



NSF Engineering Research Center  
for Computer Integrated Surgical  
Systems and Technology



LABORATORY FOR  
**Computational  
Sensing + Robotics**  
THE JOHNS HOPKINS UNIVERSITY

## Segmentation and Modeling

CIS I – 600.455/655

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Note: This lecture contains many slides from colleagues,  
including Jerry Prince, Eric Grimson, and Ayushi Sinha.

I have tried to make appropriate acknowledgments on  
the sides



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## Segmentation & Modeling

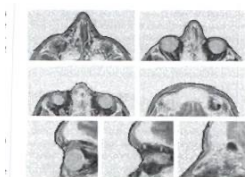


FIGURE 4.2 Here we represent the surface once we have reached a minimum of the energy  $E$ . Some vertical and horizontal cross-sections of the surface are given. They show an accurate localization of the surface at the edge points.

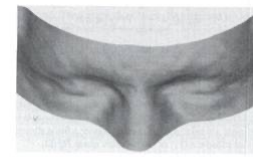


FIGURE 4.3 A 3D representation of the surface depicted in figure 4.2.

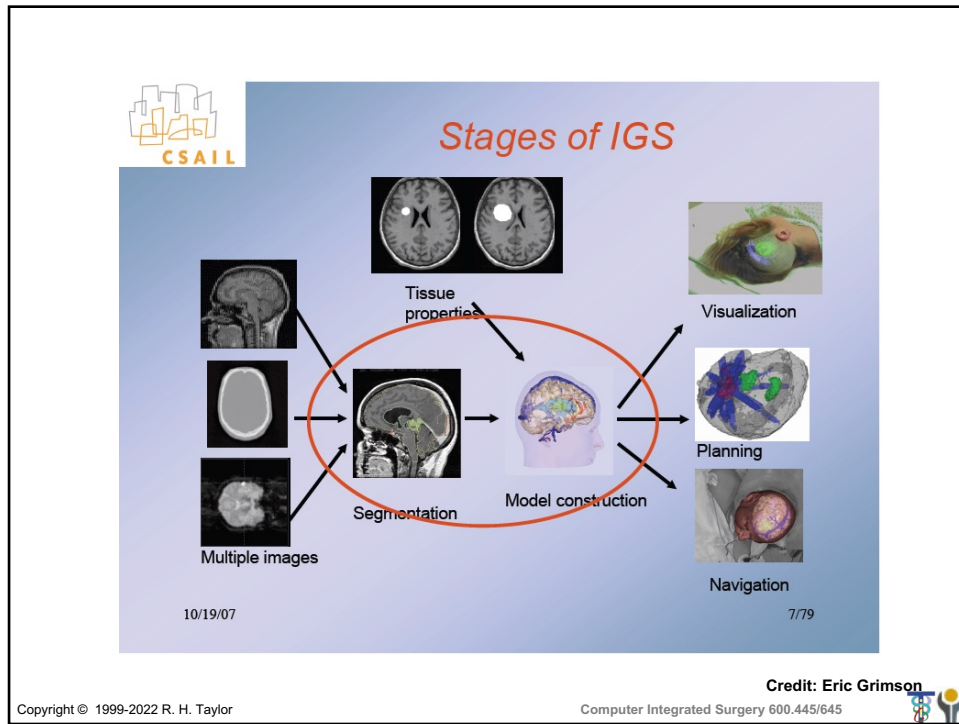
Images

Segmented  
Images

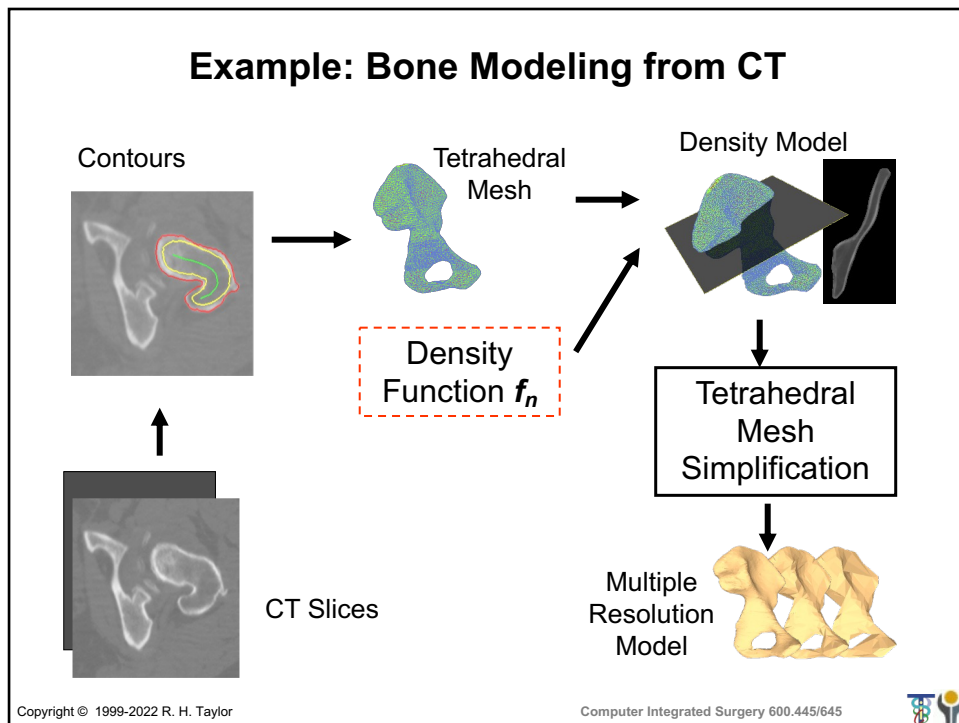
Models



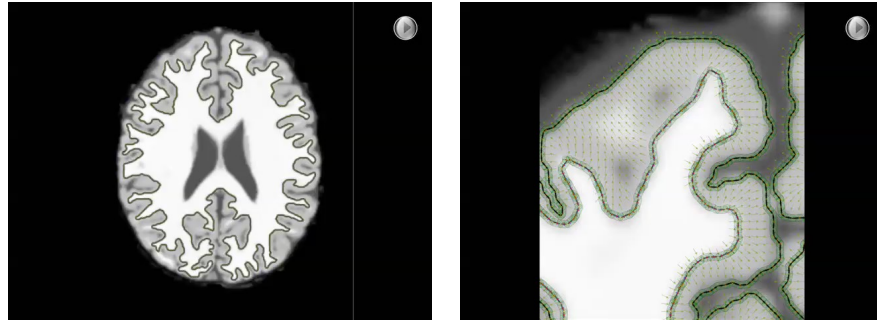
2



3



4




Brain Examples: Blake Lucas

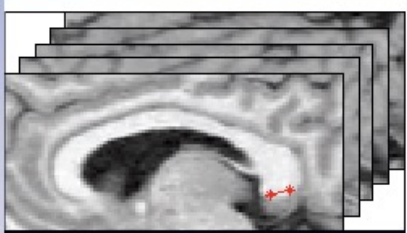


## Image Segmentation

- Process of identifying structure in 2D & 3D images
- Output may be
  - labeled pixels
  - edge map
  - set of contours




 **Manual Segmentation  
(Outlining)**



- Extremely time-consuming (~6 hours per case)
- 3D Imagery – Performed slice at a time
- Some structures near impossible (blood vessels)


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### Automation Approaches

- Pixel-based
  - Thresholding
  - Region growing
  - Machine learning approaches
- Edge/Boundary based
  - Contours/boundary surface
  - Deformable warping
  - Deformable registration to atlases

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## Thresholding

|   |   |    |    |    |    |   |
|---|---|----|----|----|----|---|
| 3 | 5 | 7  | 3  | 4  | 2  | 1 |
| 2 | 4 | 9  | 10 | 22 | 9  | 3 |
| 3 | 5 | 12 | 11 | 15 | 10 | 3 |
| 5 | 6 | 11 | 9  | 17 | 19 | 1 |
| 2 | 3 | 11 | 12 | 18 | 16 | 2 |
| 3 | 6 | 8  | 10 | 18 | 9  | 5 |
| 4 | 6 | 7  | 8  | 3  | 3  | 1 |

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## Thresholding

|   |   |    |    |    |    |   |
|---|---|----|----|----|----|---|
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| 3 | 6 | 8  | 10 | 18 | 9  | 5 |
| 4 | 6 | 7  | 8  | 3  | 3  | 1 |

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## Thresholding

|   |   |    |    |    |    |   |
|---|---|----|----|----|----|---|
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## Thresholding

|   |   |    |    |    |    |   |
|---|---|----|----|----|----|---|
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| 4 | 6 | 7  | 8  | 3  | 3  | 1 |

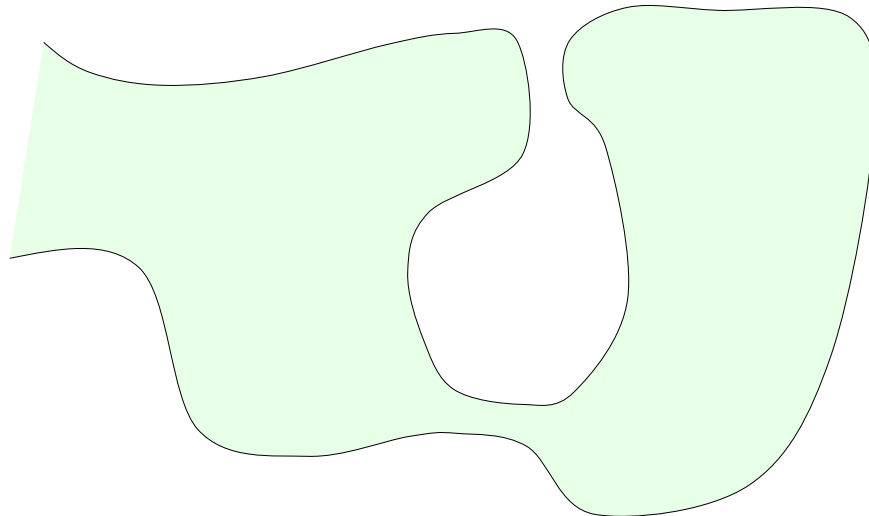
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## “Partial volume” effects



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## “Partial volume” effects

|     |     |     |    |    |     |     |     |  |
|-----|-----|-----|----|----|-----|-----|-----|--|
|     |     |     | 80 | 60 | 90  | 100 | 100 |  |
| 100 | 100 | 100 | 90 | 55 | 60  | 100 | 100 |  |
|     | 100 | 100 | 55 | 0  | 40  | 100 |     |  |
|     |     | 100 | 60 | 0  | 70  | 100 |     |  |
|     |     | 60  | 50 | 45 | 100 | 98  |     |  |

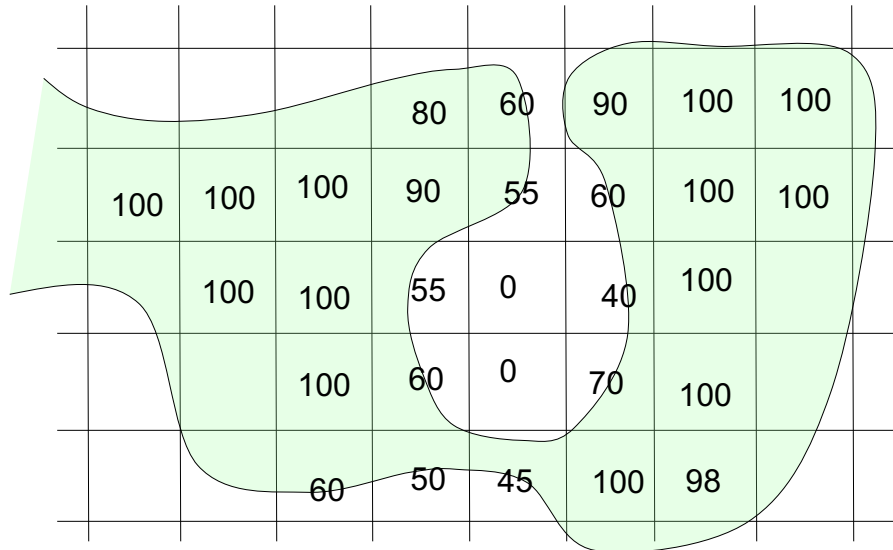
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### “Partial volume” effects



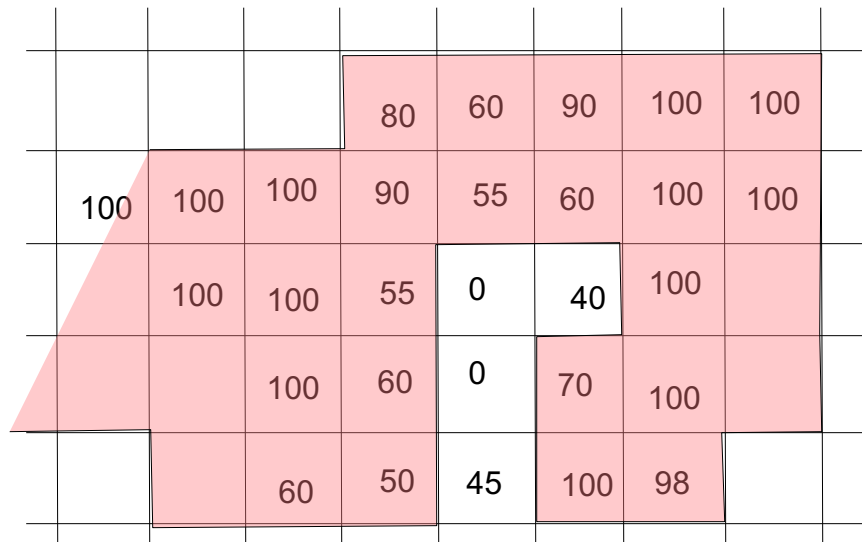
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### “Partial volume” effects



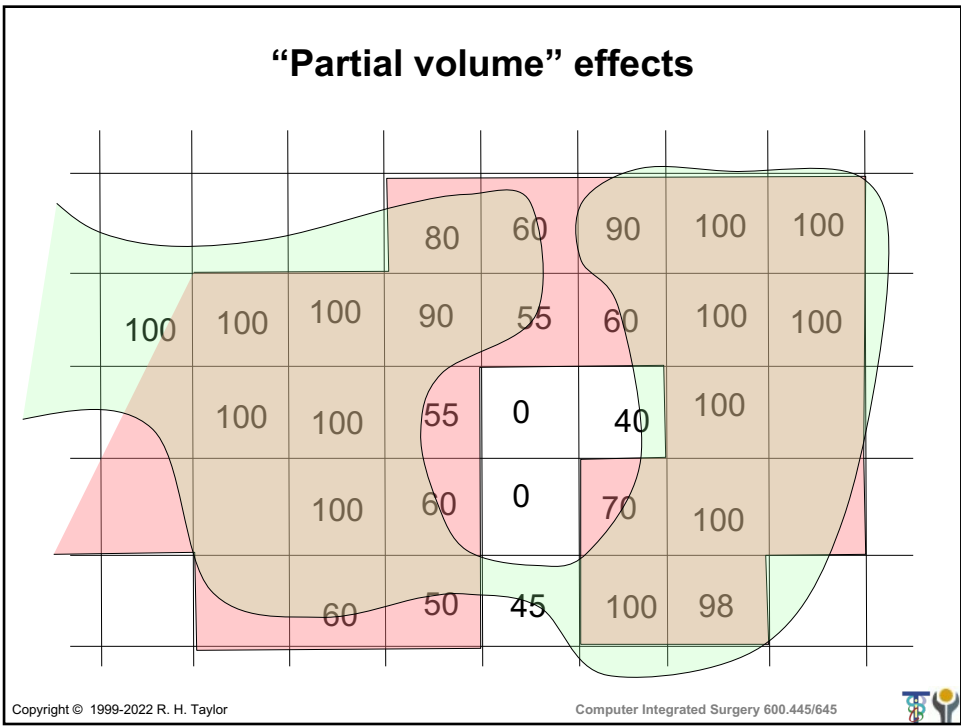
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## Segment statistically

- Measure distribution of intensities at known tissue locations
- Use nearest neighbor style classifiers for all other voxels

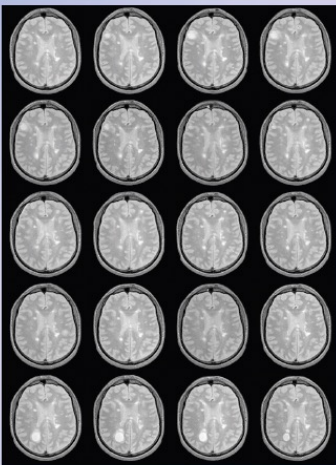
Credit: Eric Grimson

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## Standard Scans



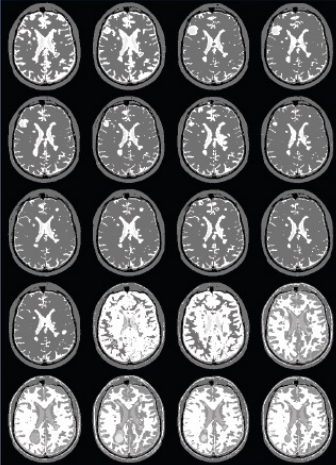
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## Statistical segmentation



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## Between Scylla and Charybdis

- Problem: imagery contains non-linear gain artifacts that shift the intensity values in a non-stationary way
- If one knew the gain field, could correct image and use standard statistical method
- If one knew the tissue types, could predict the image and find the gain field correction
- Solution: Use Expectation/Maximization method to iteratively solve for gain field and tissue class, using probabilistic models

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## EM-Segmentation [Wells 1994]

E-Step

Compute tissue posteriors using current intensity correction.



Estimate intensity correction using residuals based on current posteriors.



M-Step

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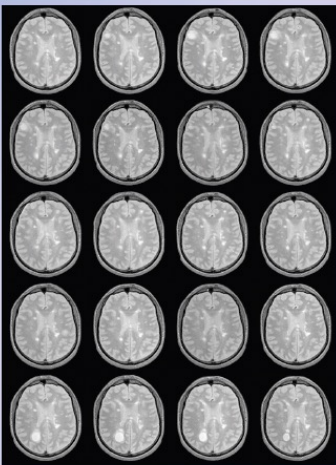
Credit: Eric Grimson




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 **CSAIL**


## Standard Scans



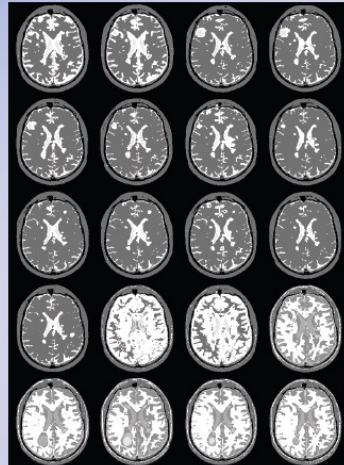
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## Statistical segmentation



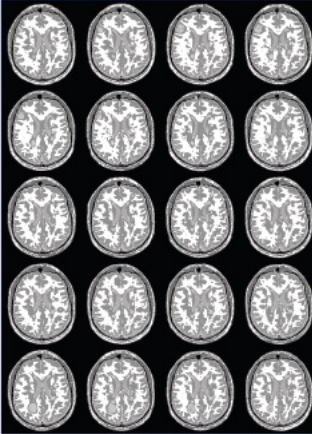
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## Gain Corrected Scans



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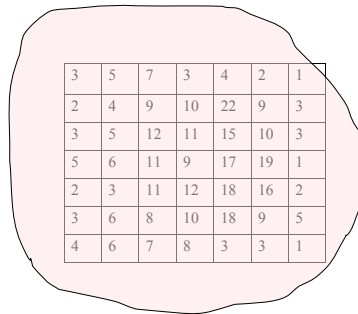
## Deformable Surfaces

|   |   |    |    |    |    |   |
|---|---|----|----|----|----|---|
| 3 | 5 | 7  | 3  | 4  | 2  | 1 |
| 2 | 4 | 9  | 10 | 22 | 9  | 3 |
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## Deformable Surfaces



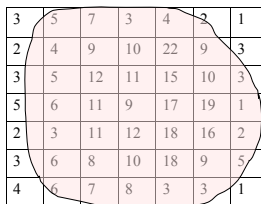
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## Deformable Surfaces



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## Deformable Surfaces

|   |   |    |    |    |    |   |
|---|---|----|----|----|----|---|
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| 4 | 6 | 7  | 8  | 3  | 3  | 1 |

- Basic concepts proposed by Demetri Terzopoulos
  - M. Kass, A. Witkin, and D. Terzopoulos, "Snakes:Active Contour Models", *Intl Journal of Computer Vision*, pp. 321-331, 1988.
- Many refinements since then

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## Traditional Active Contour

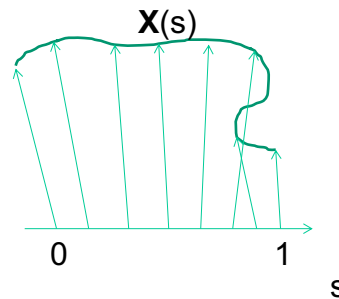
- Initialize a curve  $\mathbf{X}(s)$  around or near the object boundary
- Find  $\mathbf{X}(s)$  that minimizes:

$$E = \int_0^1 \left[ \frac{1}{2} \{ \alpha |\mathbf{X}'(s)|^2 + \beta |\mathbf{X}''(s)|^2 \} + E_{\text{ext}} \{ \mathbf{X}(s) \} \right] ds$$

- Where  $\alpha = 0.001$ ,  $\beta = 0.09$   
and

$$E_{\text{ext}}(x, y) = -\|\nabla f(x, y)\|^2$$

- How to find  $\mathbf{X}(s)$ ?



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## Dynamic Equation From E-L Equation

- Euler-Lagrange equation

$$\frac{\partial}{\partial s} \left( \alpha \frac{\partial \mathbf{X}}{\partial s} \right) - \frac{\partial^2}{\partial s^2} \left( \beta \frac{\partial^2 \mathbf{X}}{\partial s^2} \right) - \nabla P(\mathbf{X}) = 0$$

- Make  $\mathbf{X}$  dynamic:  $X(s) \rightarrow X(s, t)$

$$\mathbf{X}(s, t) = [X(s, t), Y(s, t)]$$

where  $s \in [0, 1]$

- Now set “in motion” – gradient descent

$$\gamma \frac{\partial \mathbf{X}}{\partial t} = \frac{\partial}{\partial s} \left( \alpha \frac{\partial \mathbf{X}}{\partial s} \right) - \frac{\partial^2}{\partial s^2} \left( \beta \frac{\partial^2 \mathbf{X}}{\partial s^2} \right) - \nabla P(\mathbf{X})$$

- General dynamical equation for snake:

$$\gamma \mathbf{X}_t = \mathbf{F}_{\text{int}} + \mathbf{F}_{\text{ext}}$$

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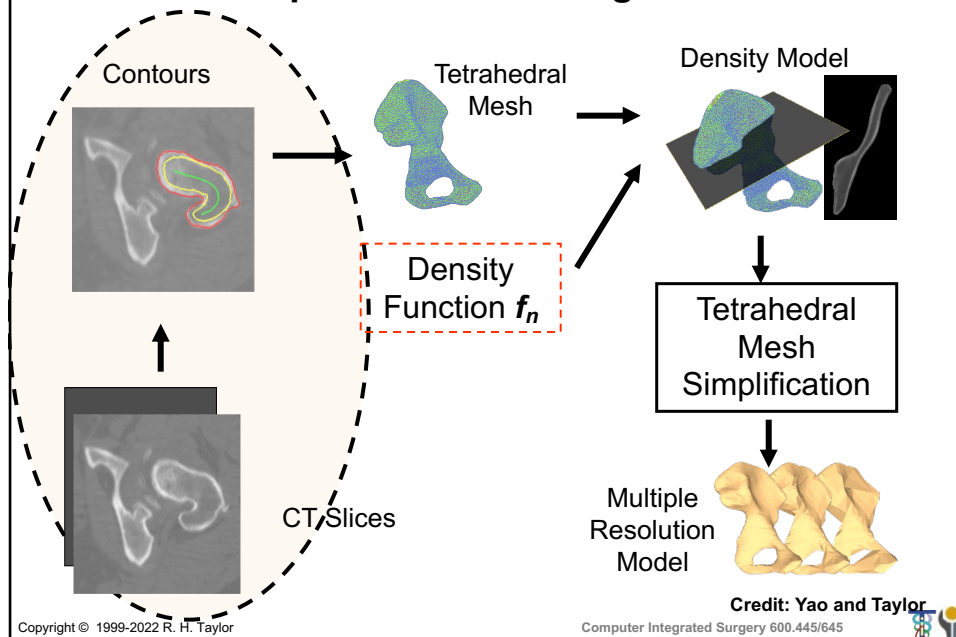
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## Example: Bone Modeling from CT

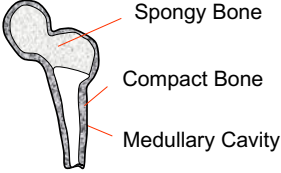


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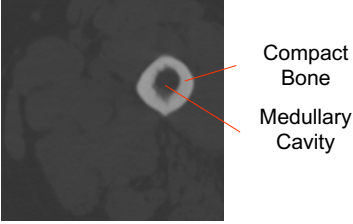
### Bone Structure


- Compact bone
- Spongy bone
- Medullary Cavity



**Bone Structure**

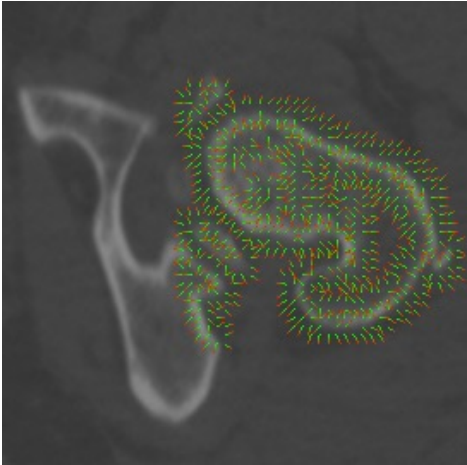




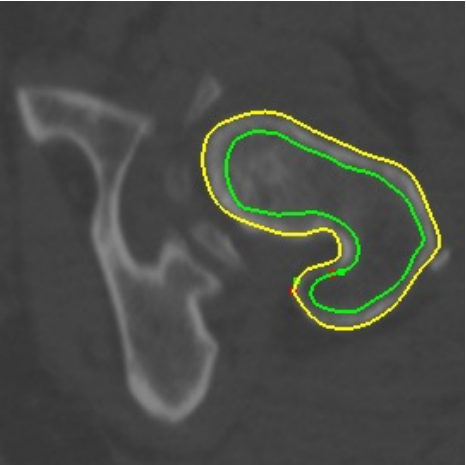
**Credit: Yao and Taylor**  
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### Bone Contour Extraction



**Needle graph of Image force**

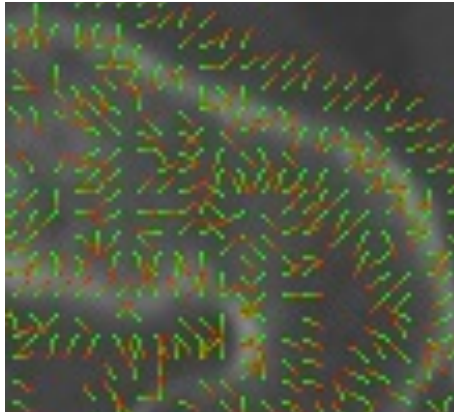


**Bone Contours**

**Credit: Yao and Taylor**  
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## Bone Contour Extraction Closer-up view



Needle graph of Image force



Bone Contours

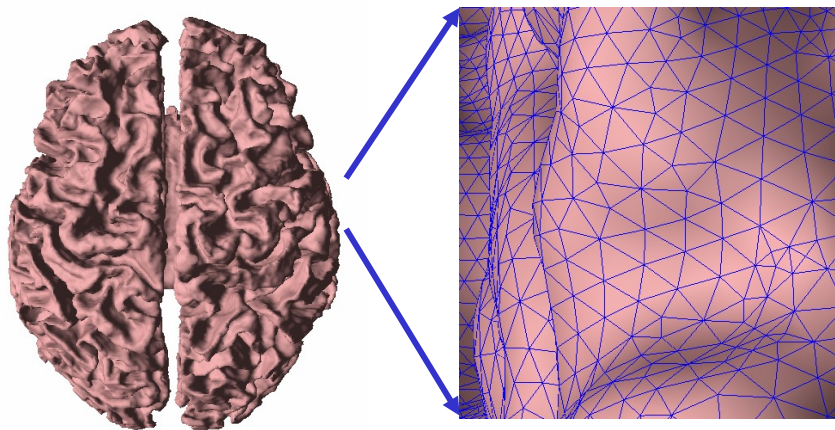
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## 3D Deformable Surface Model

Commonly done with triangle mesh



- Added complexity, time, especially to avoid self-intersection

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## Critique of Parametric Models

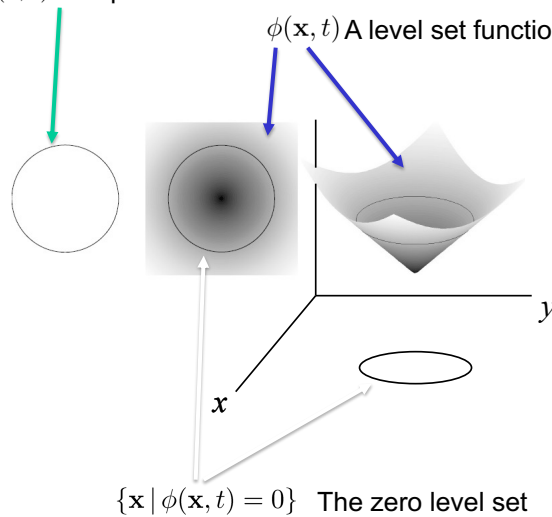
- Advantages:
  - explicit equations, direct implementation
  - automatic topology control
- Disadvantages:
  - costly to prevent overlaps
  - requires reparameterization to space out triangles



## Basic Idea of Geometric Active Contours

$\mathbf{X}(s, t)$  The parametric curve

$\phi(x, t)$  A level set function



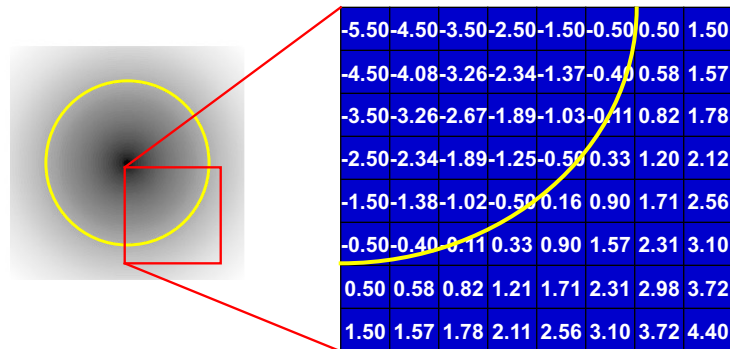
- The level set function is usually a signed distance function
- Convention:
  - positive on outside
  - negative on inside

$\{\mathbf{x} \mid \phi(\mathbf{x}, t) = 0\}$  The zero level set



## GDM: Geometric Deformable Model

- Conventional level set function  $\phi(x,t)$ 
  - signed distance function
- Change the values of  $\phi \rightarrow$  move the contour



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## Philosophy of GDMs

- Curve is not parameterized until the end of evolution
  - tangential forces are meaningless
  - forces must be derived from “spatial position” and “time” because location on the curve is meaningless
  - Final contour is an “isocurve” (2D) or “isosurface” (3D)
  - It has a “Eulerian” rather than “Lagrangian” framework
- Speed function incorporates internal and external forces
  - Design of geometric model is accomplished by selection of  $F(x)$ , the speed function
  - curvature terms takes the place of internal forces
- “Action” is near the zero level set
  - “narrowband” methods are computationally more efficient

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## Ventricle Segmentation



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## Cortical Surface Segmentation



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## Critique of Geometric Deformable Models

- Advantages:
  - Produce closed, non-self-intersecting contours
  - Independent of contour parameterization
  - Easy to implement: numerical solution of PDEs on regular computational grid
  - Stable computations
- Disadvantages:
  - topologically flexible
  - some numerical difficulties with narrowband and level set function reinitialization

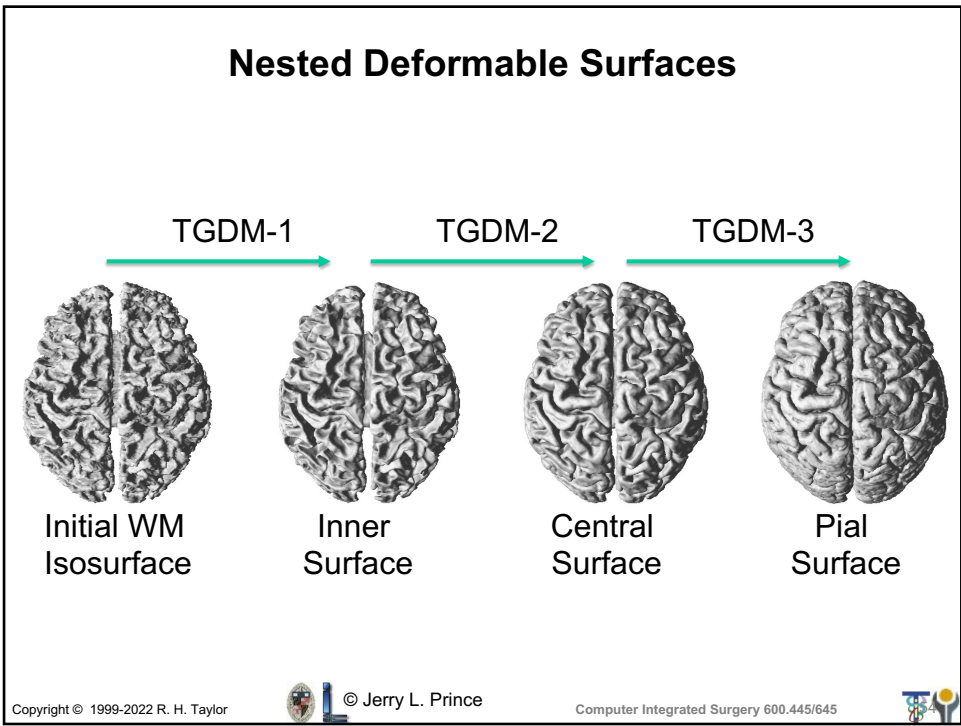


## Topology Preserving Geometric Deformable Model (TGDM)

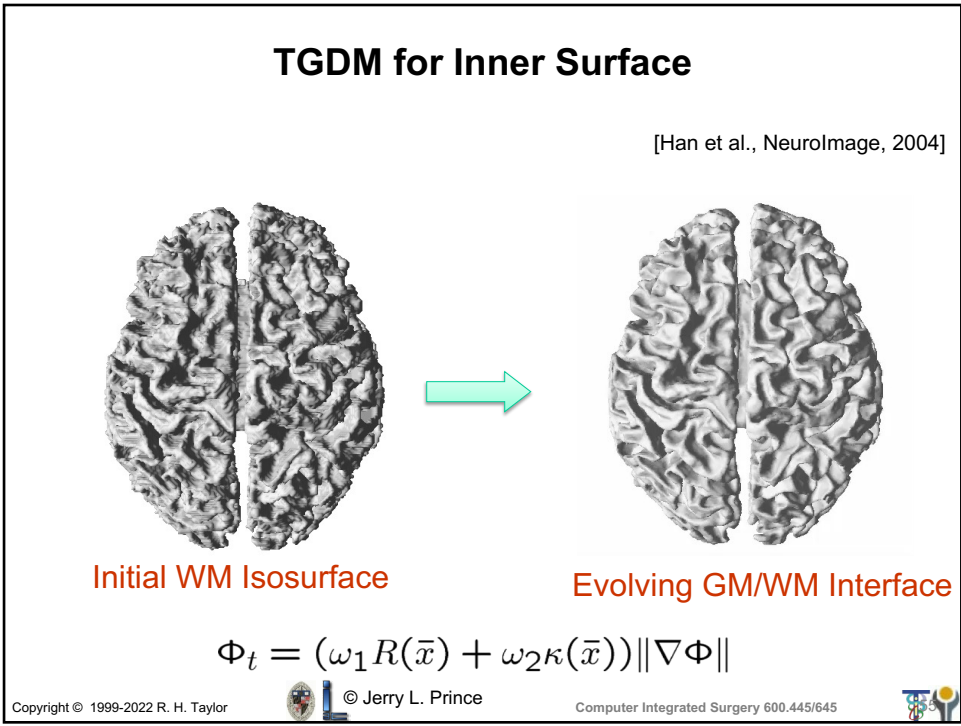
- Evolve level set function according to GDM PDE
- If level set function is going to change sign, check whether the point is a simple point
  - If simple, permit the sign-change
  - If not simple, prohibit the sign-change
  - (replace the grid value by epsilon with same sign)
  - (Roughly, this step adds 7% computation time.)
- Extract the final contour using a *connectivity consistent isocontour algorithm*

X. Han, C. Xu, and J. L. Prince, "A topology preserving level set method for geometric deformable models", IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 25- 6, pp. 755-768, 2003.



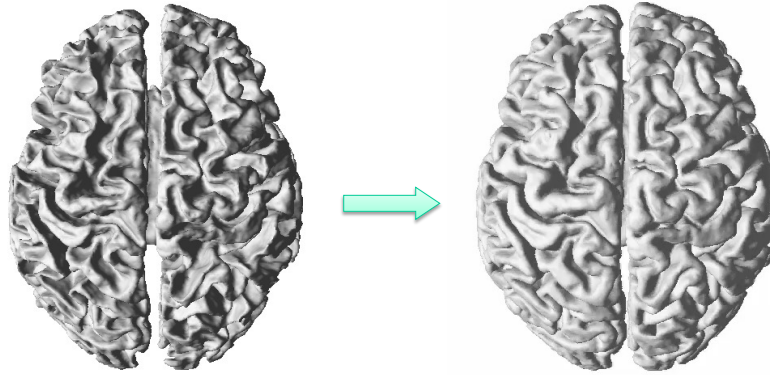


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## TGDM for Central Surface



Initialize with GM/WM surface

Evolving toward Central Surface

$$\Phi_t = (\omega_1 R(\bar{x}) + \omega_2 \kappa(\bar{x})) \|\nabla \Phi\| + \omega_3 F_{GVF}(\bar{x}) \cdot \nabla \Phi$$

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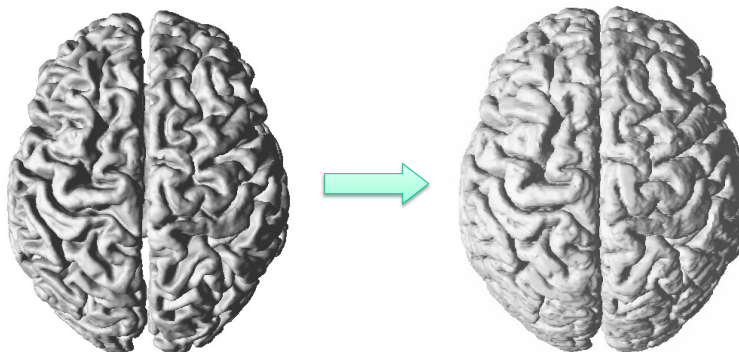
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## TGDM for Outer Surface



Start from Central Surface

Evolving toward Outer Surface

$$\Phi_t = (\omega_1 R(\bar{x}) + \omega_2 \kappa(\bar{x})) \|\nabla \Phi\| + \omega_3 F_{GVF}(\bar{x}) \cdot \nabla \Phi$$

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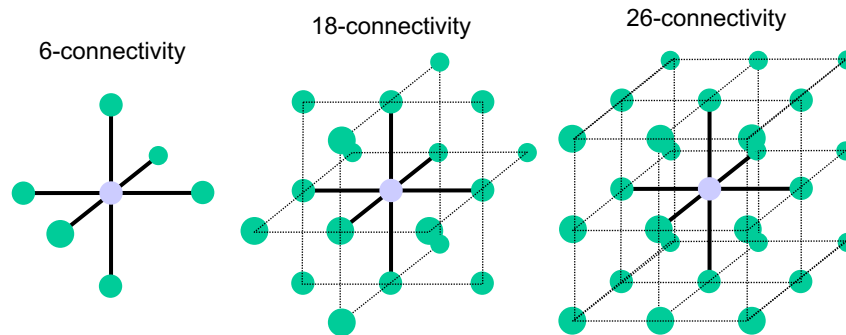


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## 3D Digital Connectivity

- In 3D there are three connectivities: 6, 18, and 26
- Four consistent connectivity pairs:  
(foreground, background)  $\rightarrow$  (6,18), (6,26), (18,6),  
(26,6)



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## Topology Preservation Principle

[Han et al., PAMI, 2003]

- Preserving topology is equivalent to maintaining the topology of the digital object
- The digital object can only change topology when the level set function changes sign at a grid point
- To prevent the digital object from changing topology, the level set function should only be allowed to change sign at *simple* points

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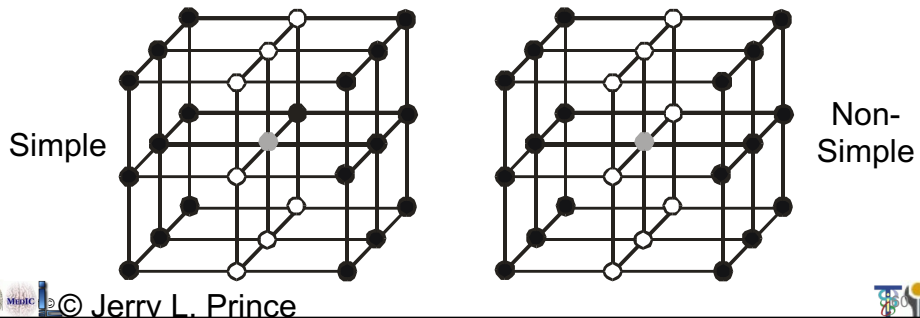
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## Simple Point

- **Definition:** a point is simple if adding or removing the point from a binary object will not change the digital object's topology
- **Determination:** can be characterized locally by the configuration of its neighborhood (8- in 2D, 26- in 3D) [Bertrand & Malandain 1994]



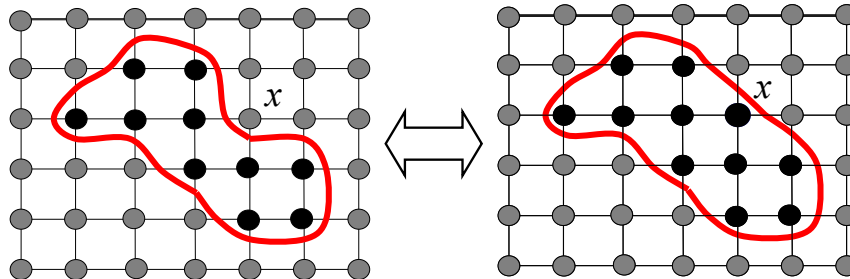
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## $x$ is a Simple Point

$$\Phi(x) > 0$$



$$\Phi(x) < 0$$



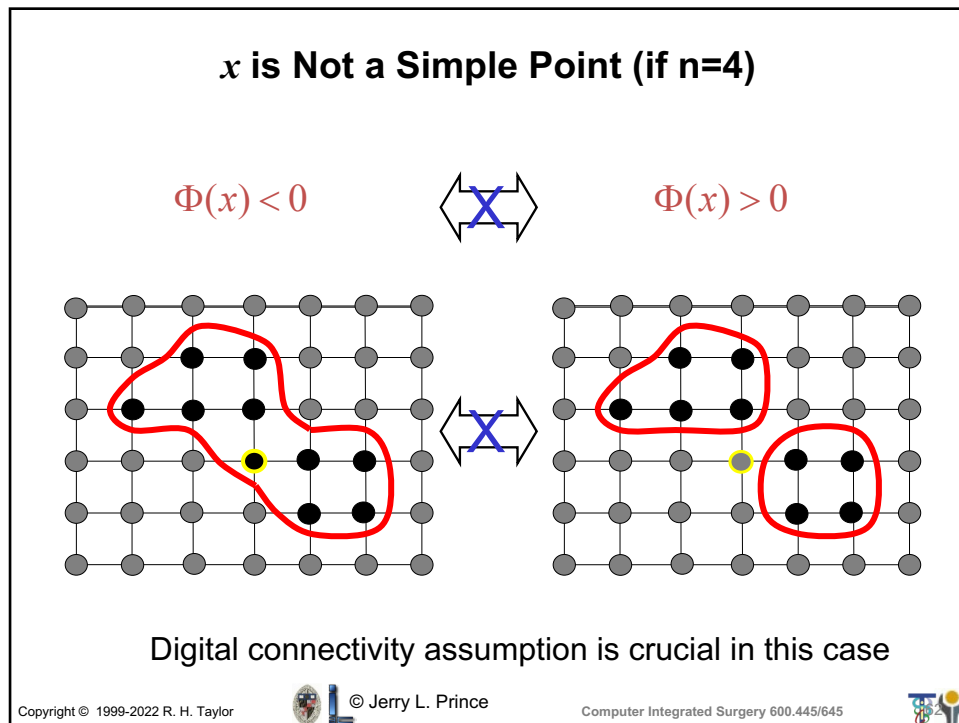
(Connectivity happens to be irrelevant in this case)

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**Topology Preserving Geometric Deformable Model (TGDM)**

- Evolve level set function according to GDM PDE
- If level set function is going to change sign, check whether the point is a simple point
  - If simple, permit the sign-change
  - If not simple, prohibit the sign-change
  - (replace the grid value by epsilon with same sign)
  - (Roughly, this step adds 7% computation time.)
- Extract the final contour using a *connectivity consistent isocontour algorithm*

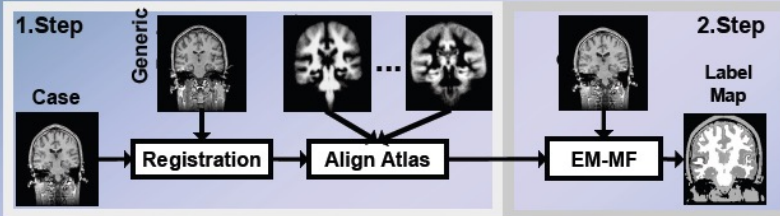
X. Han, C. Xu, and J. L. Prince, "A topology preserving level set method for geometric deformable models", IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 25- 6, pp. 755-768, 2003.

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## Segmenting with Spatial Priors



- Given standard scan, and probability maps of tissue types
- Elastically register standard scan to new case
- Apply transformation to all probability maps
- Use as prior probabilities in EM-MF segmentation
- Apply in hierarchical manner (first segment out major structures, then substructures)

10/19/07

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Credit: Eric Grimson

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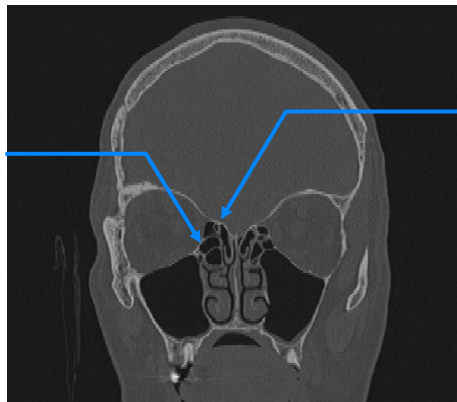


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## Example: Sinuses & Nasal Airway

### Complex structures with thin boundaries

Boundary between the sinuses and the orbit  
Thickness:  $-0.91$  mm<sup>[4]</sup>



Fovea ethmoidalis: separates the ethmoid cells from the anterior cranial fossa  
Thickness:  $-0.5$  mm<sup>[3]</sup>

[3] Kainz, J. and Stammberger, H., "The roof of the anterior ethmoid: A place of least resistance in the skull base," American Journal of Rhinology 3(4), 191-199 (1989).

[4] Tao, H., Ma, Z., Dai, P., and Jiang, L., "Computer-aided three-dimensional reconstruction and measurement of the optic canal and intracanalicular structures," The Laryngoscope 109(9), 1499-1502 (1999).

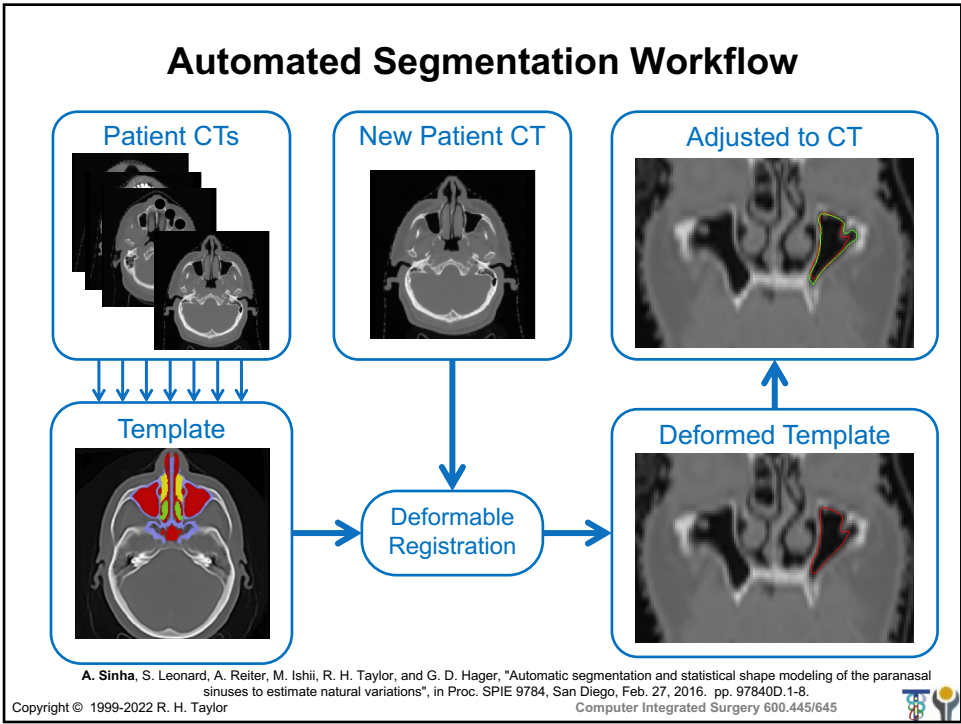
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Slide Credit: Ayushi Sinha

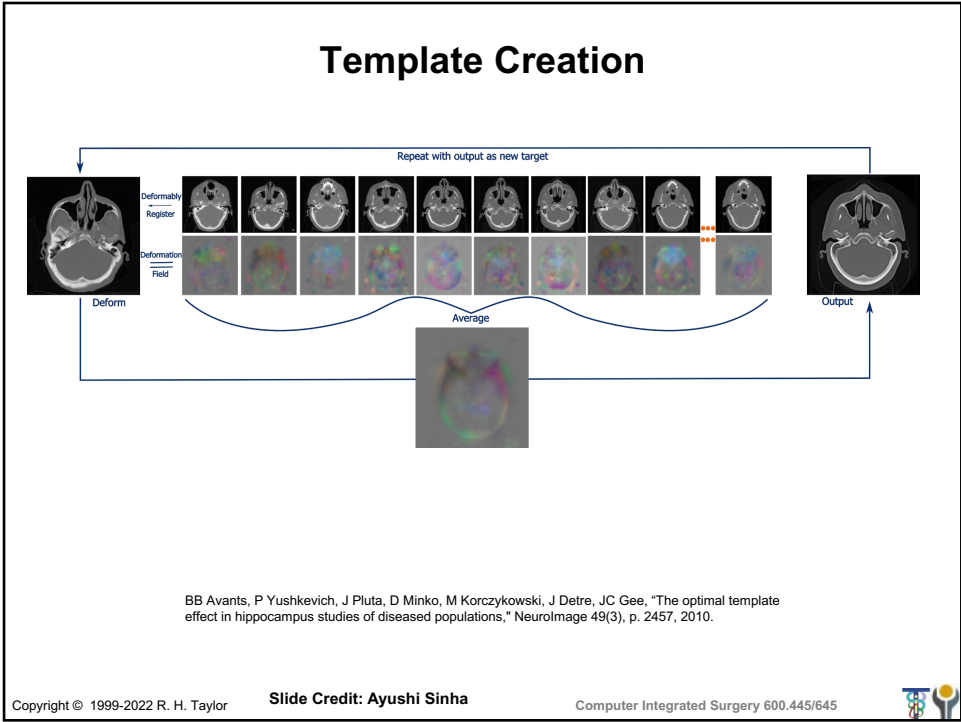
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## Deformable Registration of Template to Image



BB Avants, NJ Tustison, . Song, PA Cook, A Klein, and JC Gee, "A reproducible evaluation of ANTs similarity metric performance in brain image registration," *NeuroImage* 54(3), pp. 2033-2044, 2011.

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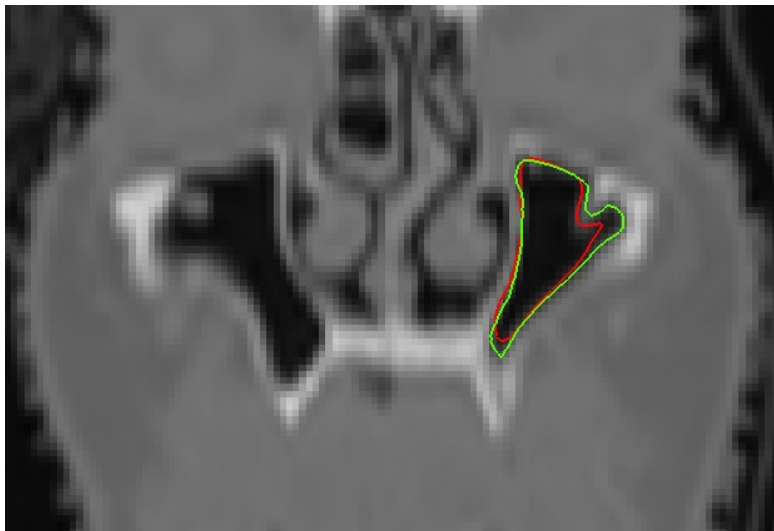
Slide Credit: Ayushi Sinha

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## Adjustment of Template to Patient CT



[10] C. Xu and J. L. Prince, "Gradient vector flow: A new external force for snakes," in *IEEE Computer Vision and Pattern Recognition*, pp. 66-71, 1997.  
[11] C. Xu and J. Prince, "Snakes, shapes, and gradient vector flow," *IEEE Transactions on Image Processing*, 7, pp. 359-369, March 1998.

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Slide Credit: Ayushi Sinha

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## Results

Red contour:  
Segmentation  
via label  
transfer using  
deformable  
registration

Blue contour:  
Hand-labeled  
gold standard

Green contour:  
Improved  
segmentation  
using our  
method

A. Sinha, A. Reiter, S. Leonard, M. Ishii, G. D. Hager, and R. H. Taylor, "Simultaneous segmentation and correspondence improvement using statistical modes", in SPIE Medical Imaging, Orlando, 2017.

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## Machine Learning Methods

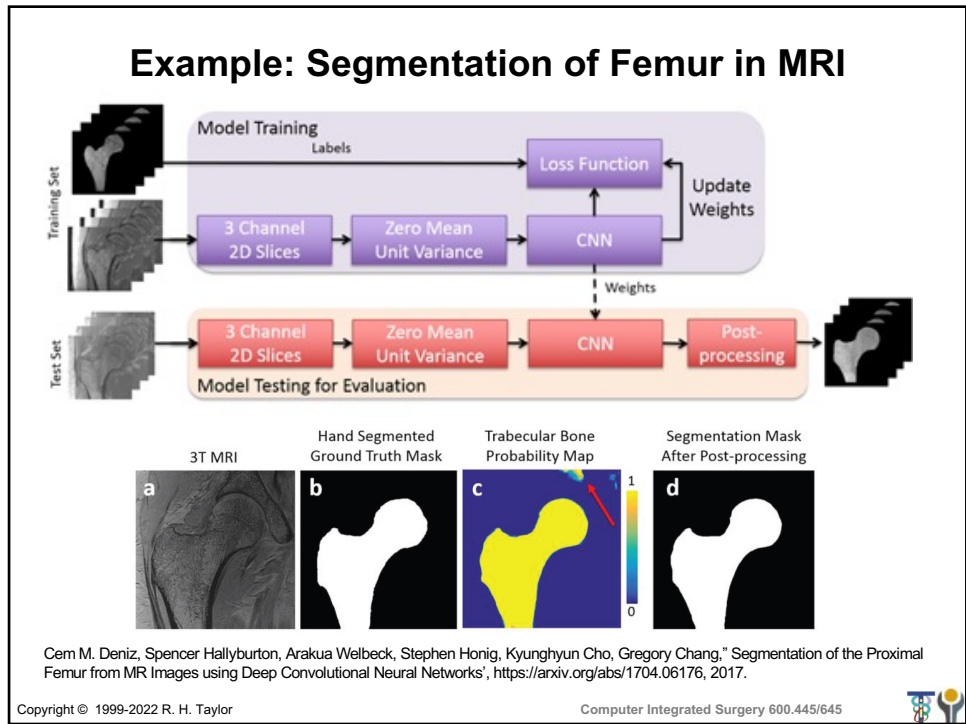
```

    graph TD
      A[Training images with known segmentations] --> B[Machine Learning Algorithm]
      B -- Parameters --> C[Classification function]
      D[Patient Image] --> C
      C --> E[Segmented Image]
  
```

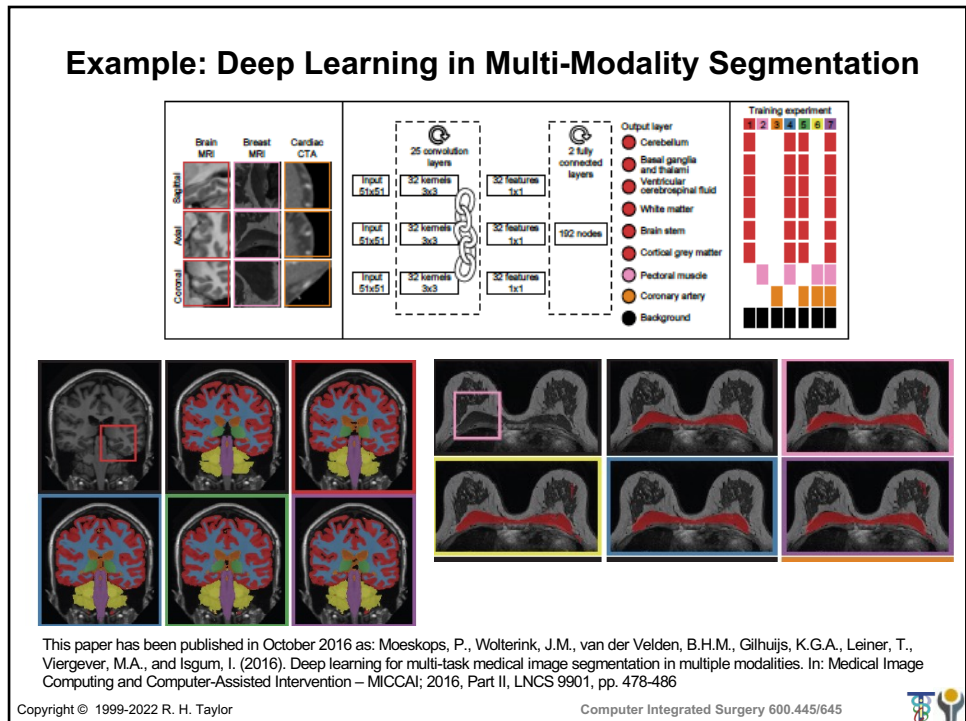
- Basic approach has been used in one form or another for many years
- Emergence of modern convolutional neural nets with GPUs has made these approaches extremely successful recently
- However, require large amounts of training data

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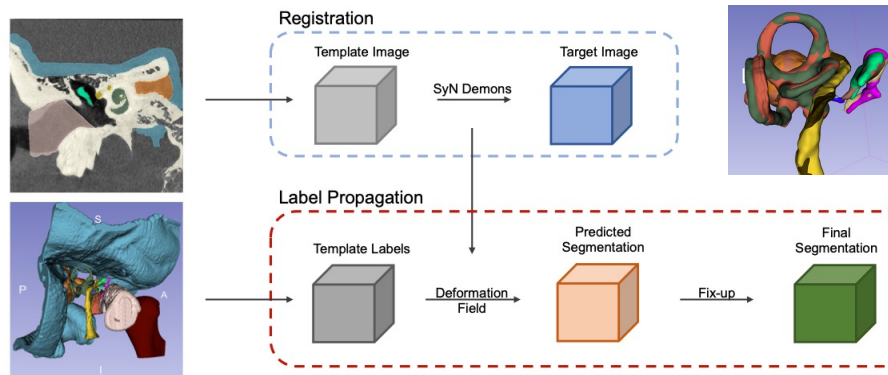
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## Automated Segmentation of Temporal Bone Structures



Andy Ding, Alex Lu, Francis Creighton, Russ Taylor

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## Modeling

- Representation of anatomical structures
- Models can be
  - Images
  - Labeled images
  - Boundary representations

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## FROM VOXELS TO SURFACES

### Representing solids:

- B-REP - surface representation,  
d/s of vertices, edges, faces.
- CSG- composition of primitive solids

**binary image**  $\longrightarrow$  **B-REP representation**

Surface construction algorithms:

- 2D-based algorithms
- 3D-based algorithms



## Surface Representations

- Implicit Representations  
 $\{\bar{x} \mid f(\bar{x}) = 0\}$
- Explicit Representations
  - Polyhedra
  - Interpolated patches
  - Spline surfaces
  - ...



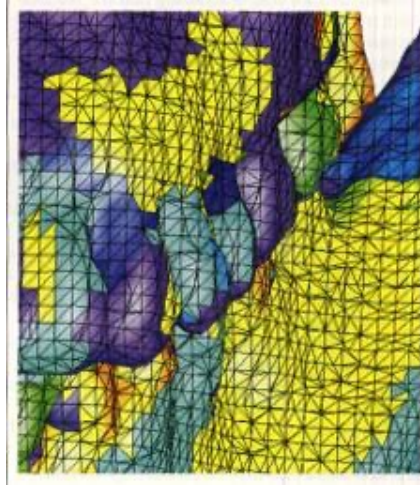
FIGURE 4.7 Segmentation of vertebra defined by a set of CT slices. Four steps of the deformation of a roughly spherical snake spline toward the vertebra are shown.

Source: CIS p 73 (Lavallee image)



## Polyhedral Boundary Reps

- Common in computer graphics
- Many data structures.
  - FEV lists
  - Winged edge
  - Connected triangles
  - etc.



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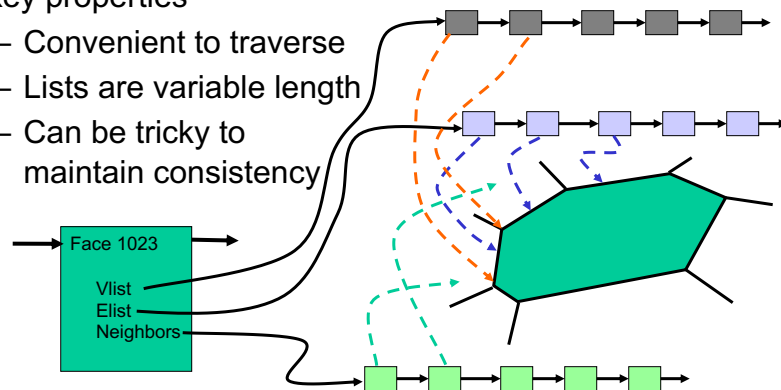
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## FEV lists

- Explicit linked lists of faces, edges, vertices
- Many variations
- Key properties
  - Convenient to traverse
  - Lists are variable length
  - Can be tricky to maintain consistency



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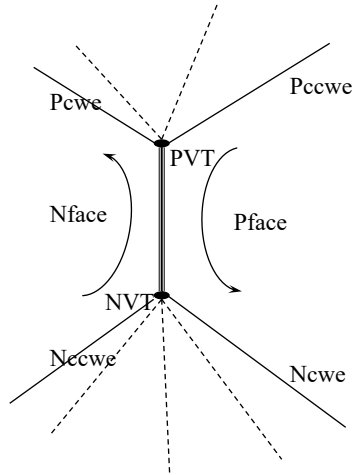
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## Winged Edge

- Baumgart 1974
- Basic data structures
  - winged edge (topology)
  - vertex (geometry)
  - face (surfaces)
- Key properties
  - constant element size
  - topological consistency

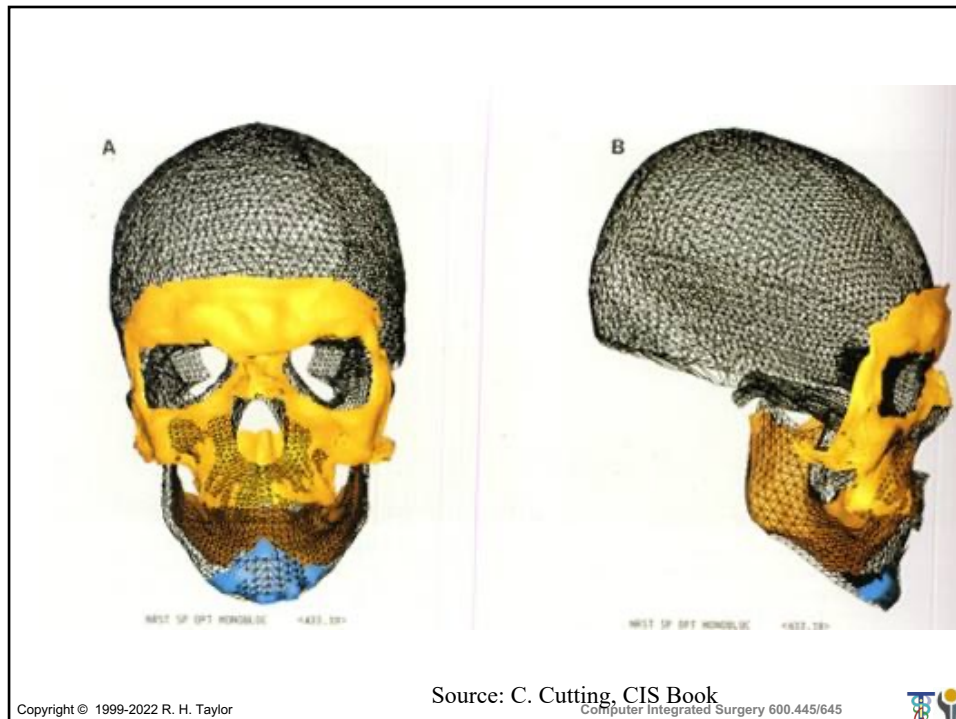


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Source: C. Cutting, CIS Book

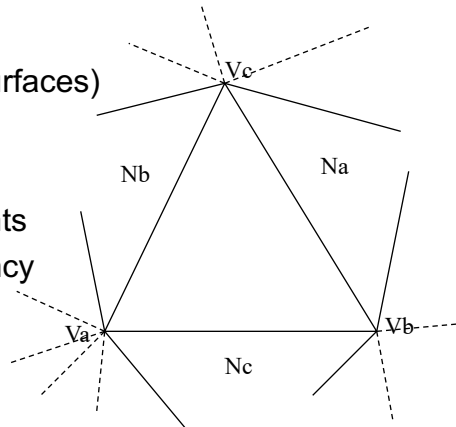
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## Connected Triangles

- Basic data structures
  - Triangle (topology, surfaces)
  - Vertex (geometry)
- Properties
  - Constant size elements
  - Topological consistency



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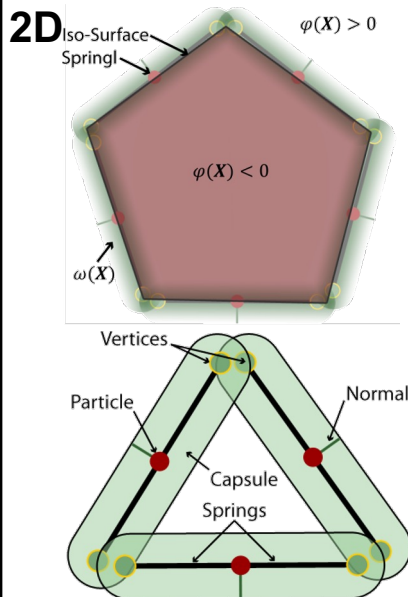
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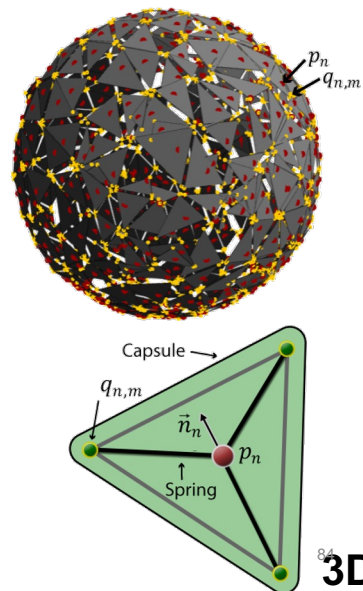
## Anatomy of a Spring

Blake Lucas



Slide credit: Blake Lucas' PhD Thesis Defense, 2012  
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**3D**

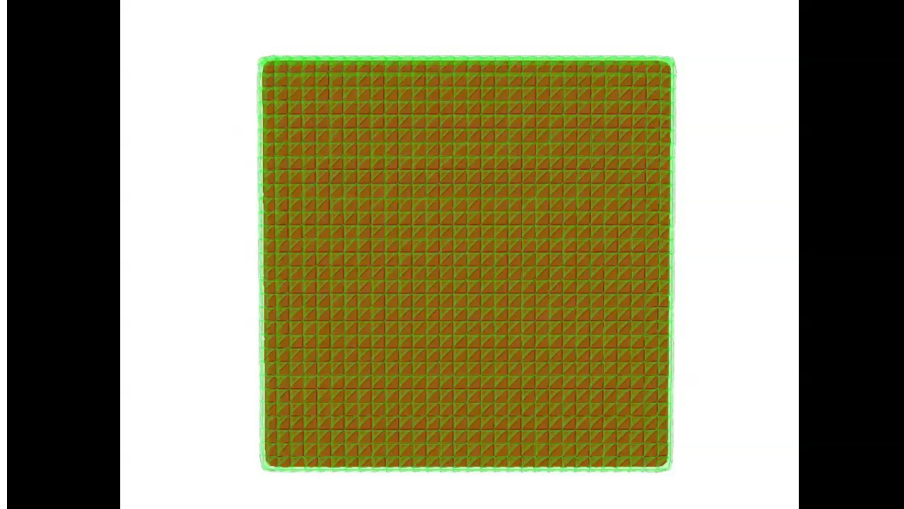


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## Deformable Surfaces & Level Sets



Blake Lucas – “Springs” (October 2010)

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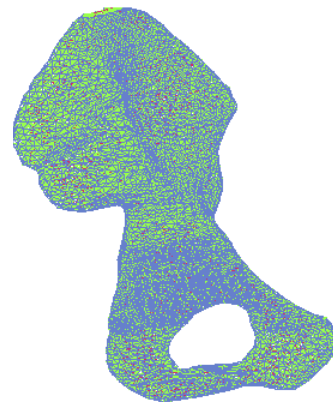
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## Tetrahedral Mesh Data Structure

- Vertex list
  - x, y, z coordinates
  - reference to one tetrahedron
- Tetrahedron list
  - references to four vertices
  - references to four face neighbors
- Properties such as density functions



Credit: Yao and Taylor

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## Advantages of Tetrahedral Mesh

- Greatest degree of flexibility
- Data structure, data traversal, and data rendering are more involved
- Ability to better adapt to local structures
- Computational steps such as interpolation, integration, and differentiation can be done in closed form
- Finite element analysis
- Hierarchical structure of multiple resolution meshes

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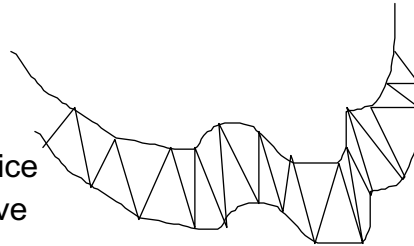
Credit: Yao and Taylor  
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## 2D-based Methods for Shape Reconstruction

- Treat 3D volume as a stack of slices
- Outline
  - Find contours in each 2D slice
  - Match contours in successive slices
  - Connect contours to create tiled surfaces (for boundary representation)
  - Use contours to guide subdivision of space between slices into tetrahedra (for volumes)



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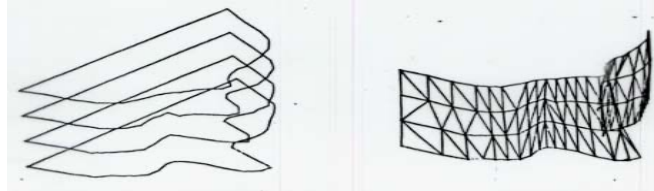
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# SURFACE CONSTRUCTION ALGORITHMS

## 2D-based algorithms

1. 2D contour extraction
2. tiling of contours

Keppel (1975), Fuchs (1978), Christiansen (1981), Shantz (1981), Ganapathy (1982), Cook (1983), Zyda (1987), Boissonnat (1988), Schwartz (1988)



### Contour extraction

- Sequential scanning
- boundary following (random access to pixels)

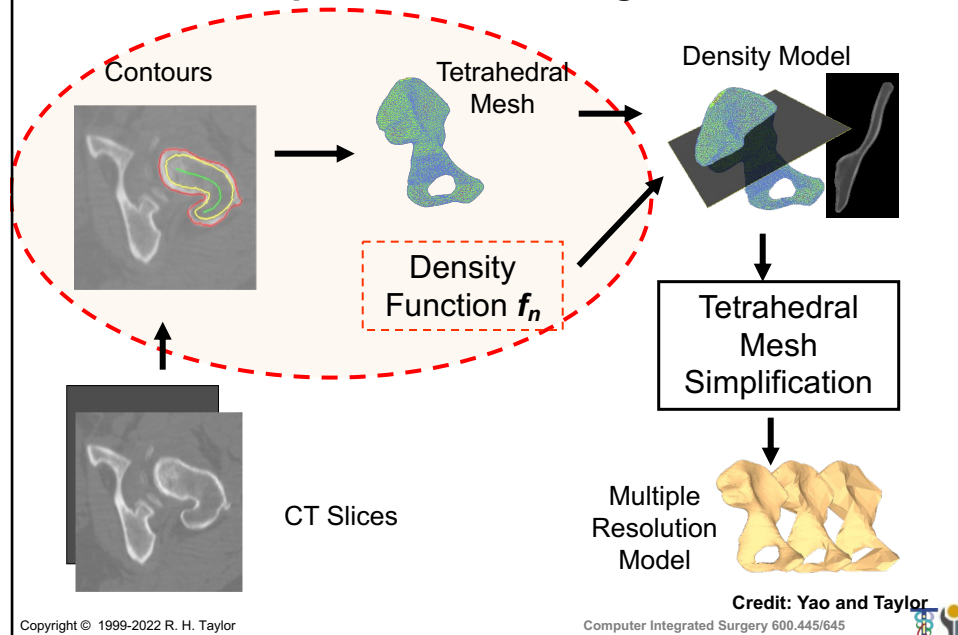
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## Example: Bone Modeling from CT



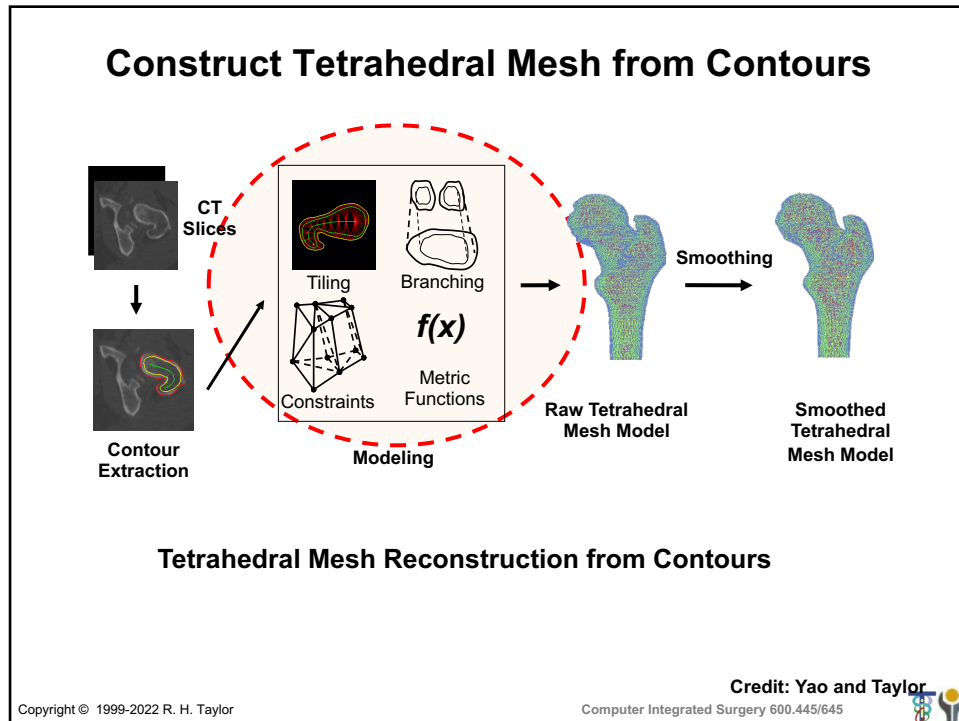
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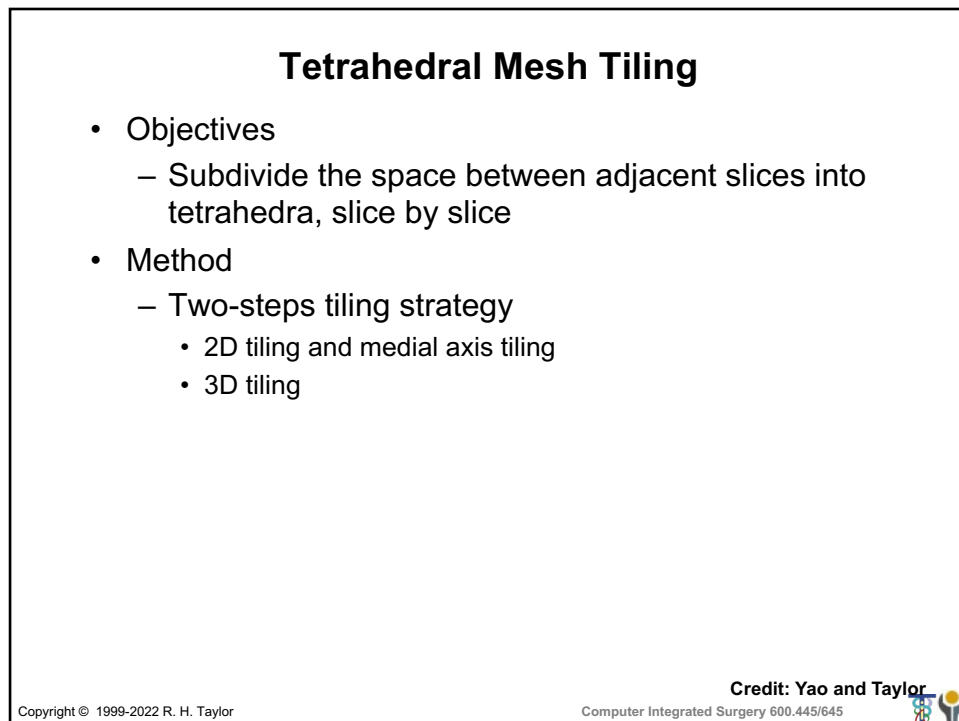


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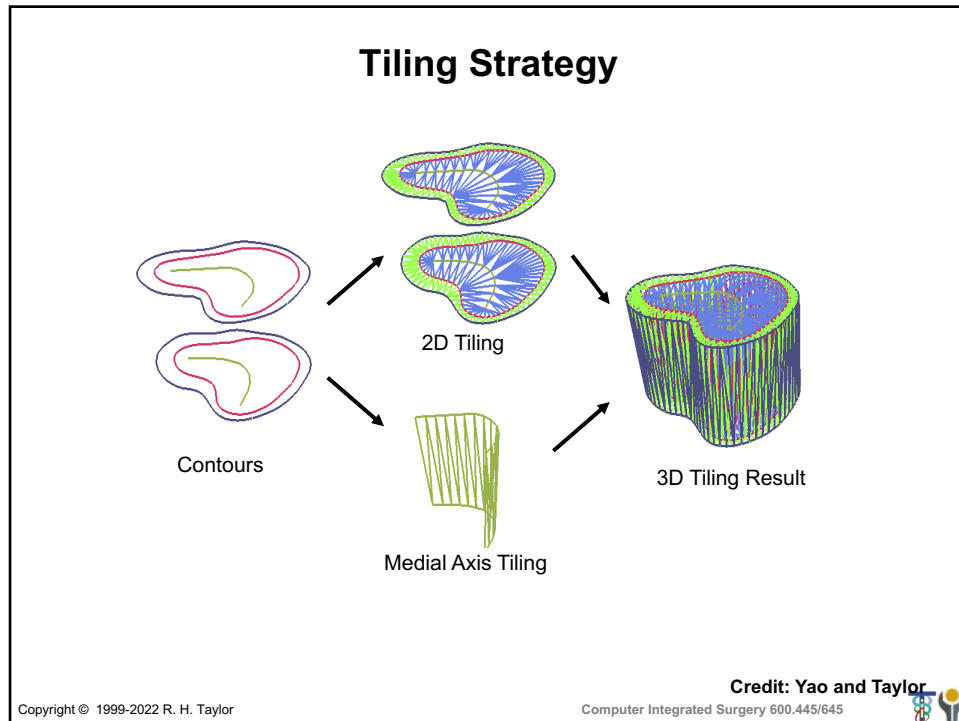




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### Metric Functions

- Maximize Volume,  $f_v$
- Minimize Area,  $f_a$
- Minimize Density Deviation,  $f_d$
- Minimize Span Length,  $f_s$

Current Metric Function:

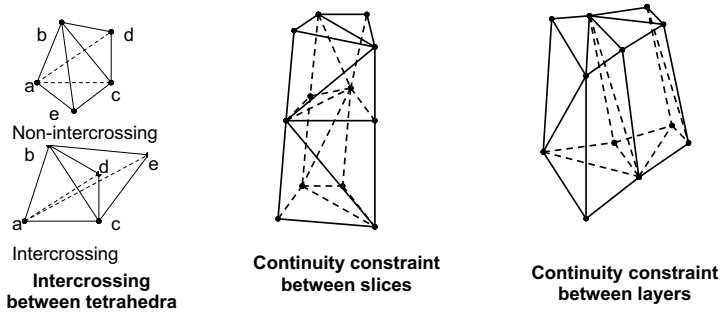
- Combination of minimizing density deviation and span length
- Minimize  $F = w_1 * f_d + w_2 * f_s$

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## Tiling Constraints

- Non-intersection between tetrahedra
- Continuity between slices
- Continuity between layers



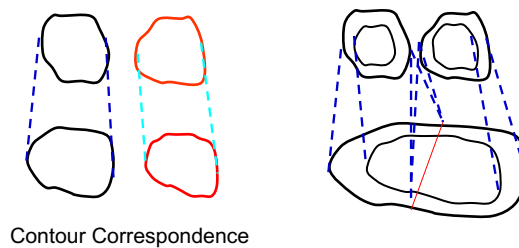
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## Correspondence Problem

- Examining the overlap and distance between contours on adjacent slices
- Graph based method



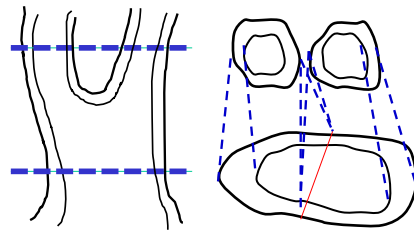
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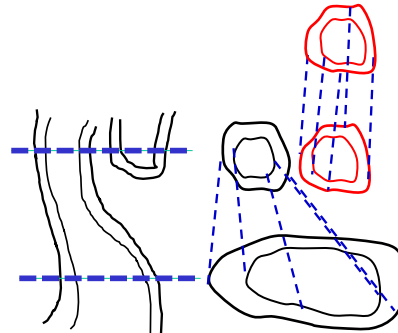
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## Branching Problem

- Branching Between layers
  - Convert to tiling of 3 contours
- Branching Between contours
  - Composite contour
  - Split contour



Split Contour



Composite Contour

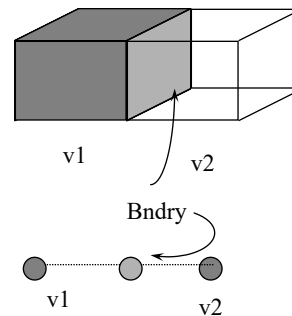
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## 3D-based methods for Surface Reconstruction

- Segment image into labeled voxels
- Define surface and connectivity structure
- Can treat boundary element between voxels as a face or a vertex



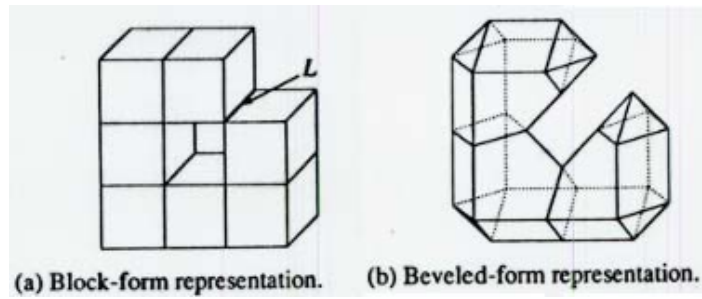
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## 3D-BASED ALGORITHMS

Block-form and Beveled-form representations of surface:



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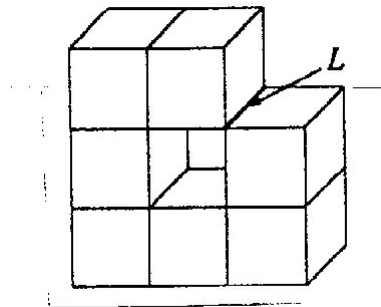
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## Block form methods

- “Cuberille”-type methods
- Treat voxels as little cubes
- May produce self-intersecting volumes
- E.g., Herman, Udupa

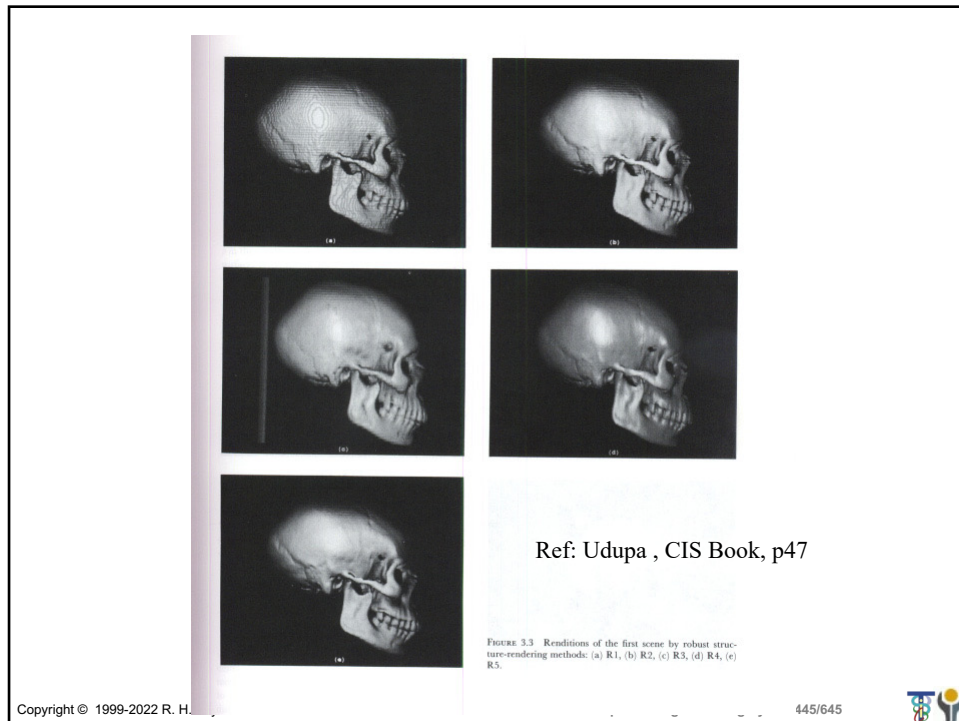


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### Beveled form methods

- “Marching cubes” type
- Voxels viewed as 3D grid points
- Vertices are points on line between adjacent grid points
- E.g. Lorensen&Cline, Baker, Kalvin, many others

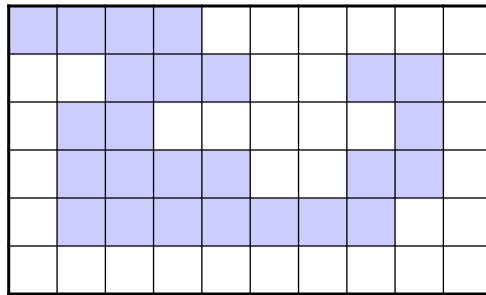
The diagram shows a 3D wireframe model of a beveled form. It consists of a series of interconnected lines that form a complex, multi-faceted shape. The lines represent the vertices and edges of a grid-based structure, where the vertices are points on the lines between adjacent grid points. The shape is composed of several interconnected volumes, each with beveled edges, creating a complex, multi-faceted form.

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## Block form to beveled form

Segmented voxels



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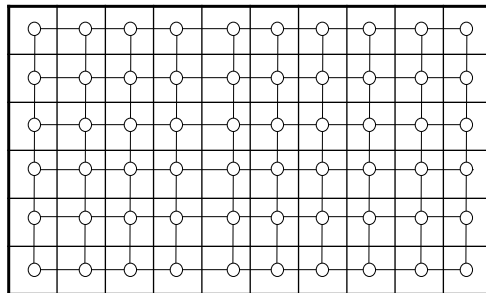
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## Block form to beveled form

Duality between voxels and vertices on adjacency graph



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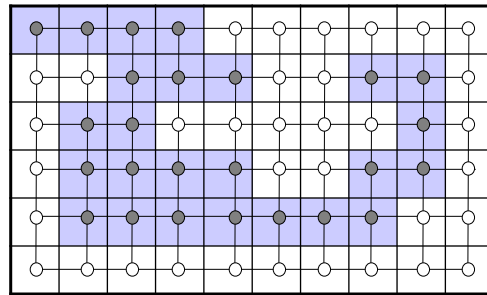
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## Block form to beveled form

Label vertices based on segmentation labels



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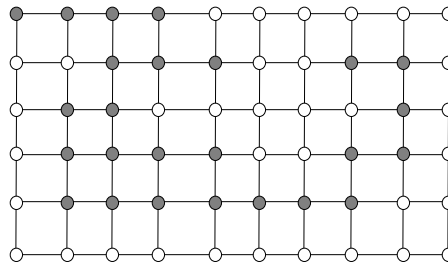
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## Block form to beveled form

Label vertices based on segmentation labels



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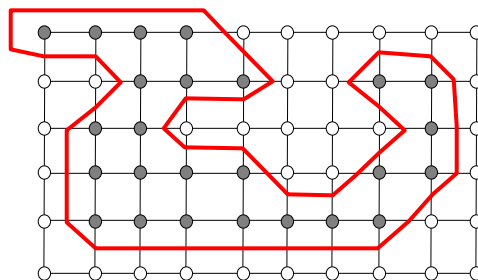


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## Block form to beveled form

Boundary crosses edges between occupied and empty vertices



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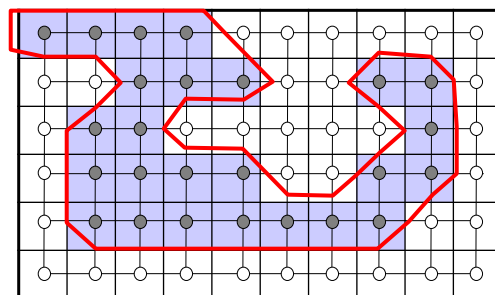
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## Block form to beveled form

Boundary crosses edges between occupied and empty vertices



**Note: Choice of exact vertex placement is somewhat arbitrary. One choice is linear interpolation along edge based on density.**

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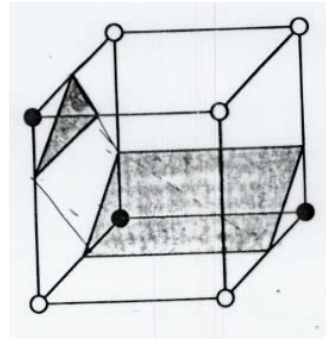
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## Beveled form basic approach

- Segment the 3D volume
- Scan 3D volume to process “8-cells” sequentially
- Use labels of 8 cells as index in (256 element) lookup table to determine where surfaces pass thru cell
- Connect up topology
- Use various methods to resolve ambiguities



Source: Kalvin survey

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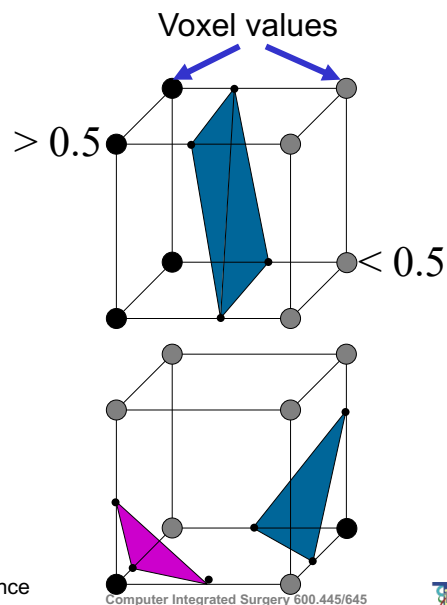
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## Marching Cubes Isosurface Algorithm

- How to “tile/triangulate” the zero level set?
- Consider values on corners of voxel (cube)
- Label as
  - above isovalue
  - below isovalue
- Determine the position of a triangular mesh surface passing through the voxel
  - Linear interpolation



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## Connectivity Errors

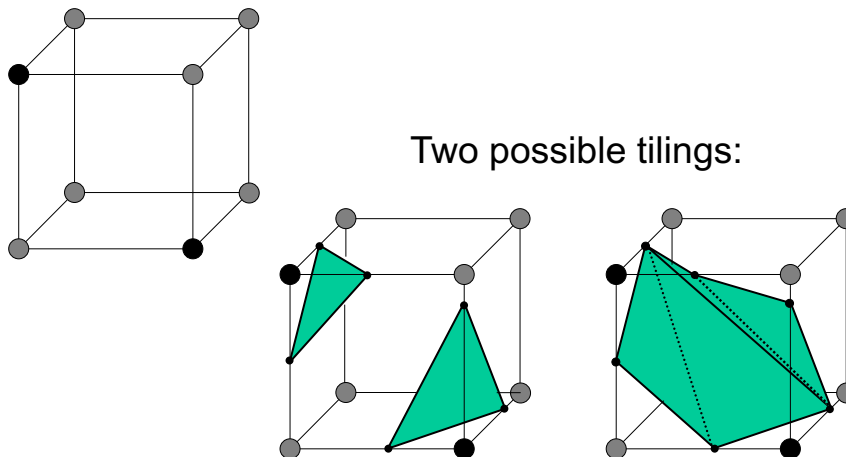
**Most isosurface codes use rules that lead to connectivity errors**

- Multiple meshes
  - typically solved by selecting the largest mesh
- Touching vertices, edges, and faces
  - typically solved isovalue choice
- Ambiguous faces and cubes
  - solved by use of a specially coded *connectivity consistent* MC algorithm



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## Ambiguous Faces



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### Ambiguous Cubes

Two possible tilings:

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### Wyvill, McPheters, Wyvill

Step 1: determine edges on each face of 8 cube

(a) (b) (c) (d)  
(e) (f) (g)

Figure 6: The seven cases for calculating vertices and edges

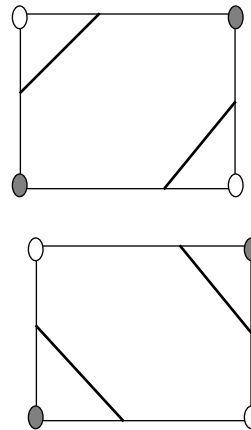
Step 2: Connect the edges up to make surfaces

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## Ambiguities

- Arise when alternate corners of a 4-face have different labels
- Ways to resolve:
  - supersampling
  - look at adjacent cells
  - tetrahedral tessellation



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## Tetrahedral Tessellation

- Many Authors
- Divide each 8-cube into tetrahedra
- Connect tetrahedra
- No ambiguities

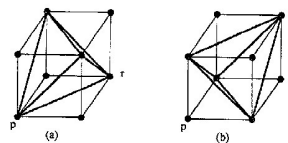


Figure 8: The two tetrahedral partitionings of an 8-cell.

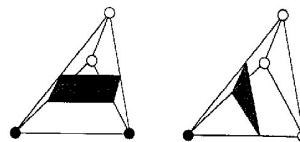


Figure 9: The two cases used for surface construction.

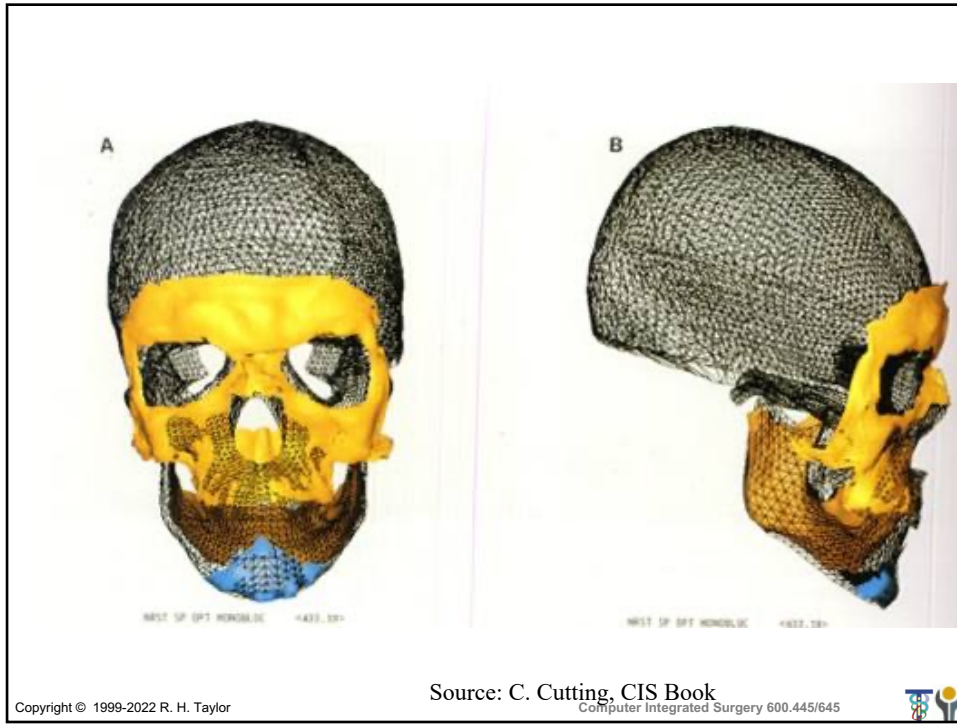
Beveled-form algorithms based on the tetrahedral decomposition of the 3D volume have been developed Payne and Toga [34], Hall and Warren [21], and Nielson *et al.* [29]. While this approach does provide a neat resolution to the ambiguous 8-cell problem, it

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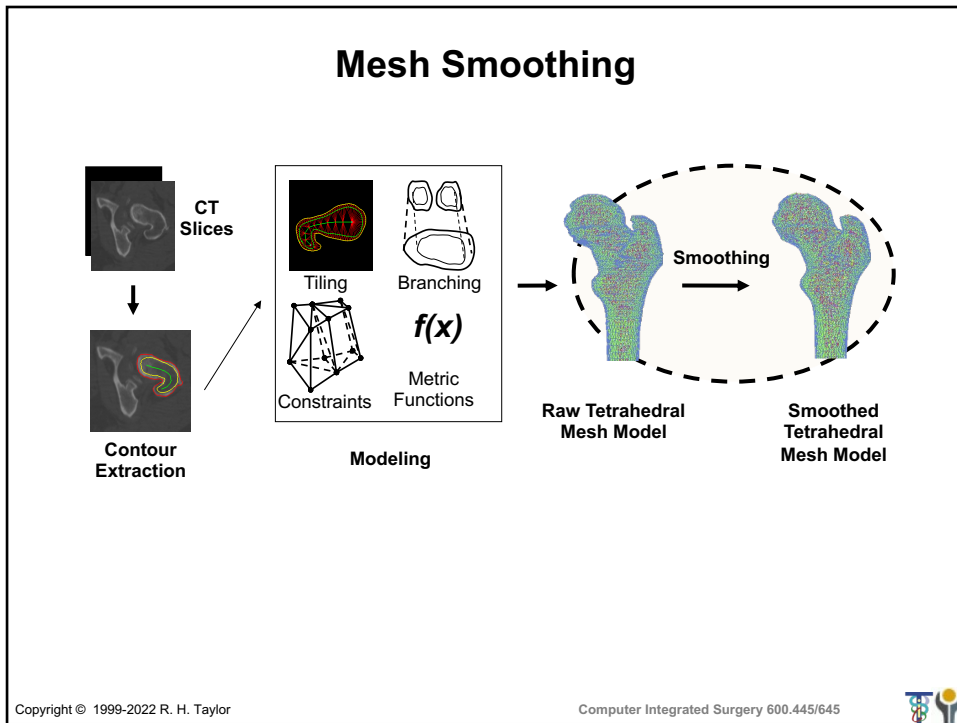
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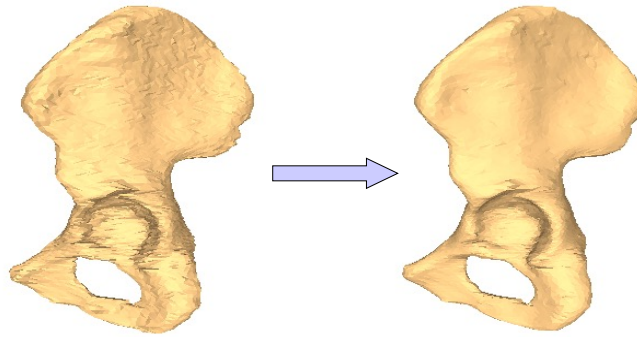
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## Mesh Smoothing

- Motivations
  - Noise/discretization in CT data set
  - Artifacts during segmentation



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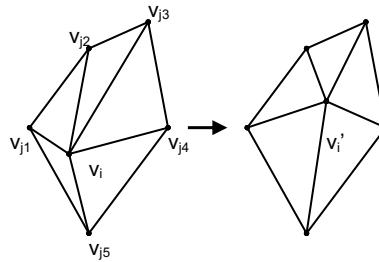


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## Classic Laplacian Smoothing Method

- Equation

$$v'_i = \frac{1}{|N_i|} \sum_{j \in N_i} v_j$$



- Advantages
  - Fast and easy
- Drawbacks
  - Shrinkage
  - Invalid elements

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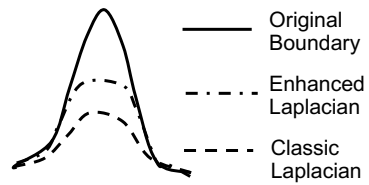


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## Enhanced Laplacian Smoothing Method

- Objective
  - Reduce shrinkage
- Method
  - Project back to boundary

$$v'_i = \text{proj}\left(\frac{1}{|N_i|} \sum_{j \in N_i} v_j\right)$$



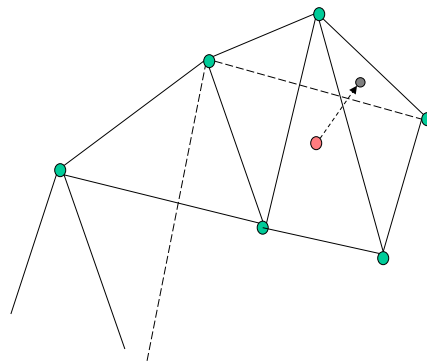
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## Average and reproject



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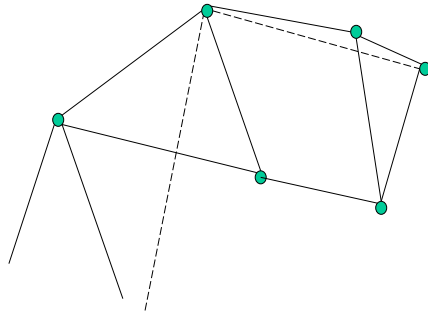
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## Average and reproject



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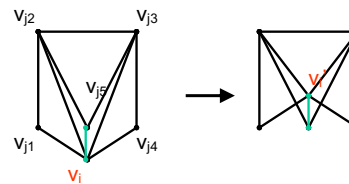
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## Enhanced Laplacian Smoothing Method

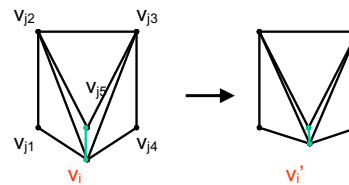
- Objective
  - Prevent invalid element
- Method
  - Iterative assignment

$$v_i^{(0)} = \text{proj}\left(\frac{1}{|N_i|} \sum_{j \in N_i} v_j\right)$$

$$v_i^{(k)} = \alpha \cdot v_i + (1 - \alpha)v_i^{(k-1)}, 0 \leq \alpha \leq 1$$



Classic Laplacian



Enhanced Laplacian

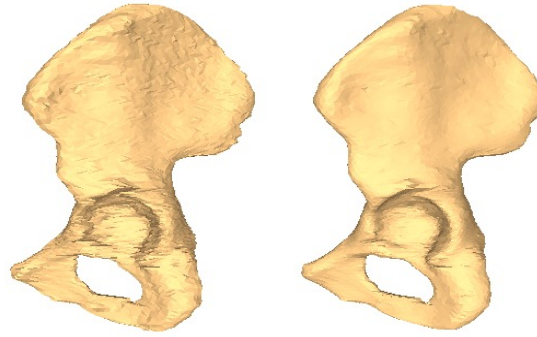
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## Mesh Smoothing Results



a) Before Smoothing

b) After Smoothing

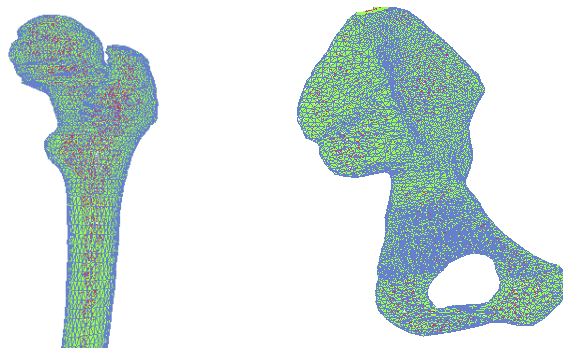
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## Tetrahedral Mesh Models



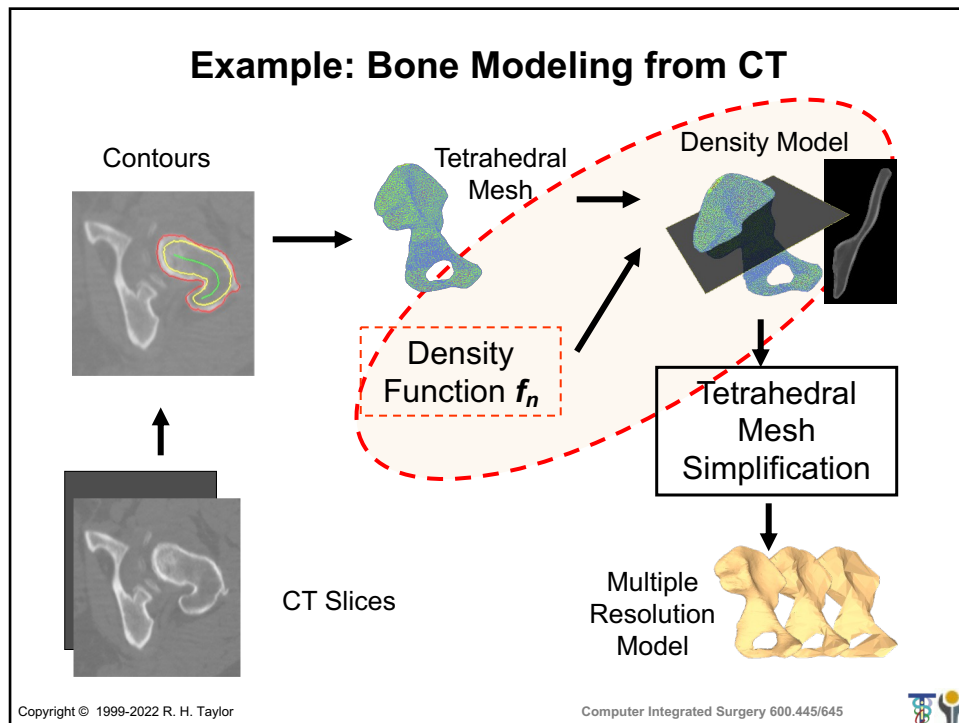
| Model  | Num of Vertices | Num of Tetrahedra | Num of Slices | Total Num of Voxels inside | Avg Num of voxels Per Tetra | Volume (mm <sup>3</sup> ) | Avg Vol. Per Tetra (mm <sup>3</sup> ) |
|--------|-----------------|-------------------|---------------|----------------------------|-----------------------------|---------------------------|---------------------------------------|
| Femur  | 6163            | 31,537            | 83            | 1,802,978                  | 57.1                        | 312,107                   | 9.9                                   |
| Pelvis | 8219            | 32,741            | 110           | 1,941,998                  | 59.3                        | 347,070                   | 10.6                                  |

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### Density Functions

- n-degree Bernstein polynomial in barycentric coordinate

$$D(\mu) = \sum_{i+j+k+l=n} C_{i,j,k,l} B_{i,j,k,l}^n(\mu)$$

$C_{i,j,k,l}$  polynomial coefficient

$$B_{i,j,k,l}^n(\mu) = \frac{n!}{i!j!k!l!} \mu_x^i \mu_y^j \mu_z^k \mu_w^l \text{ barycentric Bernstein basis}$$

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## Barycentric Coordinate of Tetrahedron

- Local coordinate system
- Symmetric and normalized
- Every 3D position can be defined by an unique coordinate  $(x, y, z, w)$

$$V = x*V_a + y*V_b + z*V_c + w*V_d$$

$x+y+z+w=1$ ,  $V_a, V_b, V_c, V_d$  are coordinate of Tetrahedron vertices

$x, y, z, w$  within  $[0, 1]$  if  $V$  is inside the tetrahedron



## Density Functions

- Advantages
  - Efficient in storage
  - Continuous function
  - Explicit form
  - Convenient to integrate, to differentiate, and to interpolate



## Fitting Density Function

- Minimize the density difference between the density function and CT data set

$$\min_{\rho_i \in \Omega} \sum \left( \left( \sum_{i+j+k+l=n} C_{i,j,k,l} B_{i,j,k,l}^n(\mu_{\rho_i}) \right) - T(\mu_{\rho_i}) \right)^2$$

$\Omega$  is the set of sample voxels,  
 $T(\mu_{\rho_i})$  is the density value from the CT data set.

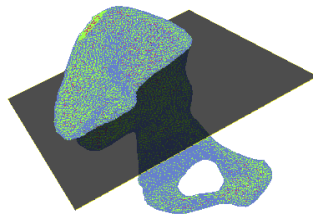
$$\begin{bmatrix} B_1(\mu_{\rho_1}) & B_2(\mu_{\rho_1}) & \dots & B_m(\mu_{\rho_1}) \\ B_1(\mu_{\rho_2}) & B_2(\mu_{\rho_2}) & \dots & B_m(\mu_{\rho_2}) \\ \vdots & \vdots & \vdots & \vdots \\ B_1(\mu_{\rho_s}) & B_2(\mu_{\rho_s}) & \dots & B_m(\mu_{\rho_s}) \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_m \end{bmatrix} = \begin{bmatrix} T(\mu_{\rho_1}) \\ T(\mu_{\rho_2}) \\ \vdots \\ T(\mu_{\rho_s}) \end{bmatrix}$$

$s$ : number of sample voxels  
 $m$ : number of density function coefficient,  
 $s > 2m$

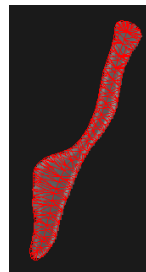


## Accuracy vs Degree of Density Function

- Use CT data set as ground truth
- Cut an arbitrary plane through the model



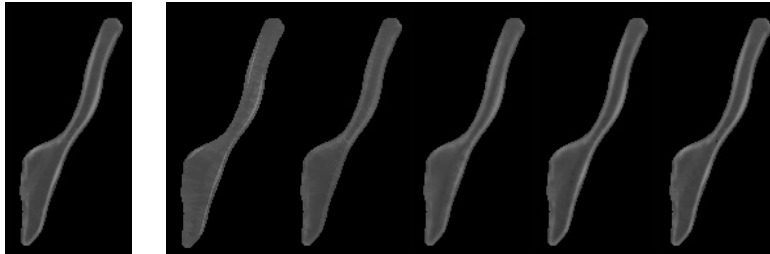
Arbitrary Cutting Plane



Partitions by tetrahedra on cutting plane



## Accuracy vs Degree of Density Function (cont')



Ground Truth      n=0      n=1      n=2      n=3      n=4

|                      |       |       |       |       |       |       |       |       |       |
|----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Degree               | 0     | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     |
| Coeff Number         | 1     | 4     | 10    | 20    | 35    | 56    | 84    | 120   | 165   |
| Avg. Density Err (%) | 3.291 | 1.583 | 0.766 | 0.442 | 0.298 | 0.216 | 0.167 | 0.149 | 0.128 |

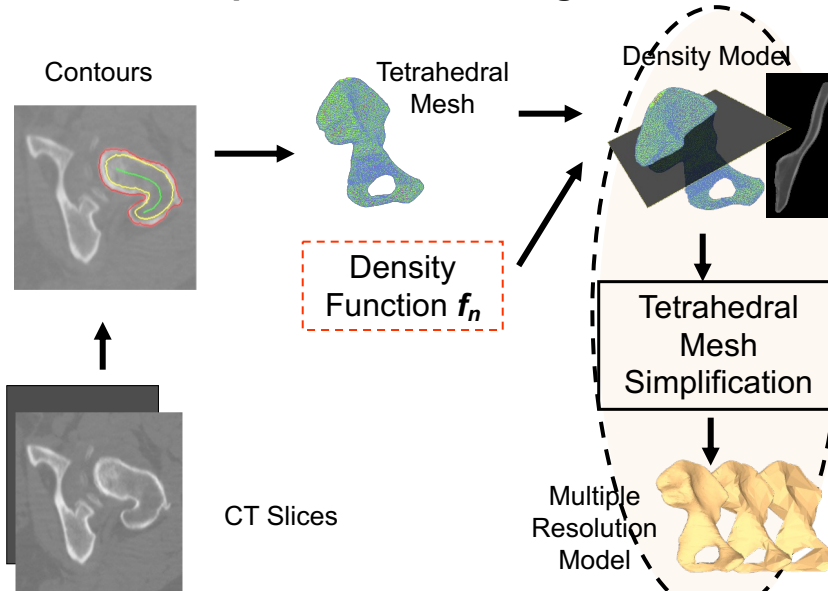
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## Example: Bone Modeling from CT



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## Model Simplification

- Models used in CIS must be guaranteed to be accurate within known bounds
- But 3D models from medical images often are very complex, and require representations with large data structures.
- Algorithms using models are often computationally expensive, and computation costs go up with model complexity
- **PROBLEM:** reduce model complexity while preserving adequate accuracy



~350,000 triangles!

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## Model simplification

- Problem is also common in computer graphics
  - There is a growing literature
  - **But** many graphics techniques only care about appearance, and do not necessarily preserve accuracy or other properties (like topological connectivity) important for computational analysis
- Broadly, two classes of approaches
  - Top down
  - Bottom-up

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## Top down

- Active surfaces used in segmentation
- Deformable registration of an atlas to a patient
  - E.g., skull atlas discussed in craniofacial lecture had about 5000 polygons (perhaps 15-20,000 triangles)
- Recursive approximations
  - E.g., Pizer *et al.* “cores”



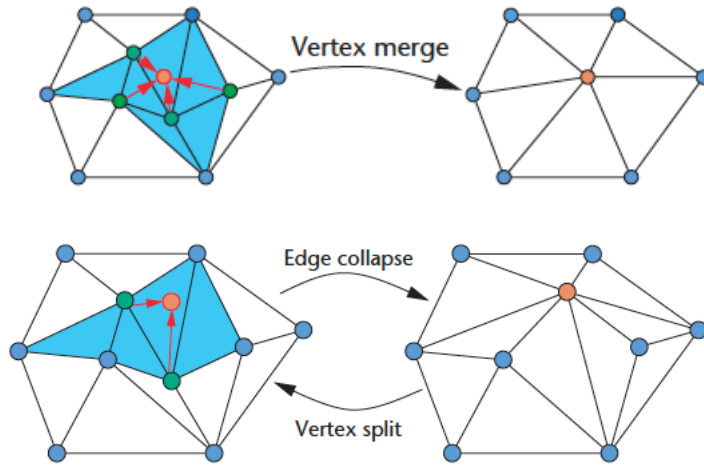
## Bottom up methods

- Typically, start with very high detail model generated from CT images
  - Large number of elements a consequence of small size of pixels & way model is created
- Then merge nearby elements into larger elements
  - E.g., “decimation” (Lorensen, et. al.)
  - E.g., “superfaces” (Kalvin & Taylor)
  - E.g., Gueziec
- An excellent tutorial may be found in:
  - David Luebke; A Developer’s Survey of Polygonal Simplification Algorithms; IEEE Computer Graphics and Application, May 2001





## Bottom-up merging



**Source:** David Luebke; A Developer's Survey of Polygonal Simplification Algorithms; IEEE Computer Graphics and Application, May 2001

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