
**SIMULATION-BASED UNCERTAINTY
PROPAGATION IN GEOMETRIC
NETWORKS FOR SURGICAL ROBOTICS**

Project Test Plan

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1 Purpose and Scope

This document defines the verification and validation (V&V) methodology for the geometric uncertainty propagation framework.

Verification addresses:

Is the mathematics implemented correctly?

Validation addresses:

Does the system behave consistently with theoretical and statistical expectations?

The test plan applies to:

- Lie group operations in $SE(3)$
- Covariance propagation rules
- Graph-level composition
- Closed-loop conditioning
- Monte Carlo statistical validation
- Numerical robustness and stability
- Performance scalability

2 Validation Philosophy

Principle 2.1. All uncertainty propagation must remain consistent with the Lie group structure of rigid-body transformations and preserve probabilistic consistency under Gaussian assumptions.

The framework rests on four primary assumptions:

1. Left-multiplicative perturbation model
2. First-order linearization
3. Gaussian uncertainty
4. Independence of edge perturbations

Each assumption is explicitly tested.

3 Mathematical Primitive Verification

3.1 Exponential and Logarithmic Consistency

Theorem 3.1 (Exp-Log Consistency). For sufficiently small perturbations $\vec{\eta}$,

$$\log(\exp(\vec{\eta})) = \vec{\eta}.$$

Test Procedure:

- Sample 1000 perturbations $\|\vec{\eta}\| < 10^{-2}$.
- Compute $T = \exp(\vec{\eta})$.
- Compute $\vec{\eta}' = \log(T)$.

Acceptance Criterion:

$$\|\vec{\eta} - \vec{\eta}'\|_2 < 10^{-8}.$$

3.2 Adjoint Consistency

Proposition 3.2 (Adjoint Inversion Identity).

$$\text{Ad}_{F^{-1}} = (\text{Ad}_F)^{-1}.$$

Acceptance Criterion:

$$\|\text{Ad}_{F^{-1}} - (\text{Ad}_F)^{-1}\|_F < 10^{-10}.$$

4 Covariance Propagation Verification

4.1 Single-Step Composition

Theorem 4.1 (First-Order Covariance Propagation). For independent perturbations,

$$C_{ac} = C_{ab} + \text{Ad}_{F_{ab}} C_{bc} \text{Ad}_{F_{ab}}^T.$$

Validation:

- Compute analytic covariance.
- Compute Monte Carlo empirical covariance.

Acceptance:

$$\frac{\|C_{emp} - C_{analytic}\|_F}{\|C_{analytic}\|_F} < 5\%.$$

4.2 Linearization Order Validation

Since first-order linearization is used:

$$\text{Error} \propto \sigma^2.$$

Procedure:

- Scale perturbation magnitude.
- Measure Monte Carlo deviation.
- Verify quadratic scaling in log-log space.

5 Network-Level Validation

5.1 Open-Chain Propagation

Construct multi-edge chains and verify:

- Monotonic increase in uncertainty.
- Agreement with Monte Carlo.

5.2 Disconnected Node Detection

Attempt invalid queries.

Principle 5.1. Disconnected node queries must produce structured exceptions.

6 Closed-Loop Conditioning Validation

6.1 Posterior Covariance Reduction

Theorem 6.1 (Uncertainty Reduction Under Conditioning).

$$C_{\text{post}}^{-1} = C_{\text{prior}}^{-1} + H^T C_m^{-1} H.$$

Expected Behavior:

$$\det(C_{\text{post}}) < \det(C_{\text{prior}}).$$

6.2 Rotation-Only Constraints

Verify only rotational block decreases.

6.3 Translation-Only Constraints

Verify only translational block decreases.

7 Monte Carlo Statistical Validation

7.1 Sampling Correctness

Procedure:

- Sample 10^4 perturbations.
- Compute empirical covariance.

Requirement:

Sample covariance converges to analytic covariance within 5%.

8 Numerical Robustness Testing

8.1 Positive Semi-Definiteness

Principle 8.1. All covariance matrices must remain symmetric and PSD.

Verification:

$$\lambda_i(C) \geq -10^{-10}.$$

8.2 Ill-Conditioned Jacobians

Test near-singular conditioning matrices.

System must:

- Avoid numerical crash.
- Provide controlled warning.

9 Stress Testing

9.1 Zero Covariance

Test $C = 0$ propagation.

9.2 Large Covariance

Test large translational variance.

9.3 Deep Network

Construct 100-node chain.

Verify:

- Stable runtime
- No memory leak

10 Performance Benchmarks

10.1 Query Latency

Target:

Query Time < 50 ms (50-node network).

10.2 Monte Carlo Throughput

Target:

> 5000 samples/sec.

11 Regression Testing

Every modification must:

- Pass all existing tests.
- Preserve numerical tolerances.
- Maintain PSD constraints.

12 Acceptance Criteria

The framework is considered validated when:

1. All mathematical unit tests pass.
2. Monte Carlo error $\leq 5\%$.
3. Covariances remain symmetric and PSD.
4. Conditioning reduces uncertainty volume.
5. No numerical instability observed.
6. Random network tests succeed on 100 graphs.
7. Performance benchmarks met.

13 Conclusion

This verification and validation plan ensures that the uncertainty propagation framework satisfies mathematical correctness, numerical robustness, statistical validity, and software reliability standards appropriate for research and teaching applications.