Compute $\bar{\mathbf{p}}_i \leftarrow \mathbf{R}\mathbf{p}_i + \bar{\mathbf{t}}$

Step 2: Define $\mathbf{P}_i = \text{skew}(\bar{\mathbf{p}}_i)$, $\mathbf{V}_i = \text{skew}(\bar{\mathbf{v}}_i)$

Step 3: Solve the least squares problem:

$$\varepsilon^2 = \min \left\| \begin{bmatrix} \vdots & \vdots \\ 2\mathbf{V}\mathbf{P}_i & \mathbf{V}_i \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \bar{\mathbf{u}} \\ \Delta\bar{\mathbf{t}} \end{bmatrix} - \begin{bmatrix} \vdots \\ \mathbf{V}_i(\bar{\mathbf{c}}_i - \bar{\mathbf{p}}_i) \\ \vdots \end{bmatrix} \right\|^2 \quad \text{subject to } \|\bar{\mathbf{u}}\| \leq \rho$$

where ρ is sufficiently small so that $\mathbf{I} + 2\mathbf{U}$ approximates a rotation

Step 4: Compute $\Delta\mathbf{R} = (\mathbf{I} - \mathbf{U})(\mathbf{I} + \mathbf{U})^{-1}$

Update $\mathbf{p}_i \leftarrow \Delta\mathbf{R}\mathbf{p}_i + \Delta\bar{\mathbf{t}}$; $\mathbf{R} \leftarrow \Delta\mathbf{R}\mathbf{R}$; $\bar{\mathbf{t}} \leftarrow \Delta\mathbf{R}\bar{\mathbf{t}} + \Delta\bar{\mathbf{t}}$

Step 5: If ε is small enough or some other termination condition is met, then stop. Otherwise go back to Step 2.

A. Guéziec, P. Kazanzides, B. Williamson, and R. Taylor, "Anatomy-Based Registration of CT-Scan and Intraoperative X-Ray Images for Guiding a Surgical Robot," IEEE Transactions on Medical Imaging, vol. 17, pp. 715-728, 1998.

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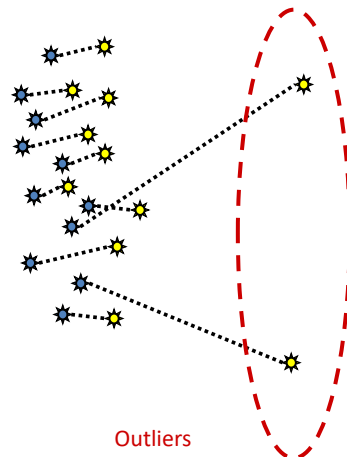
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Robust Pose Estimation ...

- Basic idea is to identify outliers and give them little or no weight.



R. Kumar and A. R. Hanson, "Robust methods for estimating pose and a sensitivity analysis," Comput. Vision, Graphics, Image Processing-IU, vol. 60, no. 3, pp. 313-342, 1994.

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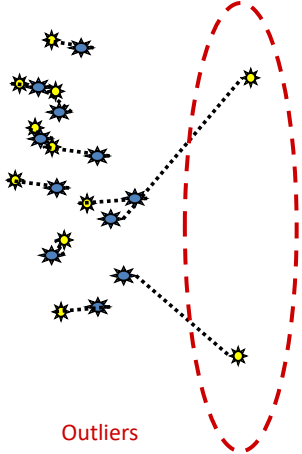
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Robust Pose Estimation ...

- Basic idea is to identify outliers and give them little or no weight.



Outliers

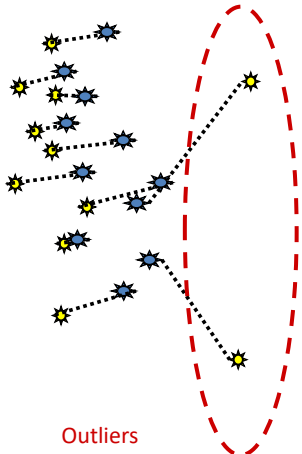
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Robust Pose Estimation ...

- Basic idea is to identify outliers and give them little or no weight.



Outliers

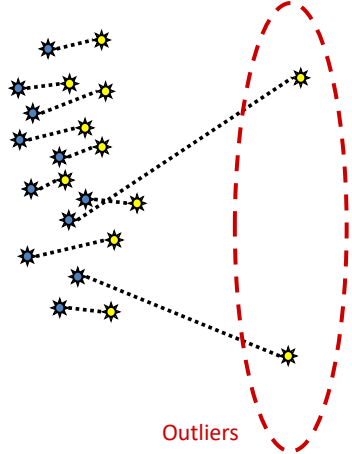
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Robust Pose Estimation ...

- Basic idea is to identify outliers and give them little or no weight.



Outliers

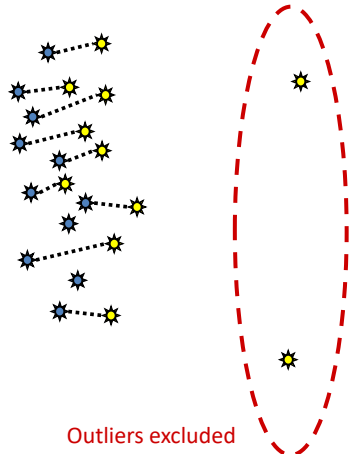
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Robust Pose Estimation ...

- Basic idea is to identify outliers and give them little or no weight.



Outliers excluded

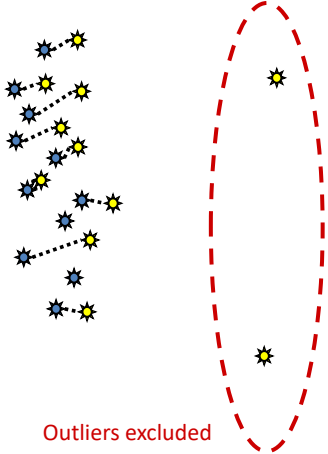
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Robust Pose Estimation ...

- Basic idea is to identify outliers and give them little or no weight.



Outliers excluded

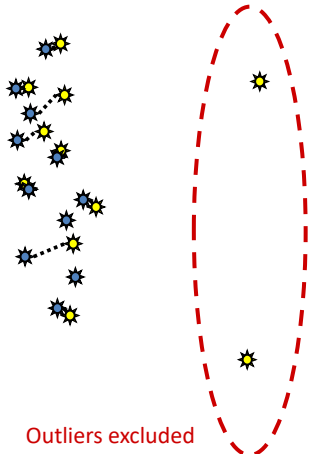
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Robust Pose Estimation ...

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Outliers excluded

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Robust M-Estimator ...

(Following development in Guezic et al., 1998)

Step 0: Make an initial guess for \mathbf{R} and $\bar{\mathbf{t}}$

Step 1: Compute $\bar{\mathbf{p}}_i \leftarrow \mathbf{R}\bar{\mathbf{p}}_i + \bar{\mathbf{t}}$

Step 2: Define $\mathbf{P}_i = \text{skew}(\bar{\mathbf{p}}_i)$, $\mathbf{V}_i = \text{skew}(\bar{\mathbf{v}}_i)$,

Step 3: Solve a robust linearized problem

$$\varepsilon = \underset{\bar{\mathbf{u}}, \Delta \bar{\mathbf{t}}}{\text{argmin}} \sum_i \rho \left(\frac{0.6745 e_i}{\text{median}\{e_i\}} \right) \quad \text{where } e_i = \|\mathbf{V}_i(\bar{\mathbf{p}}_i - \mathbf{c}_i + 2\mathbf{P}_i\bar{\mathbf{u}} + \Delta \bar{\mathbf{t}})\|$$

(See next slide)

Step 4: Compute $\Delta \mathbf{R} = (\mathbf{I} - \mathbf{U})(\mathbf{I} + \mathbf{U})^{-1}$

Update $\mathbf{p}_i \leftarrow \Delta \mathbf{R}\bar{\mathbf{p}}_i + \Delta \bar{\mathbf{t}}$; $\mathbf{R} \leftarrow \Delta \mathbf{R}\mathbf{R}$; $\bar{\mathbf{t}} \leftarrow \Delta \mathbf{R}\bar{\mathbf{t}} + \Delta \bar{\mathbf{t}}$

Step 5: If ε is small enough or some other termination condition is met, then stop. Otherwise go back to Step 2.

A. Guéziec, P. Kazanzides, B. Williamson, and R. Taylor, "Anatomy-Based Registration of CT-Scan and Intraoperative X-Ray Images for Guiding a Surgical Robot," IEEE Transactions on Medical Imaging, vol. 17, pp. 715-728, 1998.

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Robust M-Estimator ...

(Following development in Guezic et al., 1998)

Step 3.0: Set $\bar{\mathbf{u}} = \bar{\mathbf{0}}$, $\Delta \bar{\mathbf{t}} = \bar{\mathbf{0}}$

Step 3.1: Compute $e_i = \|\mathbf{V}_i(\bar{\mathbf{p}}_i - \bar{\mathbf{c}}_i + 2\mathbf{P}_i\bar{\mathbf{u}} + \Delta \bar{\mathbf{t}})\|$, $s = \text{median}\{\dots, e_i, \dots\} / 0.6745$,

Step 3.2: Solve $\mathbf{C}\bar{\mathbf{x}} = \bar{\mathbf{d}}$, where $\bar{\mathbf{x}}^t = [\bar{\mathbf{u}}^t, \bar{\mathbf{t}}^t]$

$$\mathbf{C} = \sum_i \Psi\left(\frac{e_i}{s}\right) \begin{bmatrix} 2\mathbf{P}_i\mathbf{W}_i\mathbf{P}_i & \mathbf{P}_i\mathbf{W}_i \\ 2\mathbf{P}_i\mathbf{W}_i & \mathbf{W}_i \end{bmatrix} \quad \text{and} \quad \bar{\mathbf{d}} = \sum_i \Psi\left(\frac{e_i}{s}\right) \begin{bmatrix} \mathbf{P}_i\mathbf{W}_i(\bar{\mathbf{c}}_i - \bar{\mathbf{p}}_i) \\ \mathbf{W}_i(\bar{\mathbf{c}}_i - \bar{\mathbf{p}}_i) \end{bmatrix}$$

$$\text{where } \mathbf{W}_i = \mathbf{V}_i^t \mathbf{V}_i = \mathbf{I} - \bar{\mathbf{v}}_i \bar{\mathbf{v}}_i^t \quad \Psi(\mu) = \begin{cases} \mu(1 - \mu^2 / \alpha^2)^2 & \text{if } \|\mu\| \leq \alpha \\ 0 & \text{otherwise} \end{cases}$$

(Note : We use $\alpha=2$)

Step 3.3: Iterate steps 3.1 and 3.2 until a suitable termination condition is reached.

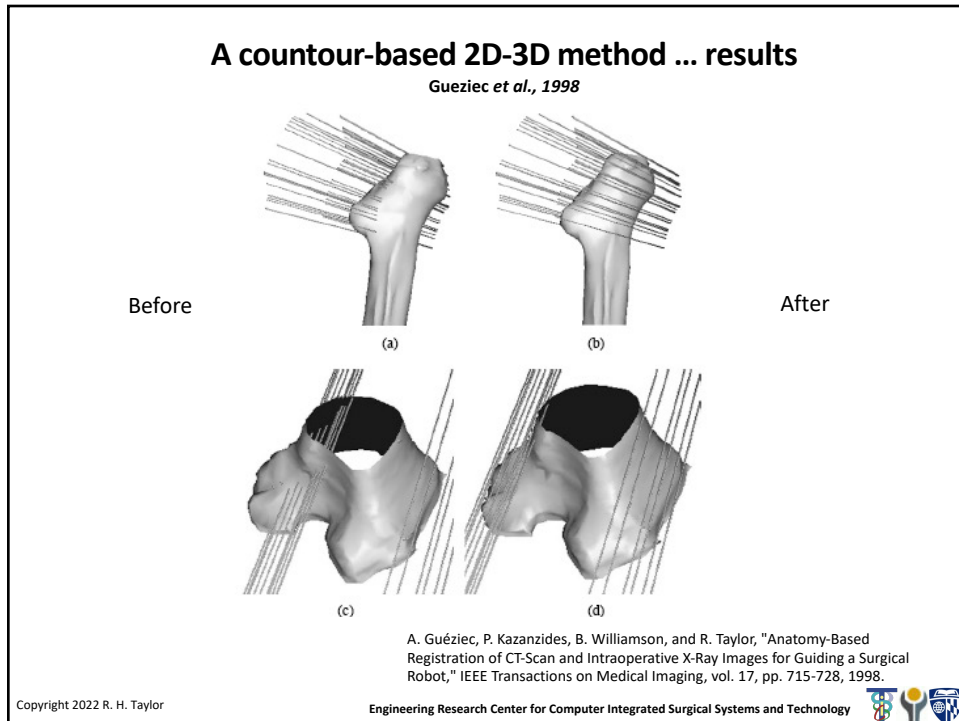
A. Guéziec, P. Kazanzides, B. Williamson, and R. Taylor, "Anatomy-Based Registration of CT-Scan and Intraoperative X-Ray Images for Guiding a Surgical Robot," IEEE Transactions on Medical Imaging, vol. 17, pp. 715-728, 1998.

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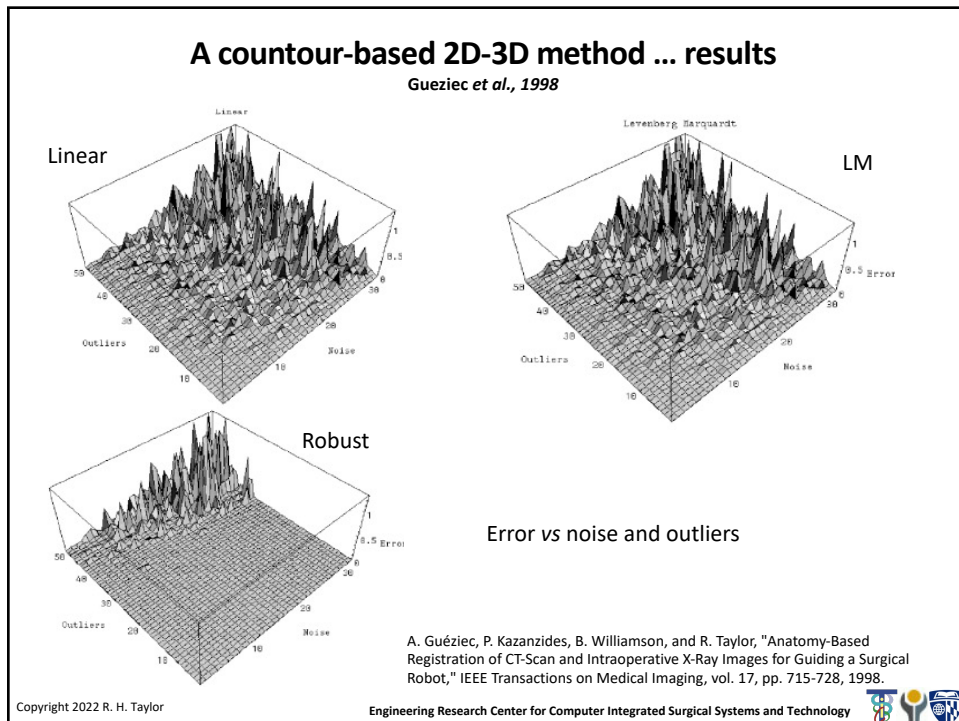
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A contour-based 2D-3D method ... times

Gueziec et al., 1998

TABLE I
 AVERAGE EXECUTION TIMES IN MS FOR THE THREE
 REGISTRATION METHODS APPLIED TO DATA SETS THAT
 COMPRISE 100 POINTS (TOP) AND 20 POINTS (BOTTOM)

Number Points/Method	LM	Linear	Robust
100 points (CPU time)	790	690	28
20 points (CPU time)	200	42	9.6

A. Guéziec, P. Kazanzides, B. Williamson, and R. Taylor, "Anatomy-Based Registration of CT-Scan and Intraoperative X-Ray Images for Guiding a Surgical Robot," IEEE Transactions on Medical Imaging, vol. 17, pp. 715-728, 1998.

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Sample Set Analysis

- **Question:** How good is a particular set of 3D sample points for the purpose of registration to a 3D surface?
- Long line of authors have looked at this question
- Next few slides are based on the work of David Simon, et al (1995)

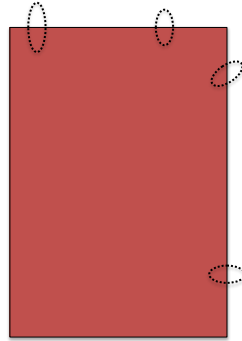
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Sample Set Selection



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Sample Set Analysis: Distance Estimates

Let

$$F(\mathbf{x}) = 0$$

be the implicit equation of a surface, then one good estimate of the distance of a point \mathbf{x} to the surface is

$$D(\mathbf{x}) = \frac{F(\mathbf{x})}{\|\nabla F(\mathbf{x})\|}$$

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Sample set analysis: sensitivity

Let \mathbf{x}_s be a point on the surface, and let $T(\bar{\eta})$ represent a small perturbation with parameters $\bar{\eta}$ with respect to the surface of point \mathbf{x}_s :

$$\mathbf{x}'_s = T(\bar{\eta})\mathbf{x}_s$$

Then we define $\mathbf{V}(\mathbf{x}_s)$ to be

$$\mathbf{V}(\mathbf{x}_s) = \frac{\partial D(T(\bar{\eta})\mathbf{x}_s)}{\partial \bar{\eta}} = \begin{bmatrix} \mathbf{n}_s \\ \mathbf{x}_s \times \mathbf{n}_s \end{bmatrix}$$

where \mathbf{n}_s is the unit normal to the surface at \mathbf{x}_s . So,

$$D(T(\bar{\eta})\mathbf{x}_s) \simeq \mathbf{V}^T(\mathbf{x}_s)\bar{\eta}$$

Squaring this gives

$$\begin{aligned} D^2(T(\bar{\eta})\mathbf{x}_s) &\simeq \bar{\eta}^T \mathbf{V}(\mathbf{x}_s) \mathbf{V}^T(\mathbf{x}_s) \bar{\eta} \\ &= \bar{\eta}^T \mathbf{M}(\mathbf{x}_s) \bar{\eta} \end{aligned}$$

Note that \mathbf{M} is 6×6 positive, semi-definite, symmetric matrix.

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Sample set analysis: sensitivity

For a region \mathcal{R} , define

$$\begin{aligned} E_R(\bar{\eta}) &= \bar{\eta}^T \left[\sum_{\mathbf{x}_s \in \mathcal{R}} \mathbf{M}(\mathbf{x}_s) \right] \bar{\eta} \\ &= \bar{\eta}^T \Psi_{\mathcal{R}} \bar{\eta} \\ &= \bar{\eta}^T \mathbf{Q} \Lambda \mathbf{Q}^T \bar{\eta} \\ &= \sum_{1 \leq i \leq 6} \lambda_i (\bar{\eta}^T \cdot \mathbf{q}_i)^2 \end{aligned}$$

- Note that the eigenvectors \mathbf{q}_i correspond to small differential transformations $\mathbf{T}(\mathbf{q}_i)$, and can sort eigenvalues so that

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_6$$

- Note that eigenvector \mathbf{q}_1 corresponds to direction of greatest constraint.
- Similarly, can also think of \mathbf{q}_6 as the least constrained direction.

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hnology



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Sample Set Analysis: Goodness Measures

- Magnitude of smallest eigenvalue (Simon)
- (Kim and Khosla)

$$\frac{\sqrt[6]{\lambda_1 \cdot \dots \cdot \lambda_6}}{\lambda_1 + \dots + \lambda_6}$$

- Nahvi

$$\frac{\lambda_6^2}{\lambda_1}$$

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Sample Set Selection

- One blind search method (similar to Simon, 1995) is:
 - Randomly select sample points on surface
 - (prune for reachability)
 - evaluate goodness of sample set using some criterion
 - repeat many times and choose the best one found

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Sample Set Selection

- Refinement of blind search (hill climbing):
 - Randomly select sample points on surface
 - (prune for reachability)
 - evaluate goodness of sample set using some criterion
 - replace a point from sample set with a randomly selected point
 - evaluate goodness
 - if better, keep it
 - else revert to original point and try again

- Variations include simulated annealing, “genetic” algorithms

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Sample Set Selection: Another Alternative

- Select large number of random points \mathbf{x}_s
- Prune for reachability
- For each point, compute constraint direction $\mathbf{V}_s = \mathbf{V}(\mathbf{x}_s)$. To a first approximation, a measurement at \mathbf{x}_s with accuracy ϵ_s constrains $\bar{\boldsymbol{\eta}}$ by

$$|\mathbf{V}_s \bar{\boldsymbol{\eta}}| \leq \epsilon_s$$

- Now select subset of the \mathbf{x}_s that minimizes, e.g.,

$$\min_{\delta_s} \max \bar{\boldsymbol{\eta}}^T \mathbf{S} \bar{\boldsymbol{\eta}}$$

subject to

$$\begin{aligned} \delta_s &\in \{0, 1\} \\ |\delta_s \mathbf{V}_s \bar{\boldsymbol{\eta}}| &\leq \epsilon_s \\ \sum_s \delta_s &\leq \text{subsetsize} \end{aligned}$$

There are various ways to do this.

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Sample Set Selection: Another Alternative (con'd)

- One can also minimize other forms, e.g.,

$$\min_s \max_i |\sigma_i \eta_i|$$

subject to similar constraints

- An alternative is to minimize the number of sample points required to ensure that some constraints on $\bar{\eta}$ are guaranteed to be met. E.g.,

$$\min_{\delta_s} \sum \delta_s$$

such that

$$\delta_s \in \{0, 1\}$$

$$\xi \leq \xi_{limit}$$

where

$$\xi = \max_{\bar{\eta}} \bar{\eta}^T \mathbf{S} \bar{\eta}$$

or some other form subject to

$$|\delta_s \mathbf{V}_s \bar{\eta}| \leq \epsilon_s$$

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Probabilistic Registration

- Registration methods typically use some optimization algorithm to find a “best” transformation between one data set and the other.
- It makes sense to try to find the “most likely” registration transformation.
- ICP minimizes sum-of-squares distances.
- This is equivalent to assuming that point-pair match probabilities are independent and symmetric Gaussian distributions based on distances
- But there are a number of other methods that explicitly consider probabilities ...

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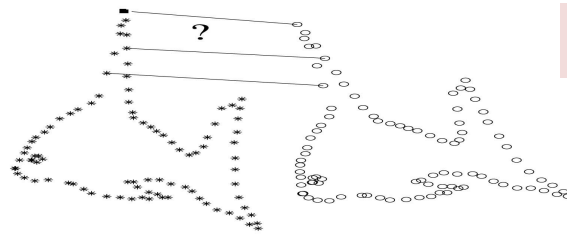
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43

Coherent Point Drift

- A. Myronenko and X. Song, "Point-Set Registration: Coherent Point Drift", *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 32- 12, pp. 2262-2275, 2010.
- Alignment of point clouds
 - Fast method follows "EM" paradigm
 - Tolerates outliers and noise
 - Transformations: Rigid, affine, general deformable



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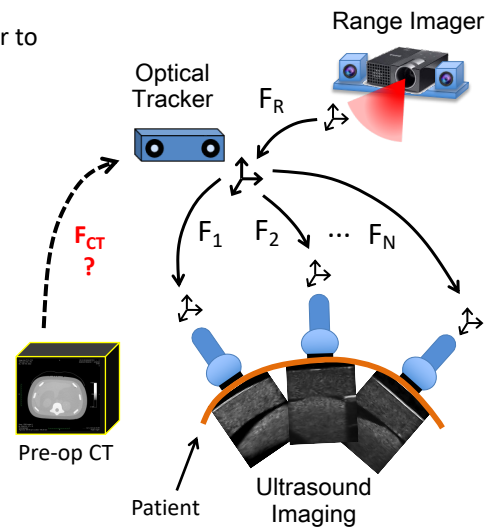
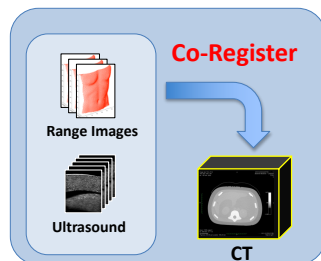
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Registration of intraoperative data to preoperative models

- Want to know registration from tracker to CT space
 - Provides tool positions relative to CT
- Data sources for registration
 - Tracked ultrasound
 - Tracked (or calibrated) range data



S. Billings and R. H. Taylor, "Iterative Most Likely Oriented Point Registration", in *Medical Image Computing and Computer-Assisted Interventions (MICCAI)*, Boston, October, 2014. (accepted).

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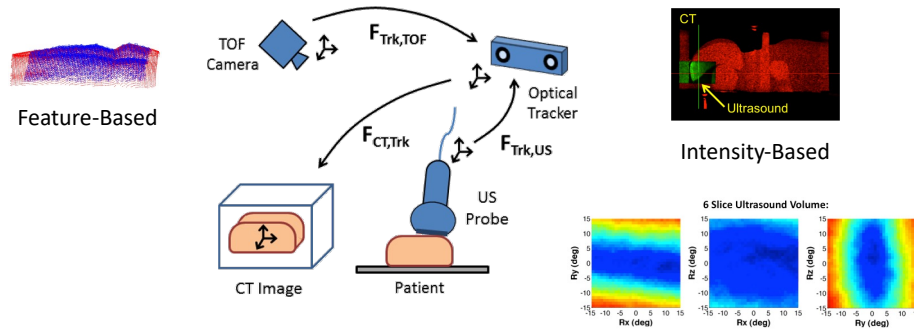
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Multi-Modal Feature-Based Registration

Question: How to combine multiple data sources, in order to improve the accuracy and robustness of registration outcomes?

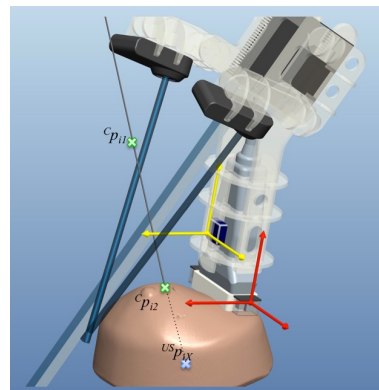


Billings S, Kapoor A, Keil M, Wood BJ, Boctor E (2011) A Hybrid Surface/Image-Based Approach to Facilitate Ultrasound/CT Registration. *SPIE, Medical Imaging 2011: Ultrasonic Imaging, Tomography, and Therapy* 7968: 79680V–79680V-12

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Example: Clear Guide Medical Navigation System

- Handheld device:
 - low cost
 - integrated on probe
 - ease of use
 - no workflow interruptions
 - in-situ guidance
 - no tool calibration
 - no sterility issues
 - high accuracy
 - real-time fusion
 - real-time quality control



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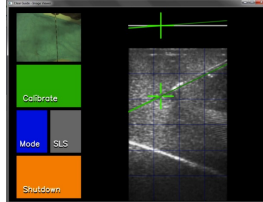
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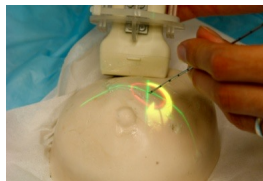
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Easy-to-Follow Guidance

- CG-1 has traditional ultrasound screen AND on-screen guidance overlay



- As well as on-patient projection



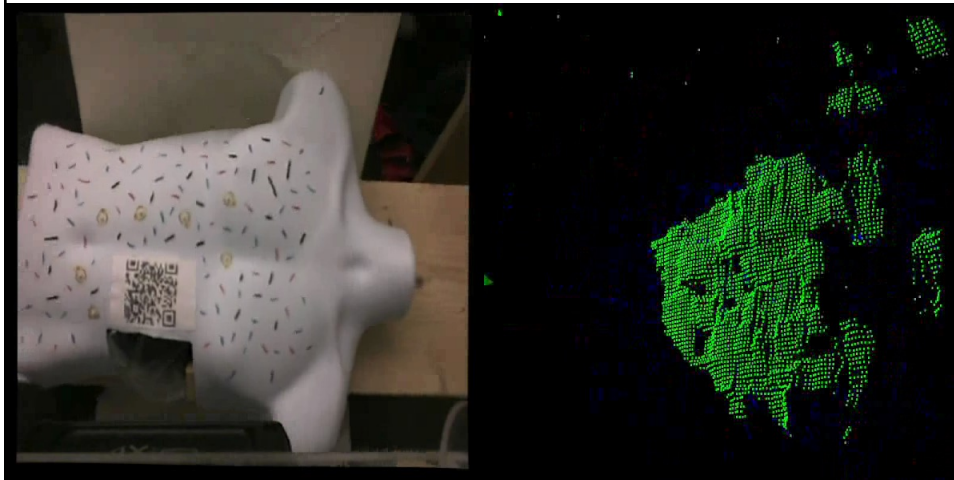
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Real-time Multi-modal Fusion

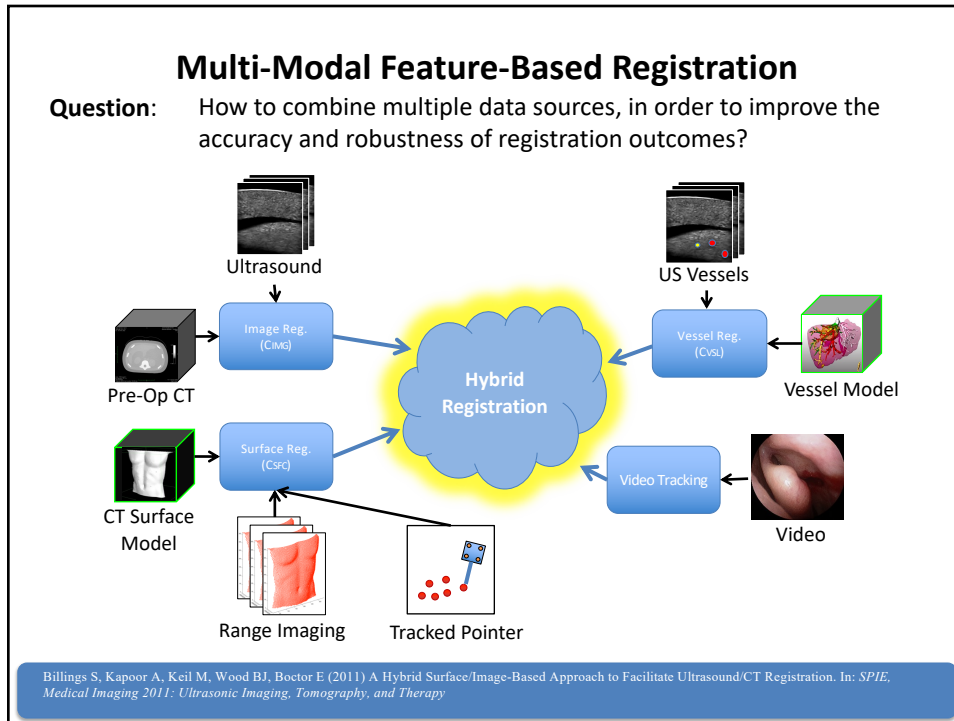


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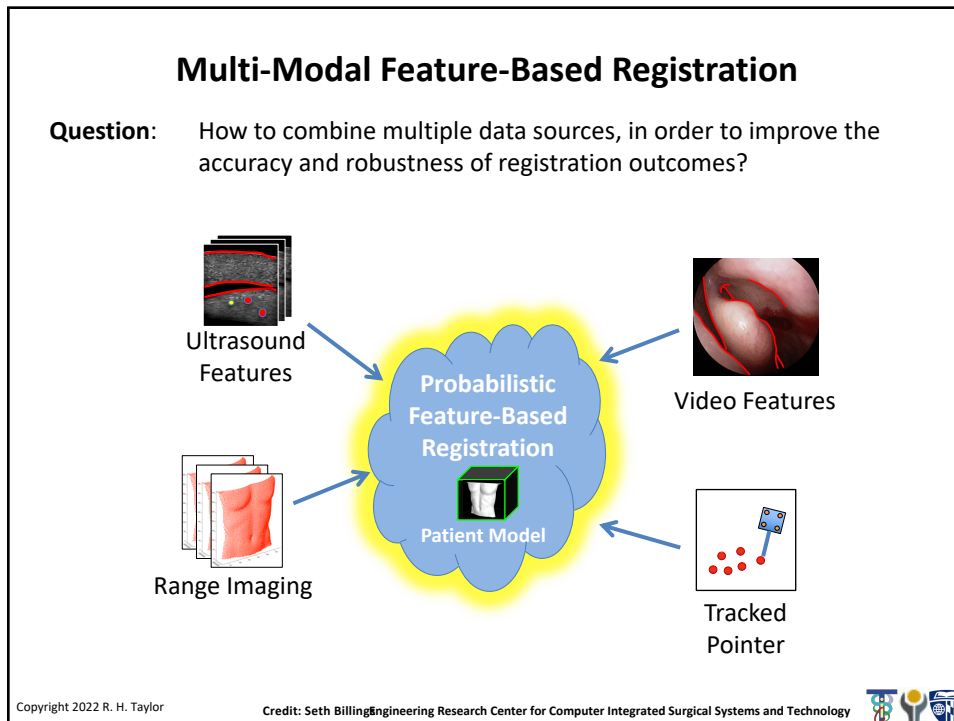
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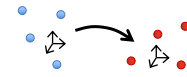


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Iterative Closest Point (ICP) Revisited

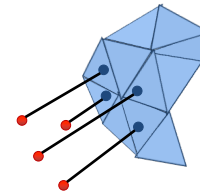


- Widely popular and useful method for point cloud to surface registration introduced by Besl & McKay in 1992
- Many variants proposed since its inception affecting all aspects of the algorithm (robustness, matching criteria, match alignment, etc.)

➤ Matching Phase:

for each point in the source shape, find the closest point on the target shape

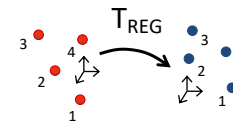
$$\mathbf{y}_i = C_{CP}(T(\mathbf{x}_i), \Psi) = \operatorname{argmin}_{\mathbf{y} \in \Psi} \|\mathbf{y} - T(\mathbf{x}_i)\|_2$$



➤ Registration Phase:

compute transformation to minimize sum of square distances between matches

$$T = \operatorname{argmin}_T \sum_{i=1}^n \|\mathbf{y}_i - T(\mathbf{x}_i)\|_2^2$$



S. Billings and R. H. Taylor, "Iterative Most Likely Oriented Point Registration", in *Medical Image Computing and Computer-Assisted Interventions (MICCAI)*, Boston, October, 2014.

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Most-Likely Point Paradigm Illustrated with ICP

1. Probability Model: isotropic Gaussian

$$f_{\text{match}}(\mathbf{x} | \mathbf{y}, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{3/2}} \cdot e^{-\frac{1}{2\sigma^2} \|\mathbf{y} - \mathbf{x}\|^2}$$

2. Match Phase:

$$\begin{aligned} \mathbf{y}_i &= \operatorname{argmax}_{\mathbf{y}_i \in \Psi} f_{\text{match}}(T(\mathbf{x}_i) | \mathbf{y}_i, \sigma^2) \\ &= \operatorname{argmax}_{\mathbf{y}_i \in \Psi} \frac{1}{(2\pi\sigma^2)^{3/2}} \cdot e^{-\frac{1}{2\sigma^2} \|\mathbf{y}_i - T(\mathbf{x}_i)\|^2} \\ &\rightarrow \operatorname{argmin}_{\mathbf{y}_i \in \Psi} \|\mathbf{y}_i - T(\mathbf{x}_i)\| \end{aligned}$$

3. Registration Phase:

$$\begin{aligned} T &= \operatorname{argmax}_T \prod_i^n f_{\text{match}}(T(\mathbf{x}_i) | \mathbf{y}_i, \sigma^2) \\ &= \operatorname{argmax}_T \prod_i^n \frac{1}{(2\pi\sigma^2)^{3/2}} \cdot e^{-\frac{1}{2\sigma^2} \|\mathbf{y}_i - T(\mathbf{x}_i)\|^2} \\ &\rightarrow \operatorname{argmax}_T \left[-n \log \left((2\pi\sigma^2)^{3/2} \right) - \frac{1}{2\sigma^2} \sum_i^n \|\mathbf{y}_i - T(\mathbf{x}_i)\|^2 \right] \\ &\rightarrow \operatorname{argmin}_T \sum_i^n \|\mathbf{y}_i - T(\mathbf{x}_i)\|^2 \end{aligned}$$

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Outline of Registration Algorithms

- ICP - Iterative Closest Point
 - isotropic position data
- **IMLP - Iterative Most Likely Point**
 - anisotropic position data
 - robust to outliers
- IMLOP - Iterative Most Likely Oriented Point
 - isotropic position & orientation data
- G-IMLOP - Generalized IMLOP
 - anisotropic position & orientation data
- P-IMLOP - Projected IMLOP
 - anisotropic position & projected orientation data

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Sources of Anisotropic Uncertainty

Tomographic Imaging

Stereo Vision

Ultrasound

Figures: [https://www.ndtportal.com/ViewContent.aspx?id=211317012101_melanopsys&lang=The Essential Physics of Medical Imaging, 3rd ed; http://www.ndttech.edu/ndt-content/uploads/2013/01/4_A-B-look-At-Robotic-3D-Ultrasound-Tests-Ultrasound-Technicians-hw-300x225.jpg](https://www.ndtportal.com/ViewContent.aspx?id=211317012101_melanopsys&lang=The%20Essential%20Physics%20of%20Medical%20Imaging,%203rd%20ed%20http://www.ndttech.edu/ndt-content/uploads/2013/01/4_A-B-look-At-Robotic-3D-Ultrasound-Tests-Ultrasound-Technicians-hw-300x225.jpg)
http://00.i.lalimg.com/photo/v0/105832128/CT_Scan_equipment.jpg

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Prior Work: Anisotropic Registration

- Generalized Total Least Squares ICP (GTLS-ICP)

Estépar RSJ, Brun A, Westin C-F (2004) Robust generalized total least squares iterative closest point registration. In: *MICCAI 2004*

- Registration Phase
 - anisotropic noise model
 - ad-hoc implementation **less accurate / efficient**; can be **unstable**
- Match Phase
 - isotropic (i.e. closest-point matching)

- Generalized ICP (G-ICP)

- Registration Phase
 - anisotropic noise model **limited** to model locally-linear surface regions surrounding each feature point of a point cloud shape
 - uses off-the-shelf conjugate gradient solver
- Match Phase
 - isotropic (i.e. closest-point matching)

Segal A, Haehnel D, Thrun S (2009) Generalized-ICP. In: *Robotics: Science and Systems V*

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Prior Work: Anisotropic Registration

- Anisotropic ICP (A-ICP)

Maier-Hein L, Franz AM, Dos Santos TR, Schmidt M, Fangerau M, et al. (2012) Convergent iterative closest-point algorithm to accommodate anisotropic and inhomogeneous localization error. *IEEE Trans Pattern Anal Mach Intell* 34: 1520–1532.

- Registration Phase
 - anisotropic noise model
 - ad-hoc implementation **does not fully account** for noise in both shapes (i.e., lacks ability to reorient the data-shape covariances during optimization)
- Match Phase
 - anisotropic noise model with **non-optimal matching** (finds minimal Mahalanobis distance match rather than most-likely match)
 - **inefficient** implementation; also **cannot guarantee** that the “best” match is found
- **Initializes registration by ICP** (due to inefficient match phase)

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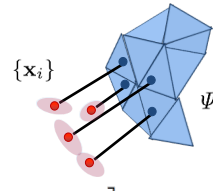
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Iterative Most Likely Point (IMLP)

Probability Model: anisotropic Gaussian

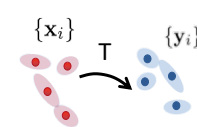
$$f_{\text{match}}(\mathbf{x} | \mathbf{y}, \Sigma_{\mathbf{x}}, \Sigma_{\mathbf{y}}) = \frac{1}{(2\pi)^{3/2} |\Sigma_{\mathbf{x}} + \Sigma_{\mathbf{y}}|^{1/2}} \cdot e^{-\frac{1}{2}(\mathbf{y}-\mathbf{x})^T (\Sigma_{\mathbf{x}} + \Sigma_{\mathbf{y}})^{-1} (\mathbf{y}-\mathbf{x})}$$

Match Phase:



$$[\mathbf{y}_i, \Sigma_{\mathbf{y}_i}] = \underset{[\mathbf{y}_i, \Sigma_{\mathbf{y}_i}] \in \Psi}{\operatorname{argmin}} \left[\log(\mathbf{R}\Sigma_{\mathbf{x}_i}\mathbf{R}^T + \Sigma_{\mathbf{y}_i}) + (\mathbf{y}_i - \mathbf{T}(\mathbf{x}_i))^T (\mathbf{R}\Sigma_{\mathbf{x}_i}\mathbf{R}^T + \Sigma_{\mathbf{y}_i})^{-1} (\mathbf{y}_i - \mathbf{T}(\mathbf{x}_i)) \right]$$

Registration Phase:

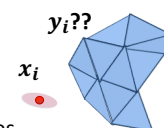


$$\mathbf{T} = \underset{\mathbf{T}=[\mathbf{R}, \mathbf{t}]}{\operatorname{argmin}} \sum_i^n (\mathbf{y}_i - \mathbf{T}(\mathbf{x}_i))^T (\mathbf{R}\Sigma_{\mathbf{x}_i}\mathbf{R}^T + \Sigma_{\mathbf{y}_i})^{-1} (\mathbf{y}_i - \mathbf{T}(\mathbf{x}_i))$$

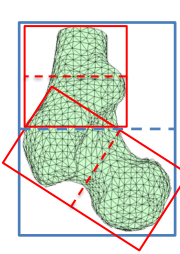
Billings SD, Boctor EM, Taylor RH (2015) Iterative Most-Likely Point Registration (IMLP): A Robust Algorithm for Computing Optimal Shape Alignment. *PLoS One* 10: e0117688

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IMLP: Match Phase



- Due to anisotropic distance metric, standard KD-tree search techniques do not apply.
- **Approach:** PD-tree search with modified node test



PD Tree Constructed by Datum Positions

Constructing the PD tree:

1. Add all datums to a root node
2. Compute covariance of datum positions within the node
3. Create minimally-sized bounding box aligned to the covariance eigenvectors
4. Partition node along the direction of greatest extent
5. Form left and right child nodes from the datums in each partition
6. Repeat from Step 2 for left and right child nodes until # datums in node < threshold or node size < threshold

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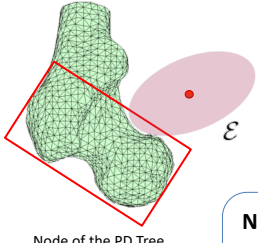
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IMLP: Match Phase

Searching the PD tree:

Assume the **current match candidate** has a match error equal to E_{best}

Question: can any feature in this node possibly provide a match error less than E_{best} ?



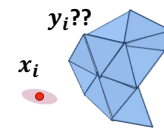
$$[y_i, \Sigma_{y_i}] = \underset{[y_i, \Sigma_{y_i}] \in \Psi}{\operatorname{argmin}} \left[\log(|\mathbf{R}\Sigma_{x_i}\mathbf{R}^T + \Sigma_{y_i}|) + (y_i - \mathbf{T}(x_i))^T (\mathbf{R}\Sigma_{x_i}\mathbf{R}^T + \Sigma_{y_i})^{-1} (y_i - \mathbf{T}(x_i)) \right]$$

True if: $(y_i - \mathbf{T}(x_i))^T (\mathbf{R}\Sigma_{x_i}\mathbf{R}^T + \Sigma_{node})^{-1} (y_i - \mathbf{T}(x_i)) < E_{best} - \log_{min}$

Node Test: if the ellipsoid

$$\mathcal{E} = \{y \mid (y - \mathbf{T}(x_i))^T (\mathbf{R}\Sigma_{x_i}\mathbf{R}^T + \Sigma_{node})^{-1} (y - \mathbf{T}(x_i)) \leq E_{best} - \log_{min}\}$$

intersects the bounding box of the node, then search the node



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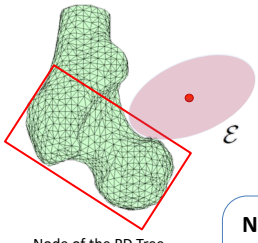
62

IMLP: Match Phase

Searching the PD tree:

Assume the **current match candidate** has a match error equal to E_{best}

Question: can any feature in this node possibly provide a match error less than E_{best} ?



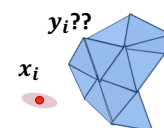
$$[y_i, \Sigma_{y_i}] = \underset{[y_i, \Sigma_{y_i}] \in \Psi}{\operatorname{argmin}} \left[\log(|\mathbf{R}\Sigma_{x_i}\mathbf{R}^T + \Sigma_{y_i}|) + (y_i - \mathbf{T}(x_i))^T (\mathbf{R}\Sigma_{x_i}\mathbf{R}^T + \Sigma_{y_i})^{-1} (y_i - \mathbf{T}(x_i)) \right]$$

True if: $(y_i - \mathbf{T}(x_i))^T (\mathbf{R}\Sigma_{x_i}\mathbf{R}^T + \Sigma_{node})^{-1} (y_i - \mathbf{T}(x_i)) < E_{best} - \log_{min}$

Node Test: if the ellipsoid

$$\mathcal{E} = \{y \mid (y - \mathbf{T}(x_i))^T (\mathbf{R}\Sigma_{x_i}\mathbf{R}^T + \Sigma_{node})^{-1} (y - \mathbf{T}(x_i)) \leq E_{best} - \log_{min}\}$$

intersects the bounding box of the node, then search the node

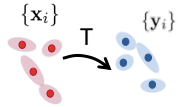


Details in Billings' Thesis

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IMLP: Registration Phase



1. Re-formulate the cost function from an unconstrained optimization

$$\mathbf{T} = \underset{[\mathbf{R}, \mathbf{t}]}{\operatorname{argmin}} \sum_{i=1}^n (\mathbf{y}_i - \mathbf{R}\mathbf{x}_i - \mathbf{t})^T (\mathbf{R}\Sigma_{x_i}\mathbf{R}^T + \Sigma_{y_i})^{-1} (\mathbf{y}_i - \mathbf{R}\mathbf{x}_i - \mathbf{t})$$

to a constrained optimization

$$\mathbf{T} = \underset{[\mathbf{R}, \mathbf{t}]}{\operatorname{argmin}} \sum_{i=1}^n (\mathbf{x}_i - \mathbf{x}_i^*)^T \Sigma_{x_i}^{-1} (\mathbf{x}_i - \mathbf{x}_i^*) + \sum_{i=1}^n (\mathbf{y}_i - \mathbf{y}_i^*)^T \Sigma_{y_i}^{-1} (\mathbf{y}_i - \mathbf{y}_i^*)$$

subject to: $F_i(\mathbf{x}_i^*, \mathbf{y}_i^*, \mathbf{R}, \mathbf{t}) = \mathbf{y}_i^* - \mathbf{R}\mathbf{x}_i^* - \mathbf{t} = 0$ Generalized Total Least Squares (GTLS)

\mathbf{x}_i^* - true (unknown) data-point position
 \mathbf{y}_i^* - true (unknown) model-point position

2. Linearize the constraints with a Taylor series centered at the measured (known) data

$$F_i(\mathbf{x}_i^*, \mathbf{y}_i^*, \mathbf{R}, \mathbf{t}) \approx F_{Li}^k(\mathbf{x}_i, \mathbf{y}_i, d\alpha, d\mathbf{t})$$

$$= F_i^0(\mathbf{x}_i, \mathbf{y}_i, \mathbf{R}_k, \mathbf{t}_k) - \mathbf{r}_{y_i} + \mathbf{R}_k \mathbf{r}_{x_i} + \operatorname{skew}(\mathbf{R}_k \mathbf{x}_i) d\alpha - d\mathbf{t} = 0$$

Note using: $\Delta \mathbf{R} \approx \mathbf{I} + \operatorname{skew}(d\alpha)$ $\mathbf{r}_{x_i} = \mathbf{x}_i - \mathbf{x}_i^*$ $\mathbf{r}_{y_i} = \mathbf{y}_i - \mathbf{y}_i^*$

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IMLP: Registration Phase

3. Apply the method of Lagrange multipliers to solve constrained optimization.

3a. Form the Lagrange function using the linearized constraints

$$\mathcal{L}(d\alpha, d\mathbf{t}, \lambda) = \sum_{i=1}^n \mathbf{r}_{x_i}^T \Sigma_{x_i}^{-1} \mathbf{r}_{x_i} + \sum_{i=1}^n \mathbf{r}_{y_i}^T \Sigma_{y_i}^{-1} \mathbf{r}_{y_i} + \sum_{i=1}^n \lambda_i^T F_{Li}^k(\mathbf{x}_i, \mathbf{y}_i, d\alpha, d\mathbf{t})$$

3b. Solve zero gradient w.r.t. the optimization parameters and the Lagrange multipliers

$$\mathbf{J}^T \Sigma^{-1} \mathbf{J} d\mathbf{p} = -\mathbf{J}^T \Sigma^{-1} \mathbf{f}^0 \quad \text{modified Gauss-Newton}$$

$d\mathbf{p} = \begin{bmatrix} d\alpha \\ d\mathbf{t} \end{bmatrix}$

$\mathbf{f}^0 = \begin{bmatrix} \mathbf{f}_1^0 \\ \vdots \\ \mathbf{f}_n^0 \end{bmatrix}$

$\mathbf{J} = \begin{bmatrix} \operatorname{skew}(\mathbf{R}_k \mathbf{x}_1) & -\mathbf{I} \\ \vdots & \vdots \\ \operatorname{skew}(\mathbf{R}_k \mathbf{x}_n) & -\mathbf{I} \end{bmatrix}$

$\Sigma = \begin{bmatrix} \mathbf{F}_x^0 \Sigma_x \mathbf{F}_x^{0T} + \Sigma_y \end{bmatrix}$

$\mathbf{F}_x^0 = \begin{bmatrix} -\mathbf{R}_k & & \\ & \ddots & \\ & & -\mathbf{R}_k \end{bmatrix}$

$\Sigma_x = \begin{bmatrix} \Sigma_{x1} & & \\ & \ddots & \\ & & \Sigma_{xn} \end{bmatrix}$

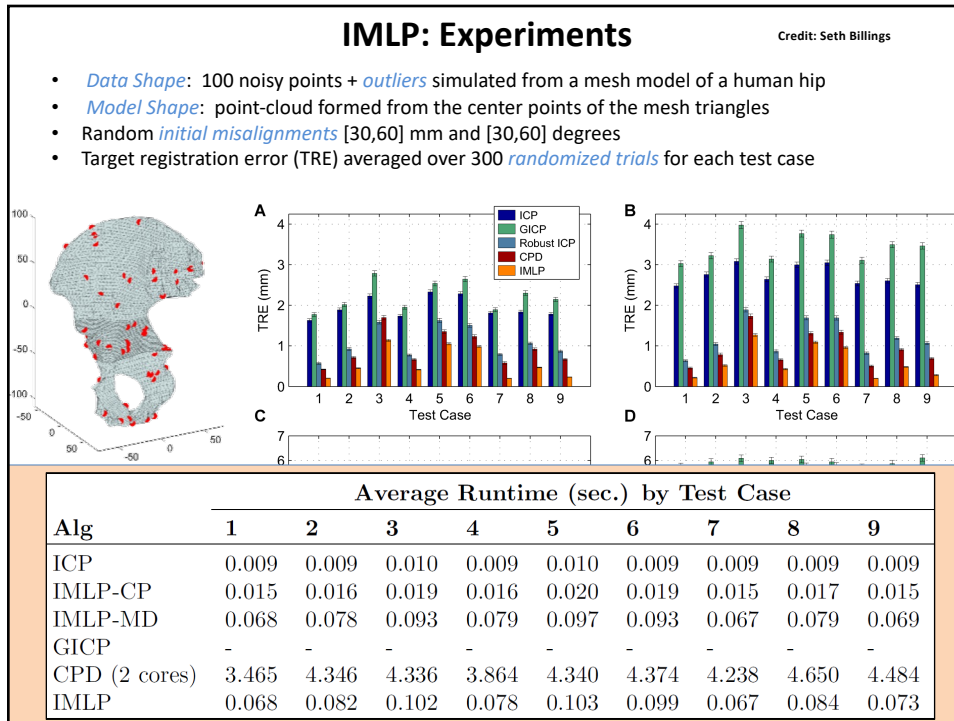
$\Sigma_y = \begin{bmatrix} \Sigma_{y1} & & \\ & \ddots & \\ & & \Sigma_{yn} \end{bmatrix}$

4. Iteratively solve 3b by linear least squares until convergence.

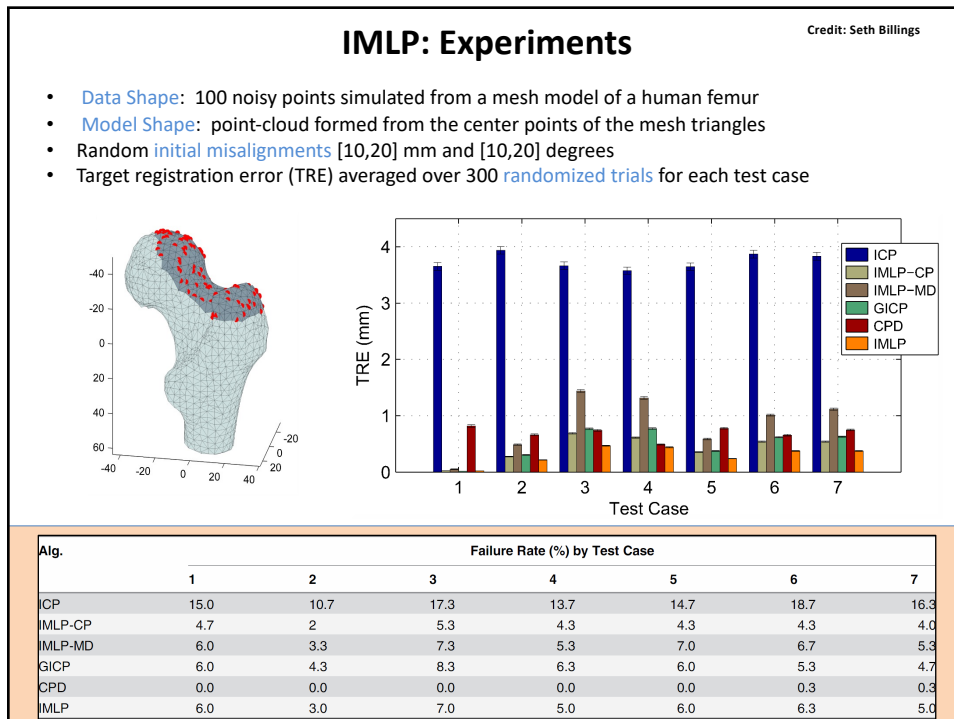
$$\mathbf{R}_{k+1} = \mathbf{R}(d\alpha)\mathbf{R}_k, \quad \mathbf{t}_{k+1} = \mathbf{t}_k + d\mathbf{t}$$

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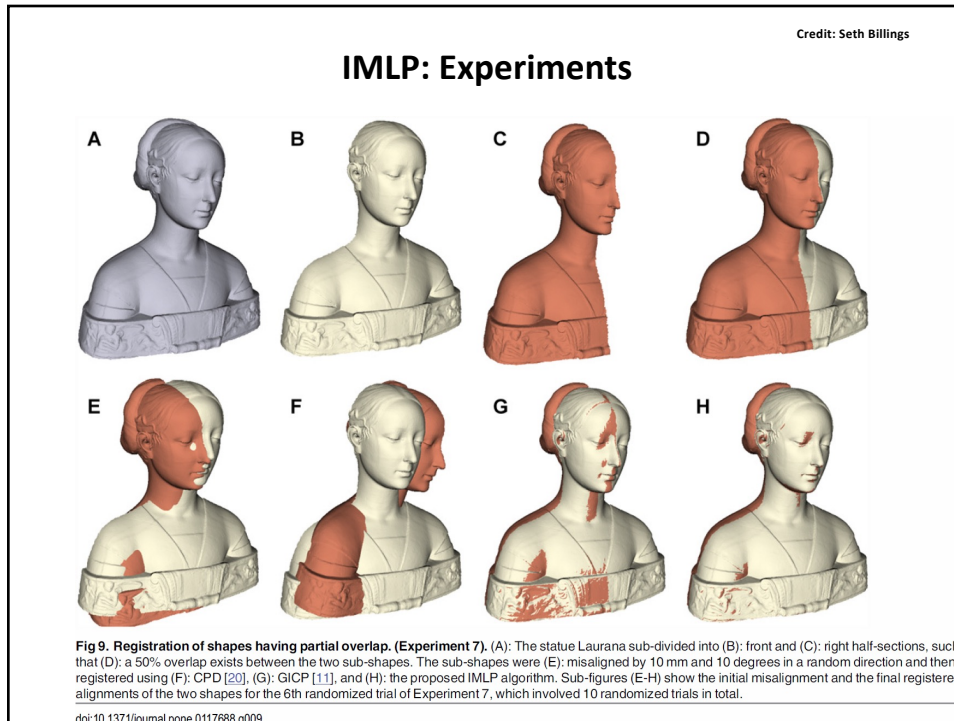
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Iterative Most Likely Oriented Point (IMLOP)

- **Matching Phase:**
for each oriented point in the source shape, find the most likely oriented point on the target shape

$$\mathbf{y}_i = C_{MLP}(T(\mathbf{x}_i), \Psi) = \operatorname{argmax}_{\mathbf{y} \in \Psi} f_{\text{match}}(T(\mathbf{x}_i), \mathbf{y})$$

- **Registration Phase:**
compute transformation to maximize the likelihood (i.e. minimize negative log-likelihood) of oriented point matches

$$T = \operatorname{argmin}_T \left(\frac{1}{2\sigma^2} \sum_{i=1}^n \|\mathbf{y}_{pi} - T(\mathbf{x}_{pi})\|_2^2 - k \sum_{i=1}^n \mathbf{y}_{ni}^T R \mathbf{x}_{ni} \right)$$

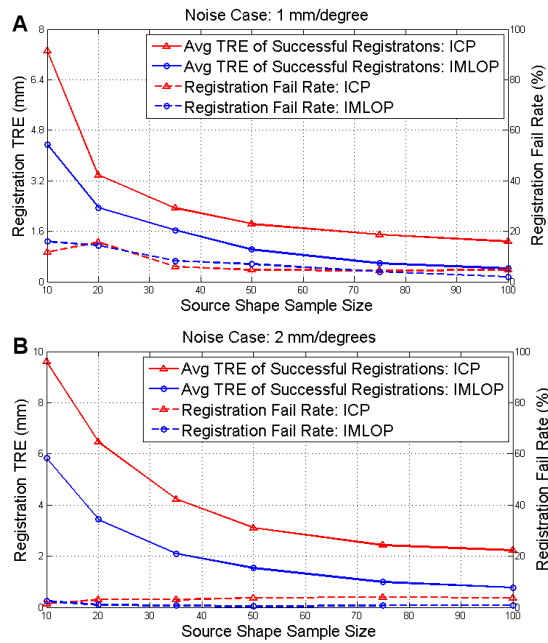
S. Billings and R. H. Taylor, "Iterative Most Likely Oriented Point Registration", in *Medical Image Computing and Computer-Assisted Interventions (MICCAI)*, Boston, October, 2014.

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Average TRE of successful registrations and registration failure rates across all sample sizes for noise levels of 1 (A) and 2 (B) mm [degrees].

Registration failure threshold set to twice the noise level for both position and orientation.



S. Billings and R. H. Taylor, "Iterative Most Likely Oriented Point Registration", in *Medical Image Computing and Computer-Assisted Interventions (MICCAI)*, Boston, October, 2014. (accepted).

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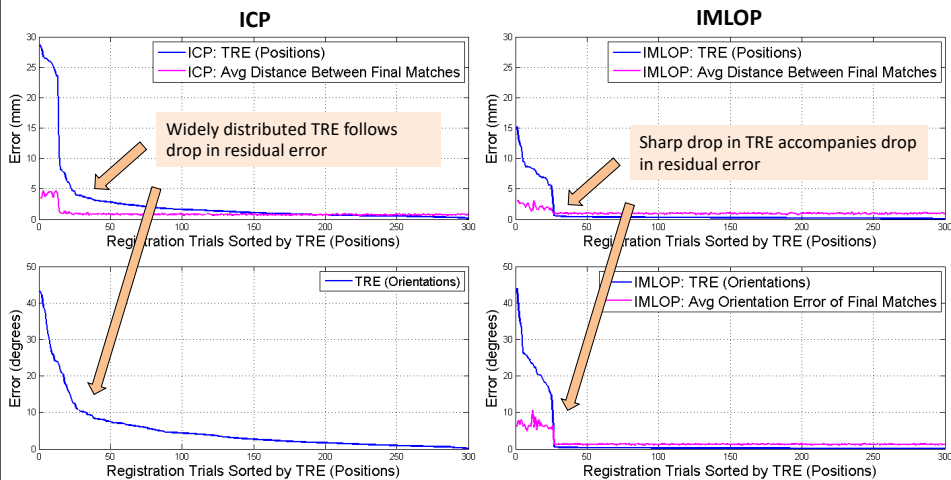
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Experiments

Results from 300 trials within a single sample size (75 points) and noise level (1.0 mm [degree]). NOTE: improved accuracy and failure detection capability for IMLOP.



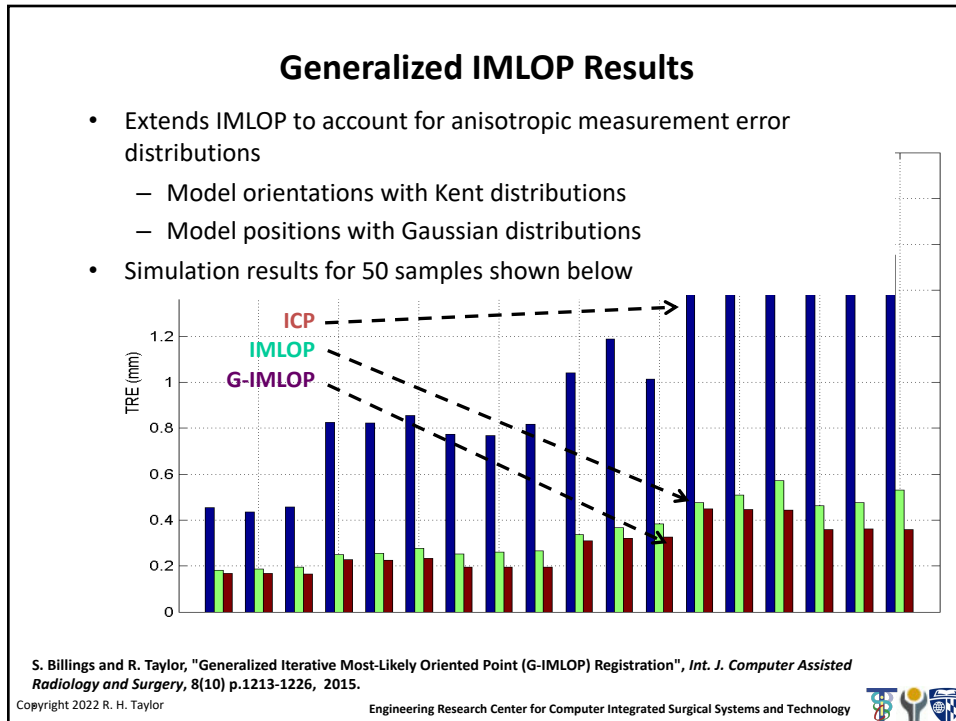
S. Billings and R. H. Taylor, "Iterative Most Likely Oriented Point Registration", in *Medical Image Computing and Computer-Assisted Interventions (MICCAI)*, Boston, October, 2014. (accepted).

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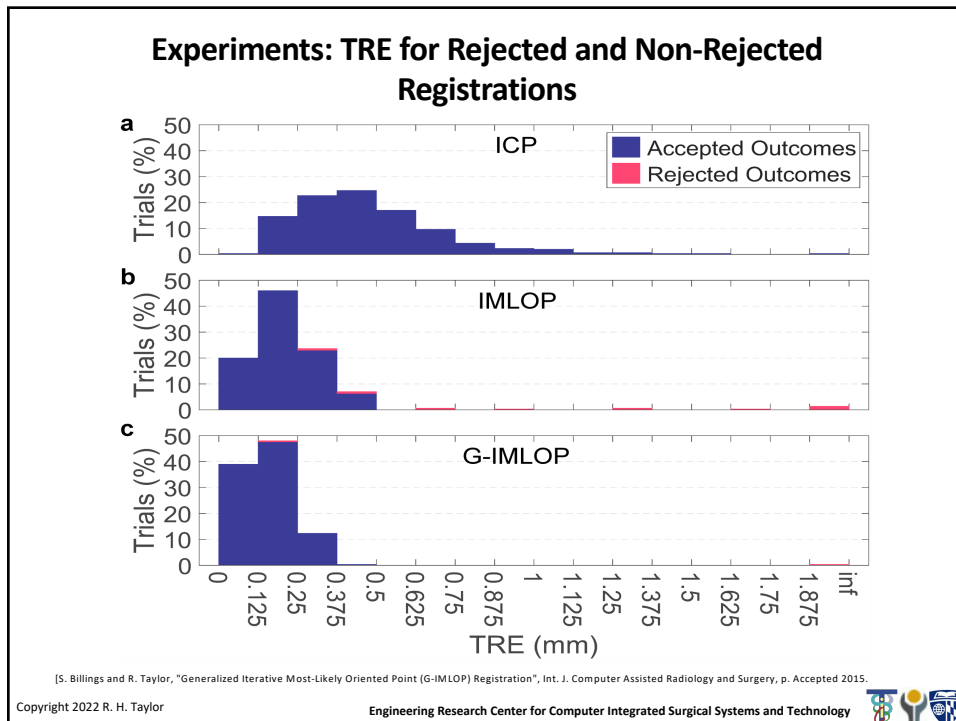
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Ultrasound-assisted Registration

(1) Generate surface model from CT

(2) Digitize proximal bone using tracked pointer

(3) Collect tracked US images of distal bone

(4) Register points/contours to surface model

S. Billings, H. J. Kang, A. Cheng, E. Boctor, P. Kazanides, and R. Taylor, "Minimally invasive registration for computer-assisted orthopedic surgery: combining tracked ultrasound and bone surface points via the P-IMLOP algorithm", Int. J. Computer Assisted Radiology and Surgery, p. (epub ahead of print), 2015. <http://dx.doi.org/10.1007/s11548-015-1188-z> DOI 10.1007/s11548-015-1188-z

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Intensity-based methods

Image 1

Image 2

$\Theta(\rho, \cdot)$

$\Theta(\rho, \text{Image 2})$

$E(\cdot, \cdot)$

Optimization Process

$\rho^* = \text{argmin } E(\text{Im 1}, \Theta(\rho, \text{Im 2}))$

ρ^*

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Intensity-based methods

- Typically performed between images
- The “features” in this case are the intensities associated with pixels (2D) or voxels (3D) in the images.
- General framework:

$$\vec{\rho}^* = \min_{\vec{\rho}} E \left(Image_1, \Theta \left(\vec{\rho}, Image_2 \right) \right)$$

- Methods differ mostly in choice of transformation function $\Theta(\cdot)$ and Energy function $E(\cdot, \cdot)$,

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Typical energy functions (not an exhaustive list)

Normalized image subtraction

$$E(\text{Im}_1, \text{Im}_2) = \sum_{\vec{k}} \frac{|\text{Im}_1[\vec{k}] - \text{Im}_2[\vec{k}]|}{\max_j (|\text{Im}_1[\vec{j}] - \text{Im}_2[\vec{j}]|)}$$

Normalized cross correlation (NCC)

$$E(\text{Im}_1, \text{Im}_2) = \frac{\sum_{\vec{k}} (\text{Im}_1[\vec{k}] - \text{avg}(\text{Im}_1)) (\text{Im}_2[\vec{k}] - \text{avg}(\text{Im}_2))}{\sqrt{\sum_{\vec{k}} (\text{Im}_1[\vec{k}] - \text{avg}(\text{Im}_1))^2} \sqrt{\sum_{\vec{k}} (\text{Im}_2[\vec{k}] - \text{avg}(\text{Im}_2))^2}}$$

Mutual information

$$\rightarrow E(\text{Im}_1, \text{Im}_2) = \sum_{p \in \text{Im}_1, q \in \text{Im}_2} \Pr(p, q) \log \Pr(p, q) - \Pr_{\text{Im}_1}(p) \log \Pr_{\text{Im}_1}(p) - \Pr_{\text{Im}_2}(q) \log \Pr_{\text{Im}_2}(q)$$

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Mutual Information

- First proposed independently in 1995 by Collignon and Viola & Wells.
- Very widely practiced
- Is able to co-register images with very different sensor modalities so long as there is a stable relationship between intensities in one modality with those in another
- Many “flavors” and variations

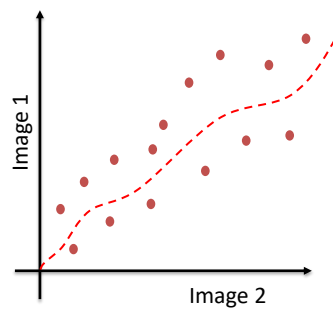
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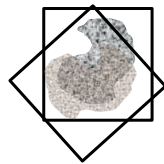


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Mutual Information



- The key idea is that the values of pixels in one image can predict the values of the pixels in the other image, even if the images come from different sensors
- The strength of this prediction will increase as the images become better aligned



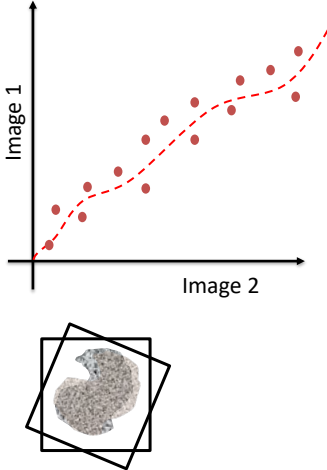
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Mutual Information



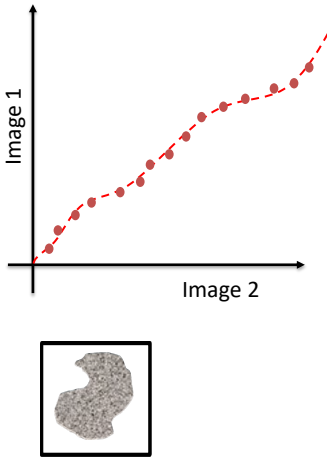
The figure shows a scatter plot with 'Image 1' on the vertical axis and 'Image 2' on the horizontal axis. Red dots represent data points, and a dashed red line indicates a positive correlation. Below the plot is a grayscale image of a kidney, which is tilted to match the slope of the trend line.

- The key idea is that the values of pixels in one image can predict the values of the pixels in the other image, even if the images come from different sensors
- The strength of this prediction will increase as the images become better aligned

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Mutual Information



The figure shows a scatter plot with 'Image 1' on the vertical axis and 'Image 2' on the horizontal axis. Red dots represent data points, and a dashed red line indicates a positive correlation. Below the plot is a grayscale image of a kidney, which is square and perfectly aligned with the axes.

- The key idea is that the values of pixels in one image can predict the values of the pixels in the other image, even if the images come from different sensors
- The strength of this prediction will increase as the images become better aligned

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Mutual Information

Entropy

$$H(a) = -\Pr(a) \log \Pr(a)$$

$$H(a,b) = -\Pr(a,b) \log \Pr(a,b)$$

Mutual Information (Viola & Wells '95, Colligen '95)

$$\text{Similarity}(A,B) = H(A) + H(B) - H(A,B)$$

Normalized mutual information (Maes et al. '97)

$$\text{Similarity}(A,B) = \frac{H(A) + H(B)}{H(A,B)}$$

Objective function

$$E(\text{Im}_1, \text{Im}_2) = -\text{Similarity}(\text{Im}_1, \text{Im}_2)$$

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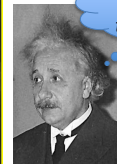
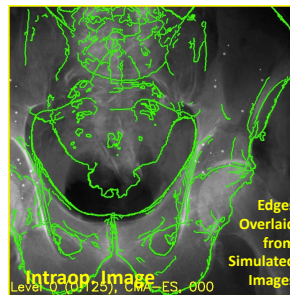
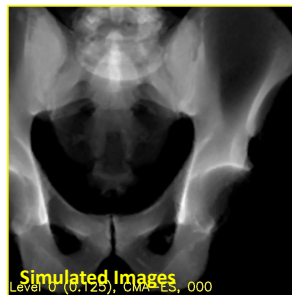


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Basic Idea of Intensity-Based 2D/3D Registration

- Assumes a pre-op CT is available
- Simulate many C-Arm images and choose the most similar to the intraoperative image
- Solves the following optimization problem:

$$\underset{\theta \in SE(3)}{\text{argmin}} \mathcal{S}(I_{\text{Intra-Op}}, \mathcal{P}(\theta, I_{\text{CT}}))$$



Do these look the same yet?

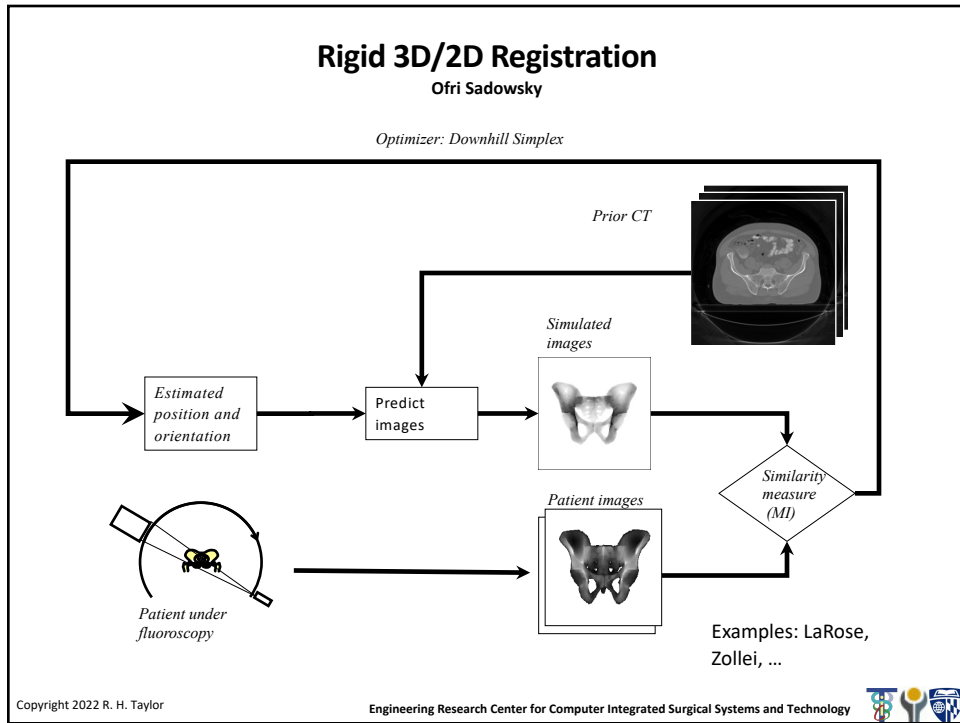
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BACS
Biomechanical and Image-guided Surgical Systems Laboratory

A clinical example (periacetabular osteotomy)

Problem: Acetabular Dysplasia

Normal hip bones Hip dysplasia

Image Source: ouh.nhs.uk

Dislocation Caused by Dysplasia

Image Source: James Heilman, MD

Slide credit: Robert Grupp

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Surgical Systems Laboratory

A clinical example (periacetabular osteotomy)

One Solution: Periacetabular Osteotomy (PAO)

Image Source: Ganz 1988

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Biomechanical and Image-guided
Surgical Systems Laboratory

A clinical example (periacetabular osteotomy)

Goal: Automatic visualization and guidance

Total Rotation: 20.5°
 Anterior/Posterior Rotation: 3.7°
 Left/Right Rotation: 16.3°
 Inferior/Superior Rotation: 12.5°

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Movement of the Osteotomy Fragment is Challenging



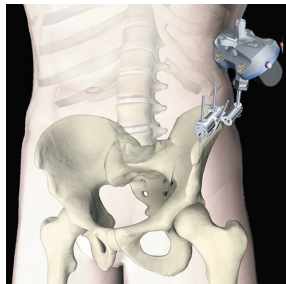
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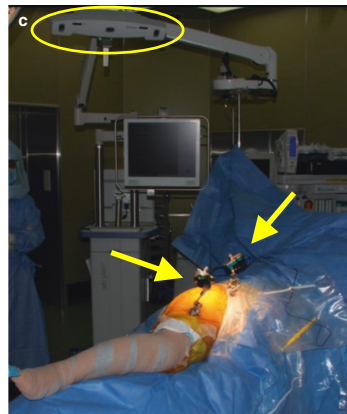


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One Approach for Computer-Assistance: Optical Tracking Devices



Source: Stiehl and Thornberry, 2016



Source: Sugano, CAOS for Hip and Knee, 2018

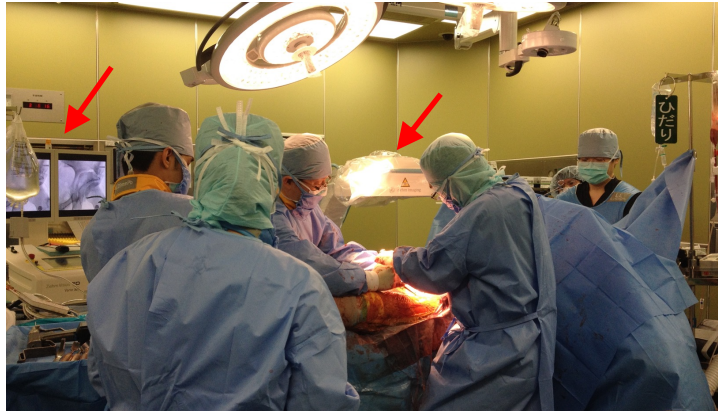
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Intraoperative Fluoroscopy is Available



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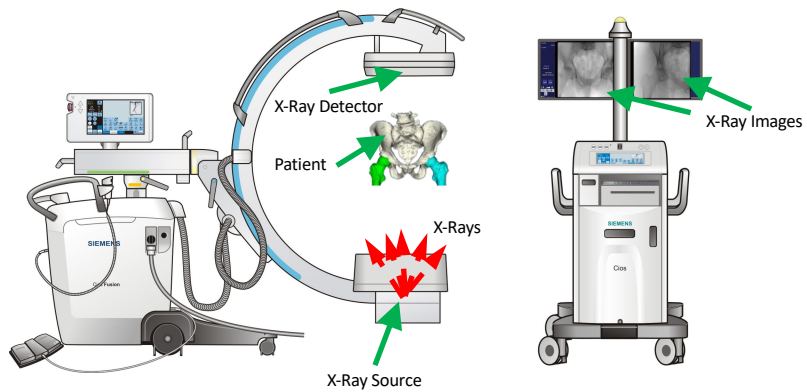
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Intraoperative X-Ray Imaging with Mobile C-Arm



C-Arm Image Source: Siemens CIOS Fusion Manual

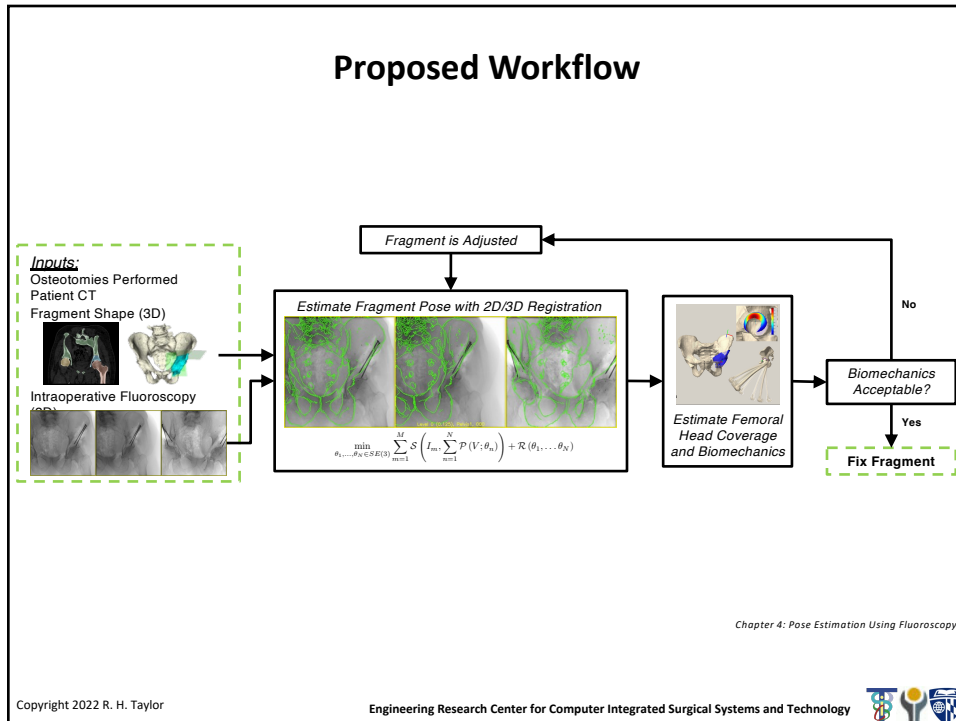
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3D-2D Registration of Osteotomy Fragments

$$\arg \min_{\theta_1, \dots, \theta_N \in SE(3)} \sum_{m=1}^M \mathcal{S} \left(I_m, \sum_{n=1}^N \mathcal{P}_m(I_{CT}; \theta_n) \right)$$

Fixed Images with Moving Image Edges


Moving Images

Level 0 (0.125), pelvis, 000


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3D-2D Registration of Osteotomy Fragments




- Compute the Sobel derivatives in the X and Y directions of the two input images:


$$\nabla_X I_1, \nabla_X I_2, \nabla_Y I_1, \nabla_Y I_2$$

- Compute NCC between the corresponding gradient images:


$$S(I_1, I_2) = NCC(\nabla_X I_1, \nabla_X I_2) + NCC(\nabla_Y I_1, \nabla_Y I_2)$$



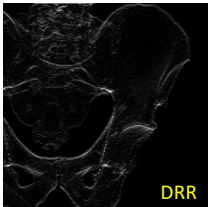
$\nabla_X I_1$



$\nabla_X I_2$



$\nabla_Y I_1$



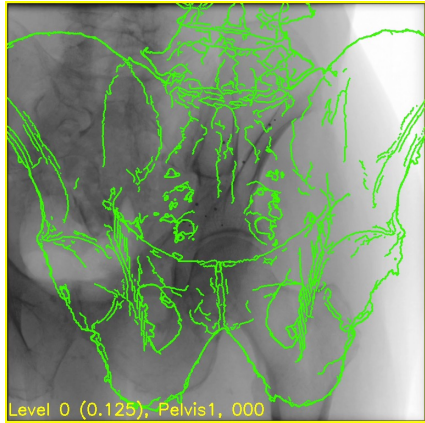
$\nabla_Y I_2$

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Initialize Using a Nominal AP View?

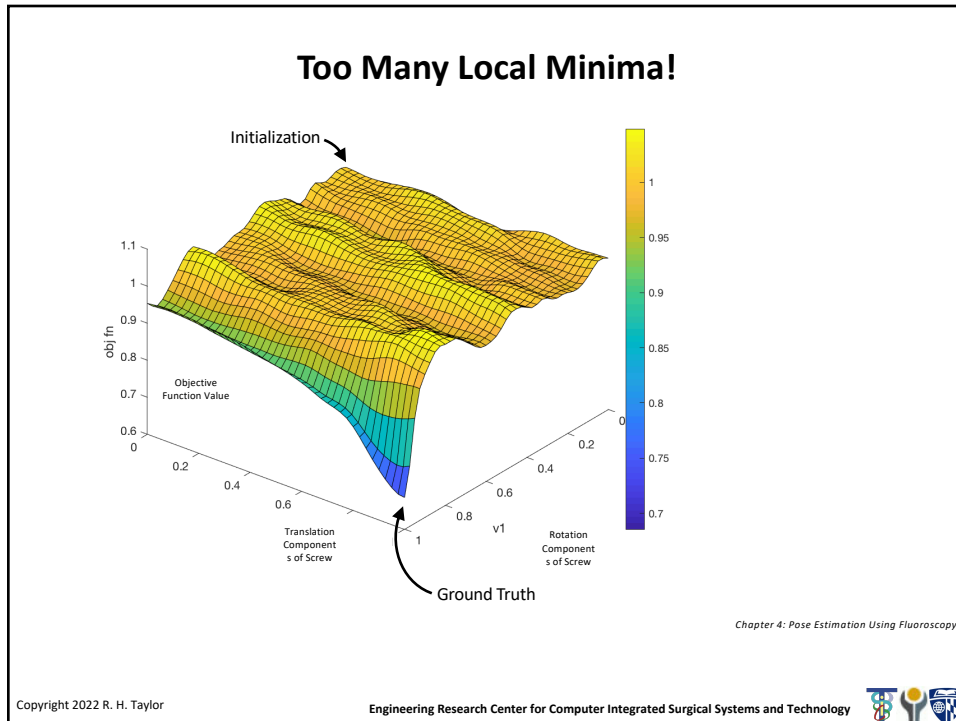


Level 0 (0.125), Pelvis1, 000

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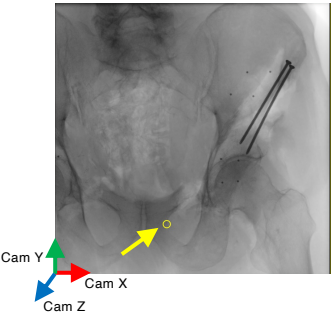
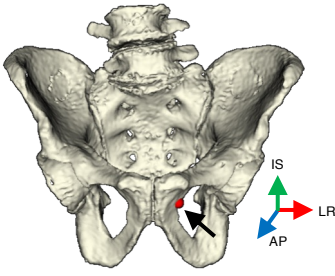
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Use a Single Landmark to Initialize Registration

- Assume the pelvis is in an AP orientation – this may be computed preoperatively
- Manually annotate a single landmark to recover translation

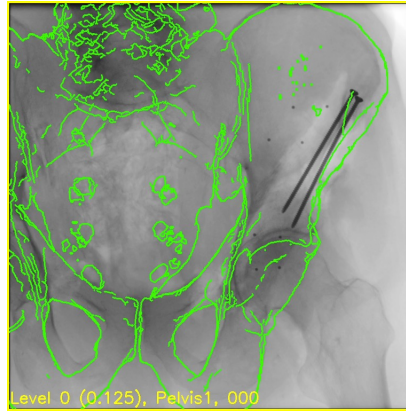
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Example of a Single Landmark Initialization



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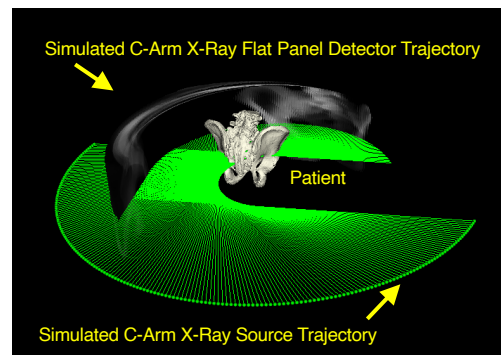
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Automatically Initialize Second and Third Views

- Constrain C-arm motion to orbital rotation
- Perform an exhaustive search over $\pm 90^\circ$ in 1° increments



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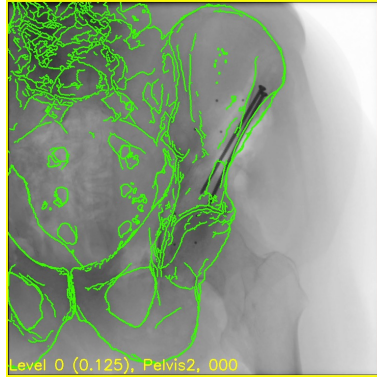
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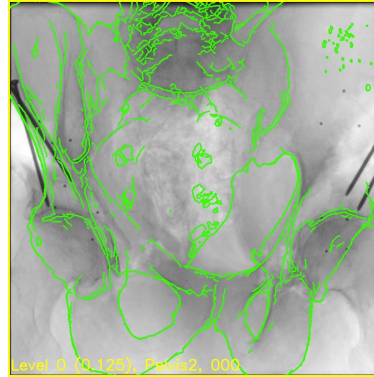
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Example Initializations From Orbital Search

View #2



View #3



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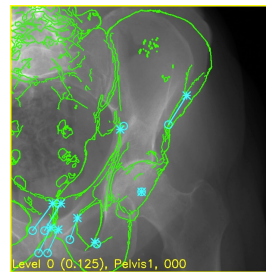
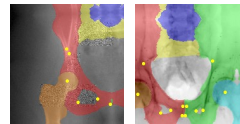
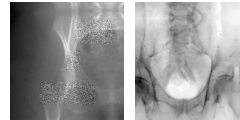
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Automatic Landmark-Based Initialization

- Train a CNN to recognize approximate landmark positions in x-ray images
- Use landmark-based 2D-3D registration to initialize registration
- Combine landmark and intensity objective functions
- Use segmentation labels to ignore intensities of irrelevant anatomy



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Why Not Simultaneously Use Intensities and Features?

- Registration objective function:

$$\min_{\theta_P, \theta_{LF}, \theta_{RF} \in SE(3)} \lambda \mathcal{S}(\mathcal{P}(\theta_P, \theta_{LF}, \theta_{RF}), I) + (1 - \lambda) \mathcal{R}(\theta_P, \theta_{LF}, \theta_{RF})$$

↑ Image Similarity Term ↑ Regularization Term

- Usually, regularization penalizes the amount of rotation and translation away from initialization
- Why not directly include the landmark re-projection as regularization?

$$\mathcal{R}(\theta_P) = \frac{1}{2\sigma_\ell^2} \sum_{l=1}^{N_\ell} \left\| \mathcal{P}\left(p_{3D}^{(l)}, \theta_P\right) - p_{2D}^{(l)} \right\|_2^2$$

- Can also think of this as running landmark registration and regularizing on image appearance

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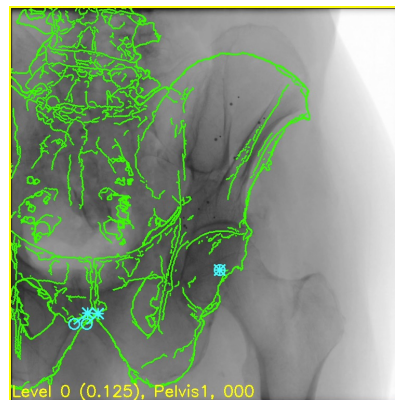
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Include Landmark Reprojection Into Objective Function

- Landmarks Detected in 2D are Shown as Cyan Circles
- Landmarks Projected from 3D are Shown as Cyan Asterisks *
- Cyan Lines Indicate Correspondence
- The Initial Pose Aligns the 2D and 3D Left Femoral Head Centers



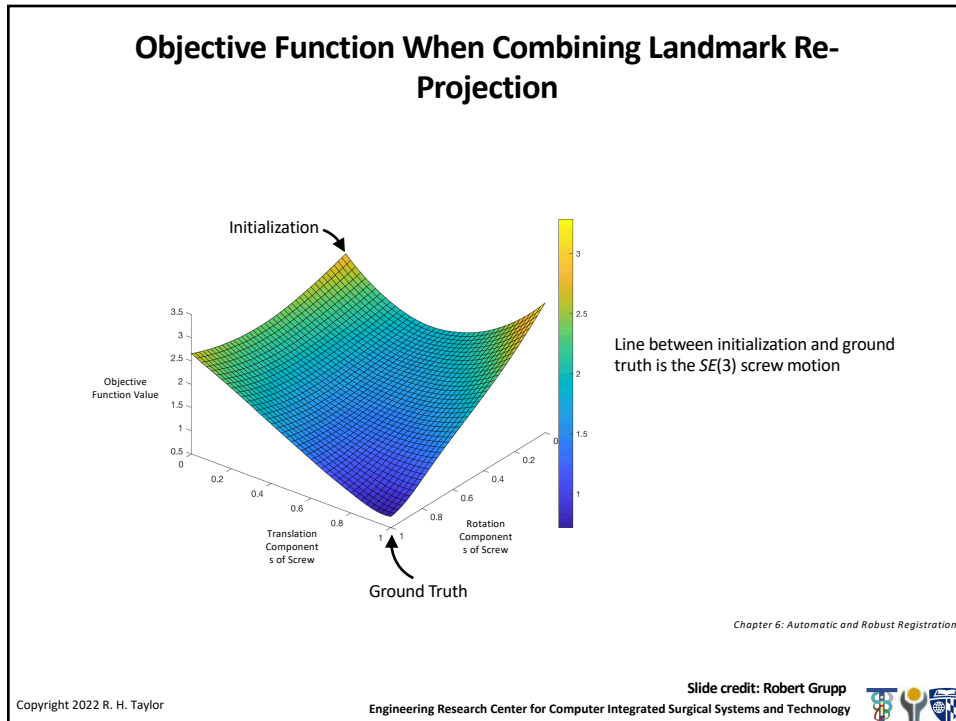
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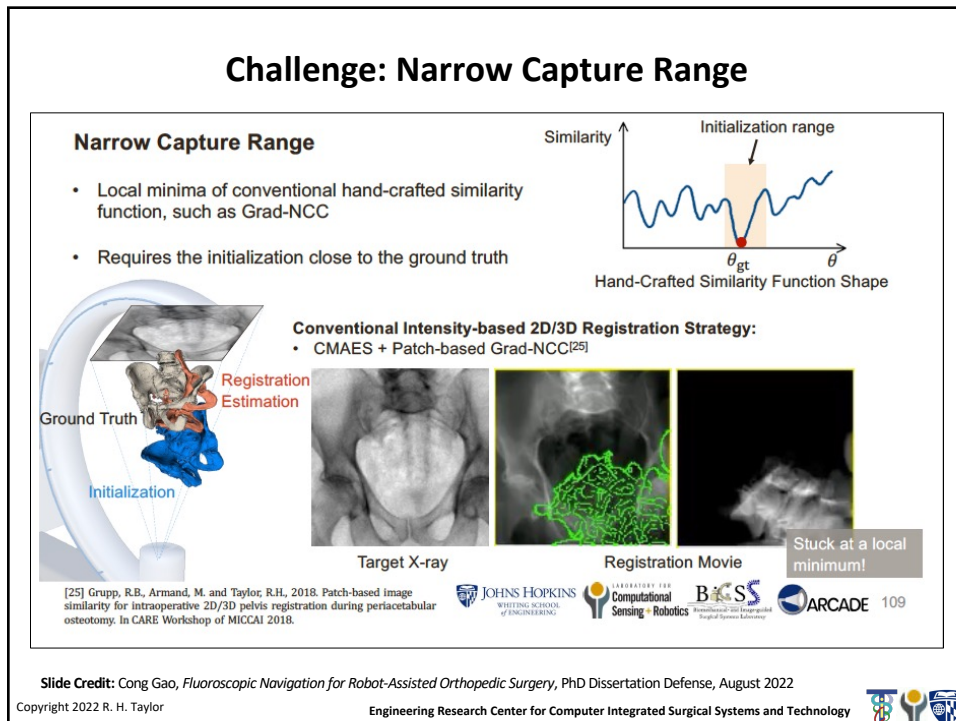
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Can CNNs Help?

Pose Regression Methods

Limitations:

- Learning a mapping function from 2D projections is an **ill-posed problem**, which is prone to **strongly overfit to training domain**
- Direct pose regression is unconstrained**, which can change dramatically if the input image appearance has a tiny difference

Target X-ray DRR → CNN Encoder → $\theta \in SE(3)$

High-level Prototype

[26] Miao, S., Wang, Z.J. and Liao, R., 2016. A CNN regression approach for real-time 2D/3D registration. IEEE transactions on medical imaging, 35(5), pp.1352-1363.

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Can CNNs Help?

A More Desired Solution

Similarity vs θ

Initialization range

θ_{gt}

Hand-Crafted Similarity Function Shape

→

Similarity vs θ

Extended capture range!

θ_{gt}

Network Similarity Function Shape

Traditional DRR Projector is not Differentiable!

- Oversized system matrix $A(\theta)$ to fit in memory
- Conventional optimization strategies are numeric-based methods, such as CMAES

3D volume: V

2D projection: $I = P(V; \theta)$

$\frac{\partial P(V, \theta)}{\partial V}$ $\frac{\partial P(V, \theta)}{\partial \theta}$

[27] Hansen, N., Müller, S.D. and Koumoutsakos, P., 2003. Reducing the time complexity of the derandomized evolution strategy with covariance matrix adaptation (CMA-ES). Evolutionary computation, 11(1), pp.1-18.

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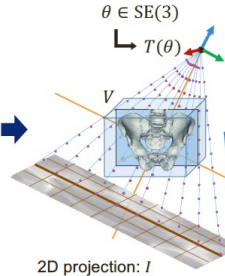
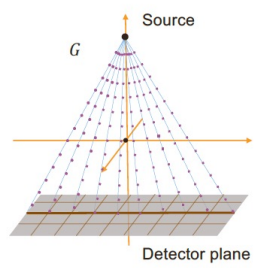
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Differentiable DRR Operator (Gao)

ProST -- Differentiable DRR Operator

Spatial Sampling Grid: G

- Follows projection geometry



Given a pose parameter θ and 3D volume V as input, the DRR projection is:

$$I = P(V, \theta) = \text{sum}(\text{interp}(V, T(\theta)G))$$

↓ Summation along the projection direction
↓ Linear interpolation
↓ Linear transformation

Breakthrough:

$$\frac{\partial P(V, \theta)}{\partial \theta} \quad \frac{\partial P(V, \theta)}{\partial V} \text{ are differentiable!}$$

[28] Gao, C., Liu, X., Gu, W., Killeen, B., Ammand, M., Taylor, R., & Unberath, M. (2020, October). Generalizing spatial transformers to projective geometry with applications to 2D/3D registration. In International Conference on Medical Image Computing and Computer-Assisted Intervention (pp. 329-339). Springer, Cham.



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ProST -- Projective Spatial Transformers (Gao)

PYTORCH
C++, CUDA backend

How do we learn a better similarity function?

Network Similarity Function Shape

Example registration using Grad-NCC similarity, optimized by PyTorch built-in SGD optimizer

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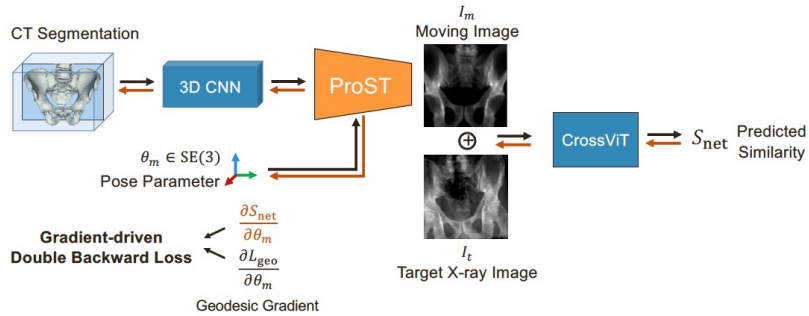


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ProST – 2D/3D Registration Pipeline (Gao)

Target Similarity Function -- Geodesic Loss

- A convex objective function with respect to SE(3) pose parameters



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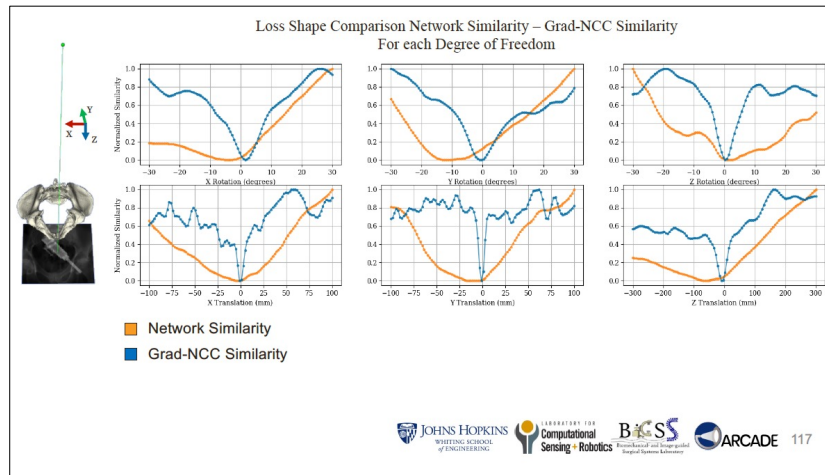
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ProST – 2D/3D Registration Pipeline (Gao)

Loss Shape Comparison Network Similarity – Grad-NCC Similarity For each Degree of Freedom



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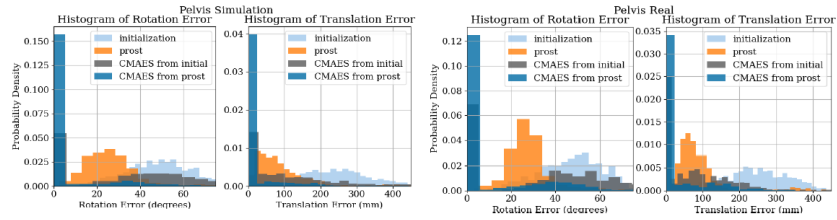
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ProST – 2D/3D Registration Pipeline (Gao)

- Our pipeline was trained using 19 CT scans, evaluated on 1,000 synthetic X-rays and 200 real X-rays



Success Rate (%): Mean Target Registration Error < 10 mm

	Simulation	Real
CMAES from Initialization	32.6	36.0
CMAES from ProST Registration	82.6	73.2



Slide Credit: Cong Gao, *Fluoroscopic Navigation for Robot-Assisted Orthopedic Surgery*, PhD Dissertation Defense, August 2022

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