Simulation-Based Uncertainty Propagation in Geometric Networks for Surgical Robotics

Computer Integrated Surgery II
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Introduction

We built a Python library for uncertainty propagation in geometric networks, validated against Monte Carlo simulation.

- Geometric Network API supporting chain, multi-path, point, distance, and loop-closure queries
- Unified Bayesian multi-path fusion via an information-form S-matrix
- 9 Monte Carlo validation scripts (Frobenius error < 1% across all scenarios)

The general problem: In surgical robotics, pose uncertainty accumulates across every sensor, link, and coordinate frame in the system. No existing tool tracks this accumulation through arbitrary network topologies — this work provides that capability.

The Problem

Surgical robotic systems chain together multiple sensors, rigid links, and coordinate frames — each introducing measurement error. In practice:

- Clinically: Uncharacterized pose uncertainty can cause tool-tip errors exceeding safe margins, especially in precise procedures such as bone cutting or needle placement
- Technically: Errors compound non-linearly through SE(3) composition; naive linear addition gives wrong answers
- Current state of the art: Systems report single end-effector accuracy numbers but do not propagate full covariance through the kinematic network, leaving surgeons and engineers blind to which component dominates the error budget

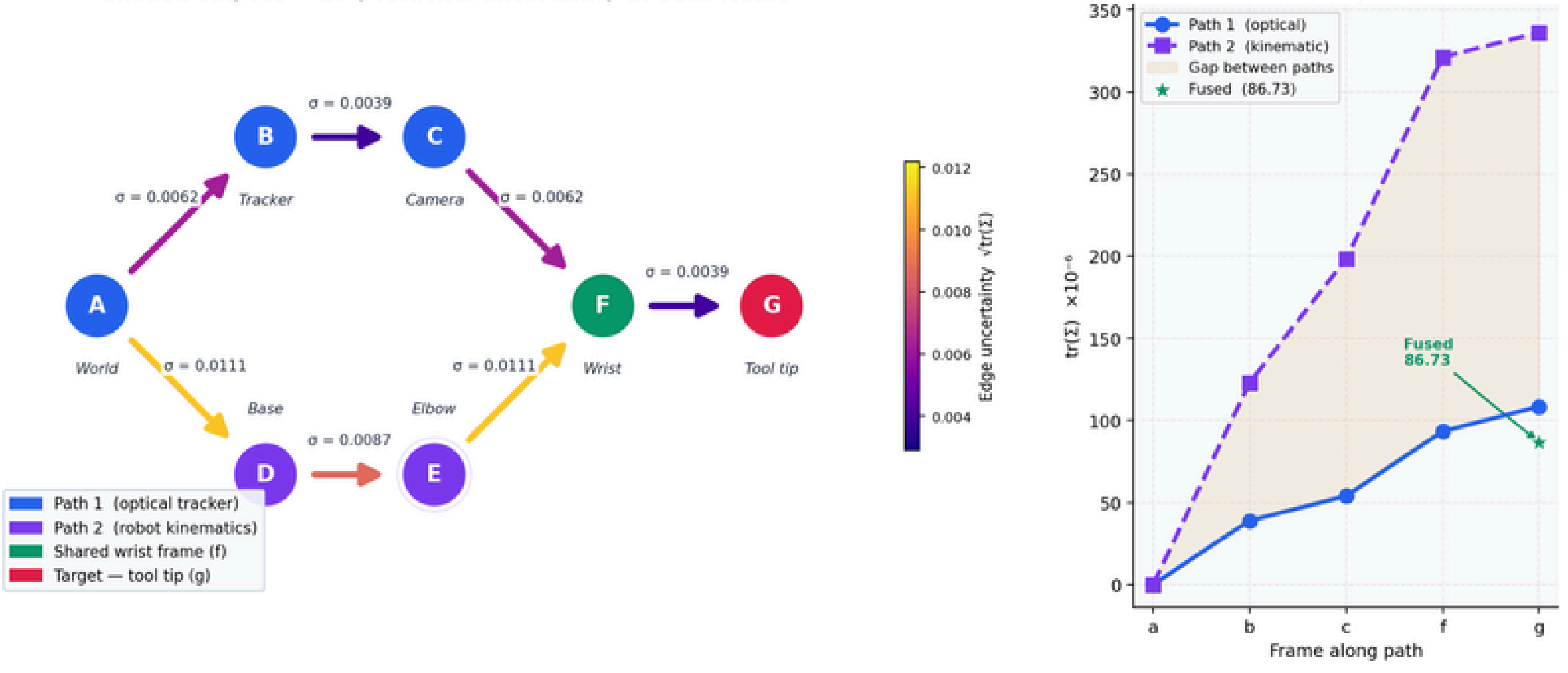
The Solution

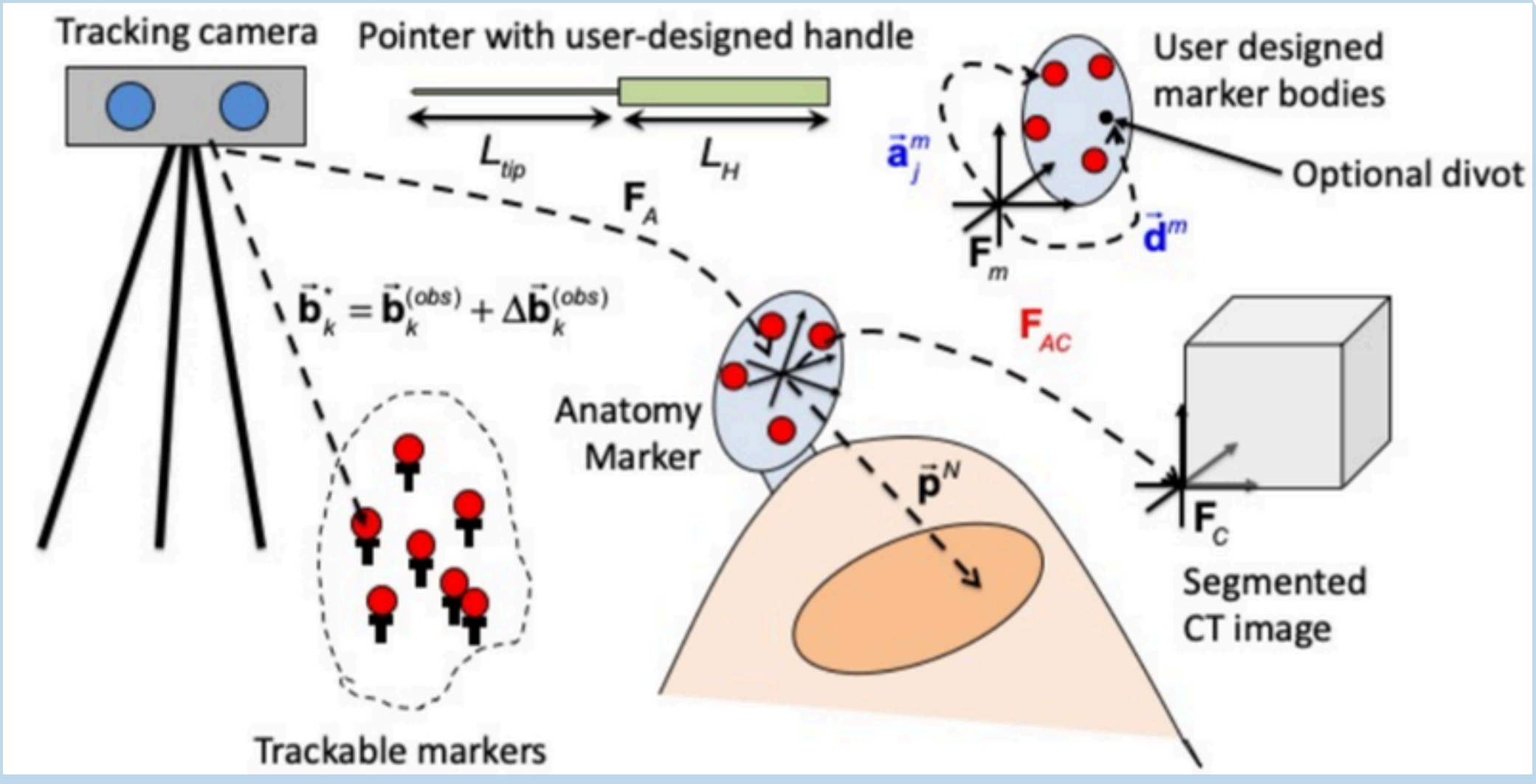
- Define uncertainty-aware geometric primitives
- Build composable operators for SE(3) chains
- Represent system as a directed **graph network**
- Support **multi-path Bayesian fusion** and **loop conditioning**
- Validate analytically and via **Monte Carlo** simulation
- Visualize in **Asynchronous multi-body framework** (**AMBF**)
- Bridge simulation to real systems via AMBF integration

Results

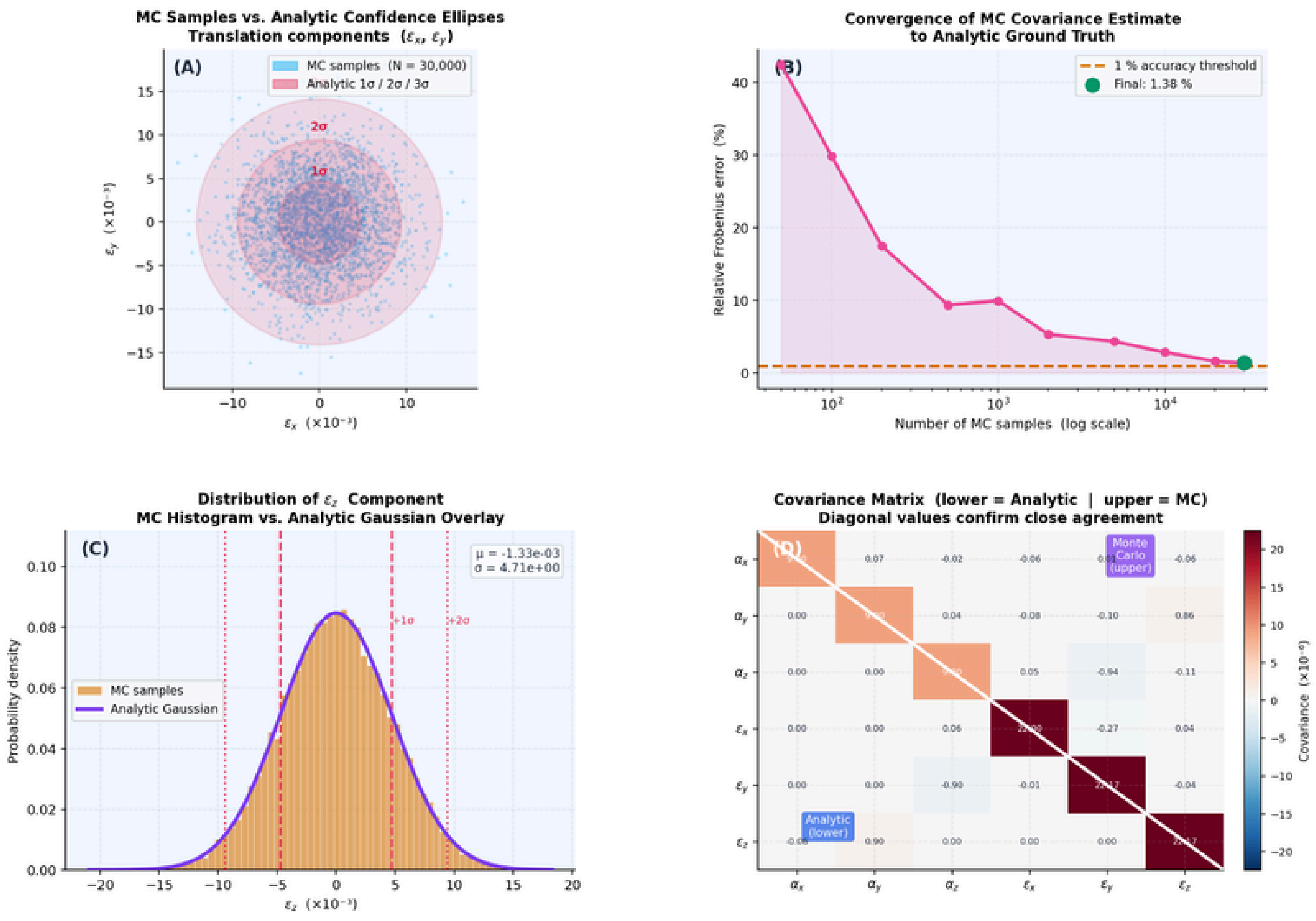
1 Results:

- Analytic covariance matches Monte Carlo (N = 30,000 samples) with **< 2% relative Frobenius error** across open chains, branching networks, and point queries
- Multi-path Bayesian fusion with two equal-uncertainty paths reduces covariance trace by **~50%**
- Closed-loop constraint conditioning reduces uncertainty trace by **> 40%** on diamond network topology
- Mixed observation types (loop + point + distance) all decrease posterior uncertainty monotonically

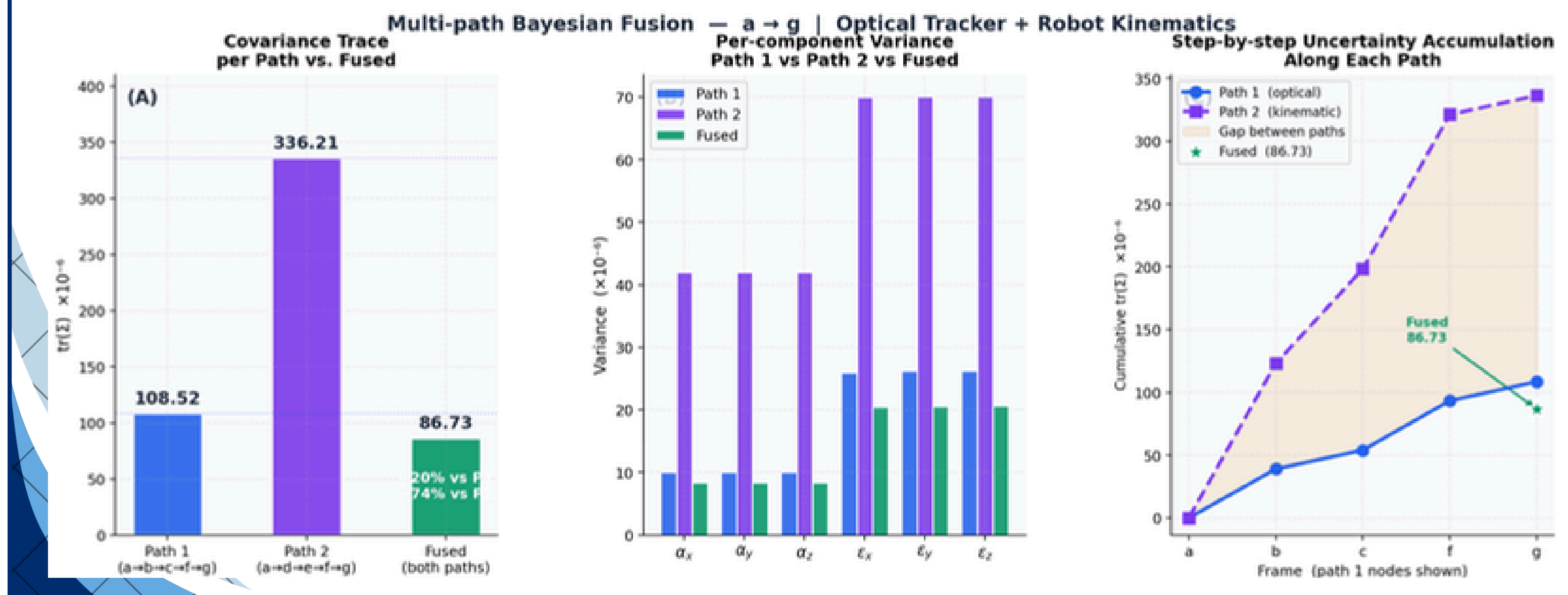




2 Monte Carlo Validation:



3 Multi-path Bayesian Fusion:



4 Lessons Learned:

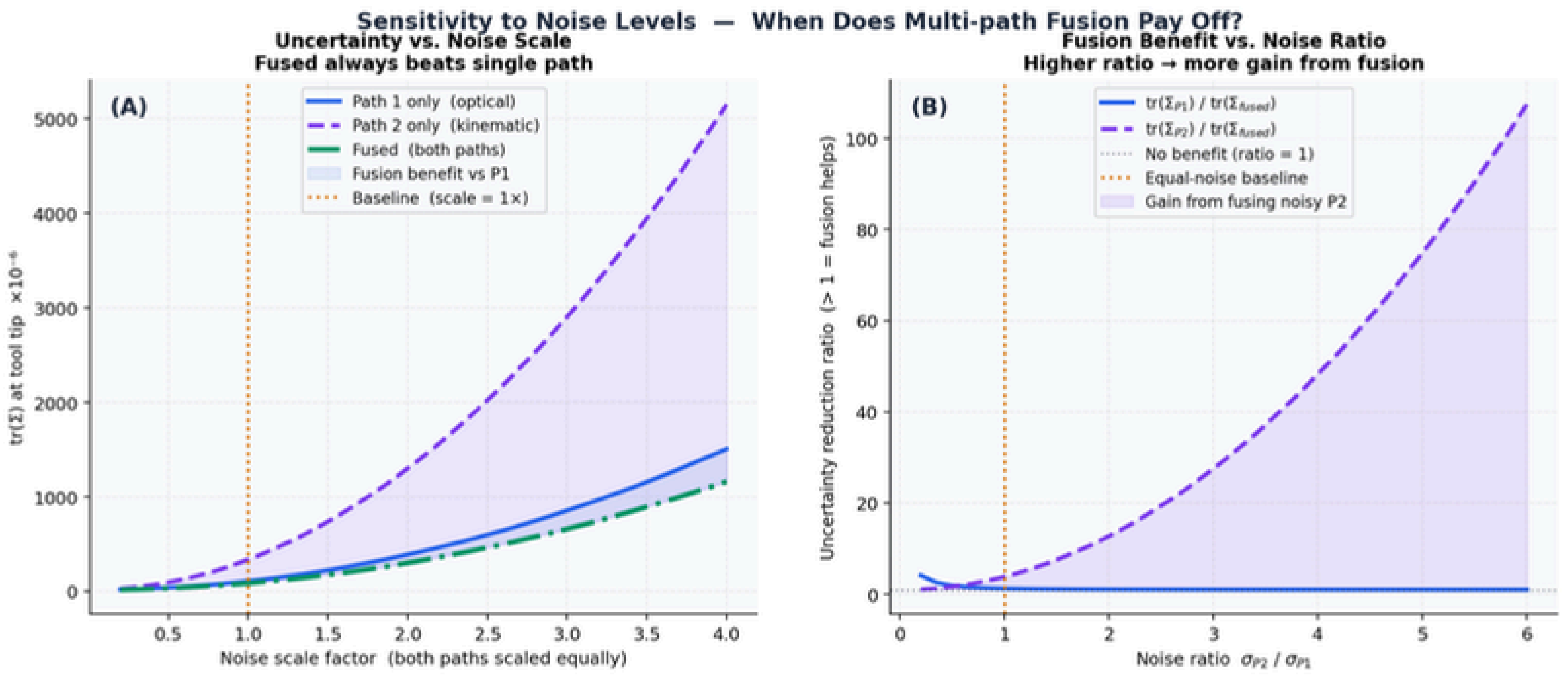
- SE(3) Lie group mathematics — learned to work directly on the manifold using exp/log maps and adjoint transport instead of treating rotation matrices as flat vectors
- Gaussian uncertainty propagation — learned to correctly propagate covariance through nonlinear rigid-body composition using first-order linearization
- Bayesian information fusion — learned to fuse multiple independent measurements in information form, and how shared structure (common edges) affects the result
- Factor graph thinking — learned to model heterogeneous constraints (loop, point, distance) as a unified observation abstraction with a joint information filter update
- Monte Carlo validation methodology — learned to design importance-weighted MC experiments and diagnose when analytic and empirical results disagree

5 Future Work:

AMBF Integration
- Integration with AMBF for simulated surgical environment and geometry interaction.

GUI
- User interface for interacting with the network, selecting nodes, visualizing paths, and inspecting uncertainty.

Full System Integration & Validation
- Combine all modules (multi-constraint, observation model, AMBF, visualization) and validate on a surgical use-case scenario.



Publications

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- T. D. Barfoot and P. T. Furgale, “Associating uncertainty with three-dimensional poses for use in estimation problems,” IEEE Transactions on Robotics, vol. 30, no. 3, pp. 679–693, Jun. 2014, doi: 10.1109/TRO.2014.2298059.
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