

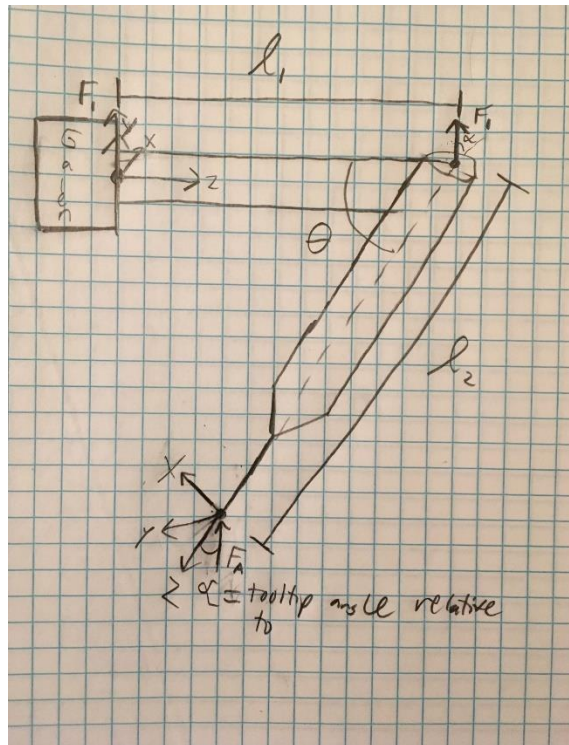
Preliminary Documentation for Calibration Procedure

General Procedure

We will measure forces at several depths in material. These depths will be controlled by the Galen system's positional control feature. We will have two resulting .txt files, one with 12 force readings from the drill and 3 from the Galen system's force sensor. Once we transform the Galen force sensor reading into a drill tip force, we solve a simple least-squares calibration problem.

Obtaining Drill Tip force

Below is a simplified diagram of the setup.



The tip forces can be written in terms of forces recorded by the Galen robot. We should be able to obtain from the Galen system the angle of rotations necessary to get the forces at the top right joint in terms of x , y , and z forces in the tip coordinate frame. We have CAD models for the drill dimensions but are still determining what data the Galen system provides.

Least-squares Calibration

We will have 12 total force readings from the drill:

$$f_{x1}, f_{y1}, f_{z1}, f_{x2}, f_{y2}, f_{z2}, f_{x3}, f_{y3}, f_{z3}, f_{x4}, f_{y4}, f_{z4}$$

The Galen will output xyz force readings that will be transformed to tip forces. The tip forces can be represented as t_x, t_y, t_z . Therefore, with m data instances, we can set up the following matrix relations: $A\vec{x} \cong \vec{b}_x, A\vec{y} \cong \vec{b}_y, A\vec{z} \cong \vec{b}_z$ where in each equation A is an m x 12 matrix.

$$A_{(m \times 12)} = \begin{pmatrix} f_{x1,1} & f_{y1,1} & \cdots & f_{z4,1} \\ \vdots & \vdots & \ddots & \vdots \\ f_{x1,m} & f_{y1,m} & \cdots & f_{z4,m} \end{pmatrix}$$

And each \vec{b} is an m x 1 column vector for each of the x, y, and z known tip forces. Therefore the objective is to solve for $\vec{x}, \vec{y}, \vec{z}$ that minimize the sum of squares error. We will use Singular value decomposition. Below, n = 12

Given an arbitrary m by n matrix **A**, there exist orthogonal matrices **U**, **V** and a diagonal matrix **S** that:

$$\mathbf{A}_{m \times n} = \mathbf{U}_{m \times m} \begin{bmatrix} \mathbf{S}_{n \times n} \\ \mathbf{0}_{(m-n) \times n} \end{bmatrix} \mathbf{V}_{n \times n}^T \quad \text{for } m \geq n$$

To solve each of the Ax = b problems follow the procedure for finding x (CIS 1 method)

$$\mathbf{A}_{m \times n} \mathbf{x} \approx \mathbf{b}$$

$$\mathbf{U}_{m \times m} \begin{bmatrix} \mathbf{S}_{n \times n} \\ \mathbf{0}_{(m-n) \times n} \end{bmatrix} \mathbf{V}_{n \times n}^T \mathbf{x} = \mathbf{b}$$

$$\begin{bmatrix} \mathbf{S}_{n \times n} \\ \mathbf{0}_{(m-n) \times n} \end{bmatrix} \mathbf{y} = \mathbf{U}_{m \times m}^T \mathbf{b} \quad \text{where } \mathbf{y} = \mathbf{V}^T \mathbf{x}$$

Solve this for **y** (trivial, since **S** is diagonal), then compute

$$\mathbf{V} \mathbf{y} = \mathbf{V} \mathbf{V}^T \mathbf{x} = \mathbf{x}$$