Snake robot calibration:

In order to develop a forward kinematic model for the snake robot (I2RIS), four general operating spaces could be defined (Figure 1). There is also a mapping between the encoder values and the motor shaft angle due to the existence of gearbox which is not shown in Figure 1 for simplicity. In our application, $q \in \mathbb{R}^2$, since there are two motors (Maxon DCX08M EB KL 4.2V) with gearhead (GPX08 A 64:1) that are responsible for bending the snake robot in two perpendicular planes, $X \in \mathbb{R}^3$ is the snake tip position described in the snake base coordinate and $\gamma \in \mathbb{R}^2$ represents the two bending angles pitch and yaw (there is no roll).
An analytical forward kinematic mapping between the encoder values to the motor shaft angles and to the cable displacements is developed in previous work by (1).

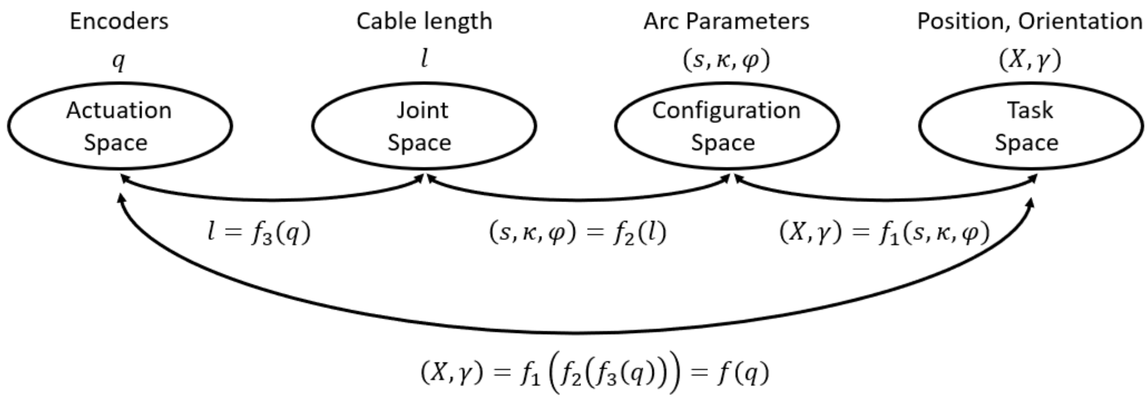

Figure 1 . Schematic view of the operating spaces of the snake robot (I2RIS)

Given that this approach models the robot kinematic based on the cable displacement, which is not measurable in practice and we can only measure the encoder values and the position/orientation using camera, we will utilize an experimental kinematic modeling approach employed by Song et al. (2). Figure 2 shows the calibration stage that is used for measuring the snake tip position and orientation (bending angle) as output of the vision-based algorithm for different encoders values as the input for five fully cycles in the snake robot's range of motion (~ -40 deg to + 40 deg).

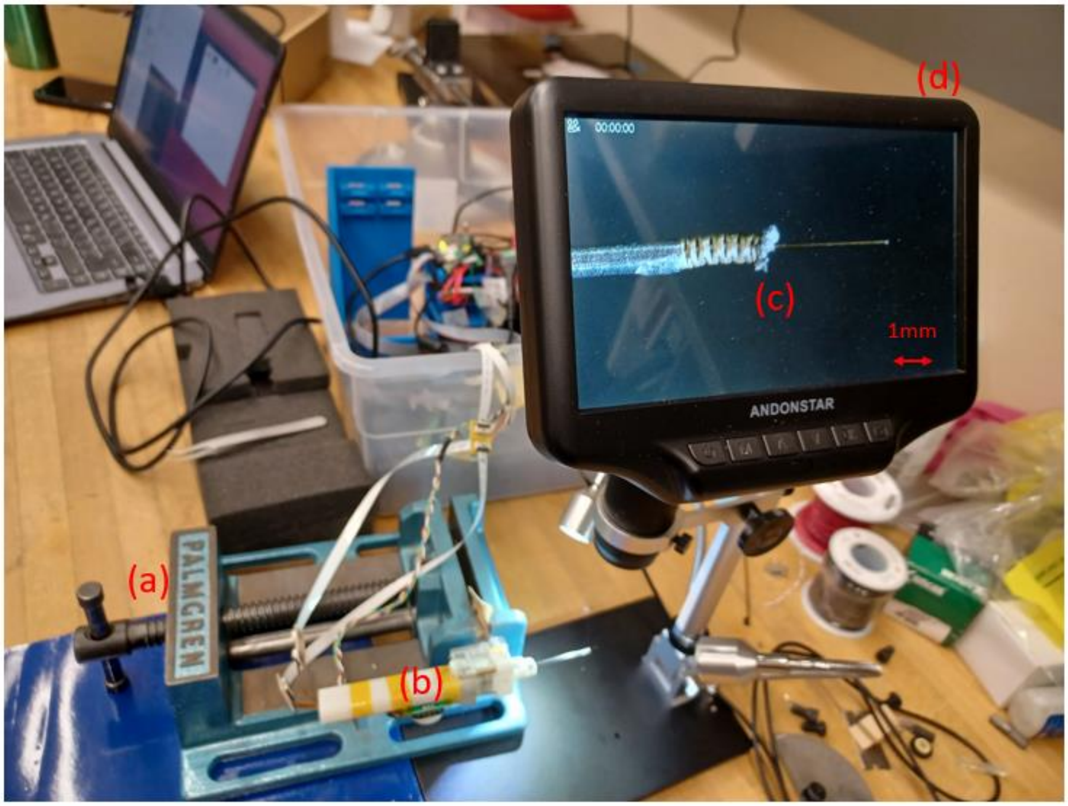

Figure 2. Calibration stage: a) fixture b) actuation unit of the snake robot including Maxon motors, derive pulley mechanism, cables etc. c) snake segment and an optical fiber passed through the snake as an indicator of measuring the bending angle as seen by the microscope d) microscope

In the first test, calibration has been done on the brass snake which shows that it has several mechanical problems such as backlash, hysteresis, and wire tension problem around the neutral axis (when the snake is oriented strait) (Figure 3 and Figure 4).

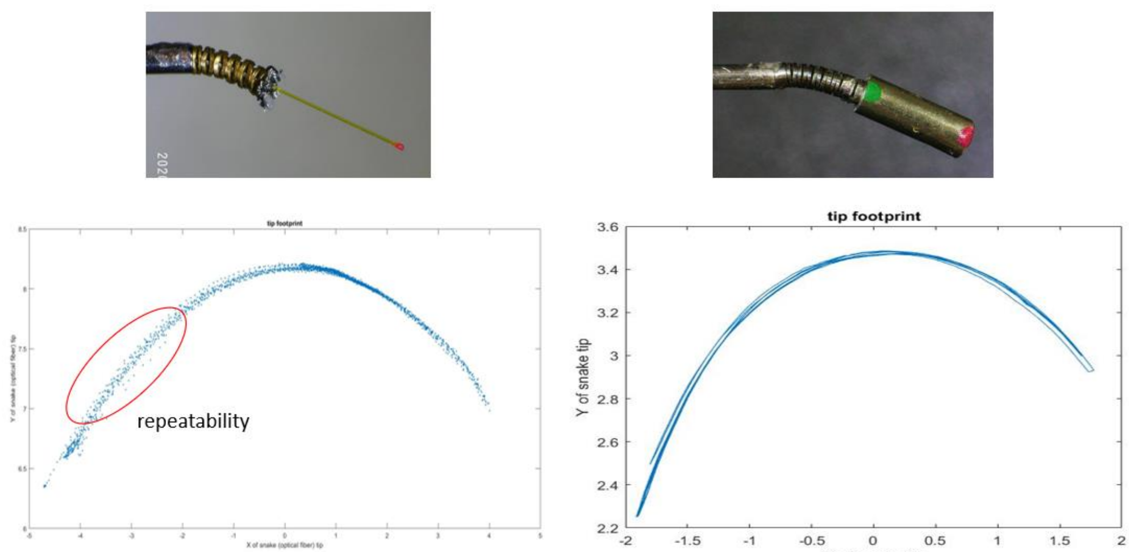

Figure 3. Left: Calibration of the brass snake $z(mm) - x(mm)$*; Right: calibration of the stainless-steel snake* $z(mm) - x(mm)$*. We can see that the brass snake has low repeatability specially for the negative values of* x*.*

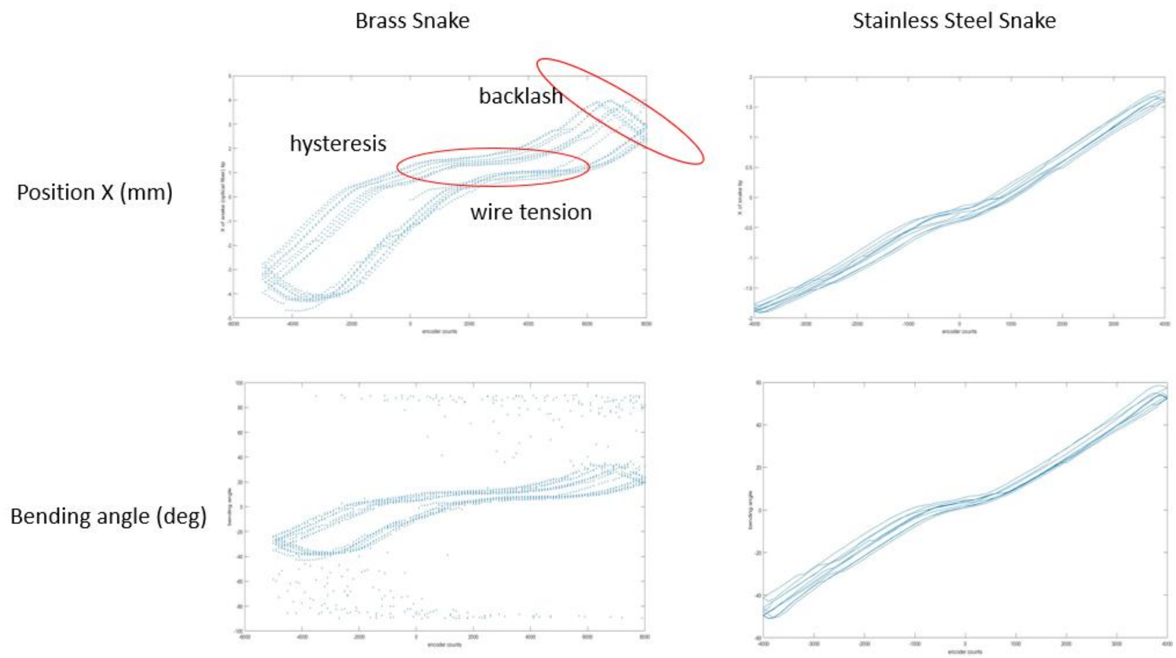

Figure 4. Left column: Brass snake calibration result, top: Snake robot tip position x (mm) vs encoder counts, bottom: snake robot bending angle γ (deg) vs encoder counts; Right column: stainless steel snake calibration result, top: Snake robot tip position x (mm) vs encoder counts, bottom: snake robot bending angle γ (deg) vs encoder counts.

Probabilistic Model (planar motion)
Gaussian Mixture Model (GMM) – Gaussian Mixture Regression (GMR):

The total dataset collected during eight reciprocating movement over the snake range of motion is defined as follows.

$$\mathcal{D} = \begin{bmatrix} q_{1,1} & q_{1,2} & \cdots & q_{c,n} & \cdots & q_{C,N} \\ \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{c,n} & \cdots & \gamma_{C,N} \end{bmatrix} \in \mathbb{R}^{D\times(C\times N)} = \mathbb{R}^{D\times\mathbb{N}}$$

In which $\gamma_{c,n}$ is the snake bending angle for n^{th} data point of the c^{th} test cycle, $q_{c,n}$ is the motor encoder counts for the n^{th} data point of the c^{th} cycle, C is the total number of calibration cycles, N is the number of data points in each cycle, $\mathbb{N}$ is the total number of data points for all (training) cycles, D is the dimension of data points and j^{th} datapoint ($j = 1,2, \dots, \mathbb{N}$) is defined as follows,

$$\xi_j = \begin{bmatrix} q_j \\ \gamma_j \end{bmatrix} = \begin{bmatrix} \xi_j^I \\ \xi_j^O \end{bmatrix} \in \mathbb{R}^{\boldsymbol{D}\times\mathbf{1}}$$

In which super scripts I and O denote input and output, respectively. Using the Gaussian Mixture Model (GMM), the probability for a datapoint ξ_j belonging to the GMM including K Gaussian distributions could be defined as follows:

$$\mathcal{P}(\xi_j) = \sum_{k=1}^{K} \mathcal{P}(k)\mathcal{P}(\xi_j|k)$$

$$\mathcal{P}(k) = \pi_k$$

$$\mathcal{P}(\xi_j|k) = \mathcal{N}(\xi_j|\mu_k, \Sigma_k) = \frac{1}{\sqrt{(2\pi)^D|\Sigma_k|}} e^{-\frac{1}{2}\left((\xi_j-\mu_k)^T \Sigma_k{}^{-1}(\xi_j-\mu_k)\right)}$$

In which $\mathcal{P}(k) = \pi_k$ is a prior probability, $\mathcal{P}(\xi_j|k)$ is the conditional probability density, μ_k and Σ_k are the mean value and covariance matrices of the k^{th} Gaussian distribution $\mathcal{N}(\xi_j|\mu_k, \Sigma_k)$.

Then We used Expectation Maximization (EM) algorithm over the dataset $\boldsymbol{\mathcal{D}}$ to iteratively optimize the parameters of the GMM ($\Theta_k = \{\pi_k \in \mathbb{R},\ \mu_k \in \mathbb{R}^2,\ \Sigma_k \in \mathbb{R}^{2\times2}\}_{k=1}^{K}$) subject to the following constraint:

$$\textstyle\sum_{k=1}^{K} \pi_k = 1 \quad , \quad \pi_k \in [0,1]$$

After training the GMM model with EM algorithm and optimizing the parameters Θ_k, it is now possible to estimate the output (snake bending angle or tip position) for any given input (encoder counts) with Gaussian Mixture Regression (GMR). The conditional probability of output ξ^O for a given input ξ^I could then be estimated in the GMR phase as

$$\mathcal{P}(\xi^O|\xi^I) \sim \sum_{k=1}^{K} h_k \mathcal{N}(\hat{\xi}_k, \hat{\Sigma}_k)$$

In which $\hat{\xi}_k$ and $\hat{\Sigma}_k$ are the expected mean values and covariance matrices of the k^{th} Gaussian distribution and are calculated as follows,

$$\hat{\xi}_k = \mu_k^O + \Sigma_k^{OI}(\Sigma_k^{OI})^{-1}(\xi^I - \mu_k^I)$$

$$\hat{\Sigma}_k = \Sigma_k^O - \Sigma_k^{OI}(\Sigma_k^I)^{-1}\Sigma_k^{IO}$$

and $h_k = \mathcal{P}(k|\xi^I)$ specifies the probability of the k^{th} Gaussian distribution being responsible for ξ^I and is calculated as

$$h_k = \frac{\mathcal{P}(k)\mathcal{P}(\xi^I|k)}{\sum_{k=1}^{K} \mathcal{P}(i)\mathcal{P}(\xi^I|i)} = \frac{\pi_k \mathcal{N}(\xi^I; \mu_k^I, \Sigma_k^I)}{\sum_{k=1}^{K} \pi_i \mathcal{N}(\xi^I; \mu_i^I, \Sigma_i^I)}$$

Finally, it is possible to approximate the conditional expectation of the output ξ^O for a given input ξ^I using a single Gaussian distribution function $\mathcal{N}(\hat{\xi}, \hat{\Sigma})$ such that

$$\hat{\xi} = \sum_{k=1}^{K} h_k \hat{\xi}_k$$

$$\hat{\Sigma} = \sum_{k=1}^{K} h_k^2 \hat{\Sigma}_k$$

Results (Figure 5) show that a GMM model with at least 3 different Gaussian distributions is capable of representing the input-output behavior of the snake with good precision.

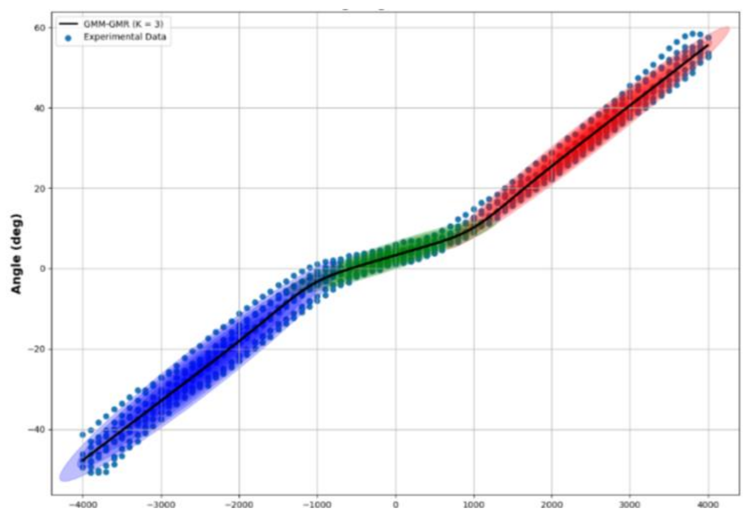

Figure 5. Probabilistic GMM-GMR algorithm for modeling the snake bending angle vs. motor encoder counts.

	Values
Coefficient of Determination (R2)	0.99032982
Root-Mean-Square Deviation (RMSD)	2.74465439
Normalized Root-Mean-Square Deviation (NRMSD)	0.0251204