Intraoperative Fiducial Tracking in TORS

CIS2   Project #15

Seminar Report: Paper Critical Review

"Active Contours Without Edges"

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1. Brief introduction to my project:

The goal of this project is to design and implement an intraoperative fiducial tracking algorithm in TORS that can accurately track the fiducial under the endoscope.

The fiducial now using are three colored spheres, yellow, white and black, and connected by a rigid green frame.

Basic technique is to locate the green frame by detecting its contours first and then detect the fiducials attached to it.

2. Paper selection:

"Active Contours Without Edges" by Tony F. Chan and Luminita A. Vese

Reason of selection:

This paper provides a model that can detect objects whose boundaries are not necessarily defined by the gradient of image, and it works well on noisy images. Besides, there have been a bunch of adaptive models upon this one which give good contour shapes, and which is related to my project. Furthermore, it's frequently cited and is basic to a many computer vision tasks.

3. Summary of the paper:

It has proposed a new model for active contours to detect objects in a given image, based on techniques of curve evolution, Mumford–Shah functional for segmentation and level sets. The model can detect objects whose boundaries are not necessarily defined by gradient, by minimizing an energy which can be seen as a particular case of the minimal partition problem. In the level set formulation, the problem becomes a "mean-curvature flow"-like evolving the active contour, which will stop on the desired boundary.

The significance of this paper is that it doesn't depend on the gradient of the image, as in the classical active contour models, but is instead related to a particular segmentation of the image.
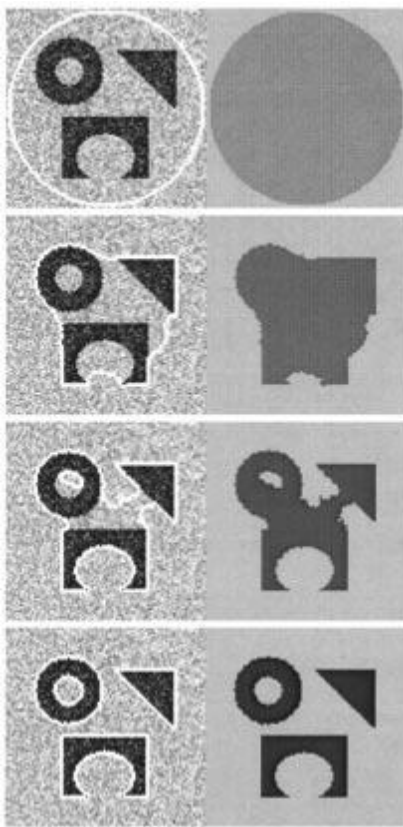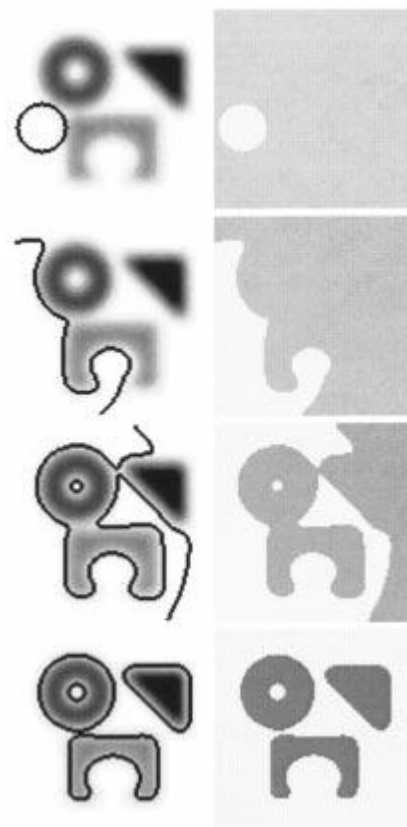
Some results shown in the paper:

Figure 1

Figure 2

The objects are differently shaped with an interior contour and it's a noisy image in Figure 1.

The objects are of different intensities and have blurred edges in Figure 2.

The model works good on both conditions.

4. Necessary background to read the paper:

Some knowledge of the Snakes, of the Mumford–Shah functional, and the level sets. (It has some reference papers talking about the Mumford–Shah functional and its solutions using level sets, which I haven't read yet and don't know how the solution is got for the minimal partition problem , but this paper can still be understood at some level.)

5. A description of the model and the experiments in the paper:

Some notations:

Ω: a bounded open subset of R^2,

$\partial\Omega$: the boundary of Ω,

$u_0: \Omega \to R$: a given image,

C: $[0,1] \to R^2$: a parameterized curve, and which would evolve to the contour, also is the

boundary of an open subset ω of Ω

inside C: the region of ω

outside C: the region of $\Omega\backslash\,\omega$

First, a simple term, as an energy function, is defined :

$$F_1(C) + F_2(C) = \int_{inside(C)} |u_0(x,\,y) - c_1|^2 \, dx\, dy$$
$$+ \int_{outside(C)} |u_0(x,\,y) - c_2|^2 \, dx\, dy$$

And $\inf\limits_{C}\{F_1(C) + F_2(C)\} \approx 0 \approx F_1(C_0) + F_2(C_0)$, so the boundary of the object is the curve that minimizes $F_1(c) + F_2(c)$.

By adding the length of the curve C , and (or) the area of the region inside C as regularizing terms,

$$F(c_1, c_2, C) = \mu \cdot \text{Length}(C) + \nu \cdot \text{Area}(inside(C))$$
$$+ \lambda_1 \int_{inside(C)} |u_0(x, y) - c_1|^2 \, dx \, dy$$
$$+ \lambda_2 \int_{outside(C)} |u_0(x, y) - c_2|^2 \, dx \, dy,$$

also the c_1 and c_2 and C that minimizes F is the contour of the object.

In the paper, the parameters are set $\lambda_1 = \lambda_2 = 1$ and v=0.

So, it is a case of the minimal partition problem regarding to the Mumford–Shah functional, which can be solved using the level set method.

In the level set method, C is represented by the zero level set of a Lipschitz function $\phi:\Omega \to R$, such that

$$\begin{cases} C = \partial\omega = \{(x, y) \in \Omega: \phi(x, y) = 0\}, \\ inside(C) = \omega = \{(x, y) \in \Omega: \phi(x, y) > 0\} \\ outside(C) = \Omega \setminus \overline{\omega} = \{(x, y) \in \Omega: \phi(x, y) < 0\} \end{cases}$$

and use

$$H(z) = \begin{cases} 1, & \text{if } z \geq 0 \\ 0, & \text{if } z < 0, \end{cases} \quad \delta_0(z) = \frac{d}{dz} H(z)$$
,

The terms in the energy function $F(c_1, c_2, C)$ could be wrote as:

$$\text{Length}\{\phi = 0\} = \int_\Omega |\nabla H(\phi(x, y))| \, dx \, dy$$
$$= \int_\Omega \delta_0(\phi(x, y)) |\nabla \phi(x, y)| \, dx \, dy \quad \text{Area}\{\phi \geq 0\} = \int_\Omega H(\phi(x, y)) \, dx \, dy$$
,

$$\int_{\phi>0} |u_0(x, y) - c_1|^2 \, dx \, dy$$
$$= \int_\Omega |u_0(x, y) - c_1|^2 H(\phi(x, y)) \, dx \, dy.$$
, and

$$\int_{\phi<0} |u_0(x, y) - c_2|^2 \, dx \, dy$$
$$= \int_\Omega |u_0(x, y) - c_2|^2 (1 - H(\phi(x, y))) \, dx \, dy.$$
,

So,

$$F(c_1, c_2, \phi)$$
$$= \mu \int_\Omega \delta(\phi(x, y))|\nabla\phi(x, y)| \, dx \, dy$$
$$+ \nu \int_\Omega H(\phi(x, y)) \, dx \, dy$$
$$+ \lambda_1 \int_\Omega |u_0(x, y) - c_1|^2 H(\phi(x, y)) \, dx \, dy$$
$$+ \lambda_2 \int_\Omega |u_0(x, y) - c_2|^2 (1 - H(\phi(x, y))) \, dx \, dy$$

Using the level set formulation, (here I think uses the Euler Lagrange method to take the partial derivative)

$$c_1(\phi) = \frac{\int_\Omega u_0(x, y)H(\phi(x, y)) \, dx \, dy}{\int_\Omega H(\phi(x, y)) \, dx \, dy}$$
, when $\int_\Omega H(\phi(x, y)) \, dx \, dy > 0$

$$c_2(\phi) = \frac{\int_\Omega u_0(x, y) (1 - H(\phi(x, y))) \, dx \, dy}{\int_\Omega (1 - H(\phi(x, y))) \, dx \, dy}$$
, when $\int_\Omega (1 - H(\phi(x, y))) \, dx \, dy > 0$,

And so c_1 , c_2 can be given by

$$\begin{cases} c_1(\phi) = \text{average}(u_0) \text{ in } \{\phi \geq 0\} \\ c_2(\phi) = \text{average}(u_0) \text{ in } \{\phi < 0\} \end{cases}$$

To compute for ϕ, using a slightly regularized versions of H and δ_0, plug in F, minimize it with respect to ϕ, get (still a little confused here) :

$$\frac{\partial\phi}{\partial t} = \delta_\varepsilon(\phi)\left[\mu \text{ div}\left(\frac{\nabla\phi}{|\nabla\phi|}\right) - \nu - \lambda_1(u_0 - c_1)^2\right.$$
$$\left. + \lambda_2(u_0 - c_2)^2\right] = 0 \text{ in } (0, \infty) \times \Omega,$$
$$\phi(0, x, y) = \phi_0(x, y) \text{ in } \Omega,$$
$$\frac{\delta_\varepsilon(\phi)}{|\nabla\phi|} \frac{\partial\phi}{\partial\vec{n}} = 0 \text{ on } \partial\Omega$$

, and it is the model of the paper.

For the numerical approximation of the model part, the paper uses the $C^\infty(\overline{\Omega})$ regularization of H,

$$H_{2,\varepsilon}(z) = \frac{1}{2}\left(1 + \frac{2}{\pi}\arctan\left(\frac{z}{\varepsilon}\right)\right)$$

, to avoid local minimizer and to automatically detect the interior contours.

In all, the principle steps in the algorithm for this model are :

• Initialize ϕ^0 by ϕ_0

• Compute $c_1(\phi^n)$ and $c_2(\phi^n)$

• Solve the PDE in ϕ to obtain ϕ^{n+1} .

• Reinitialize ϕ locally to the signed distance function to the curve (this step is optional).

• Check whether the solution is stationary. If not, n=n+1 and repeat.

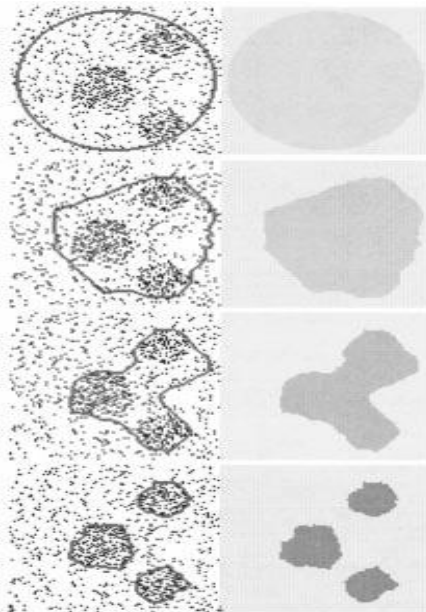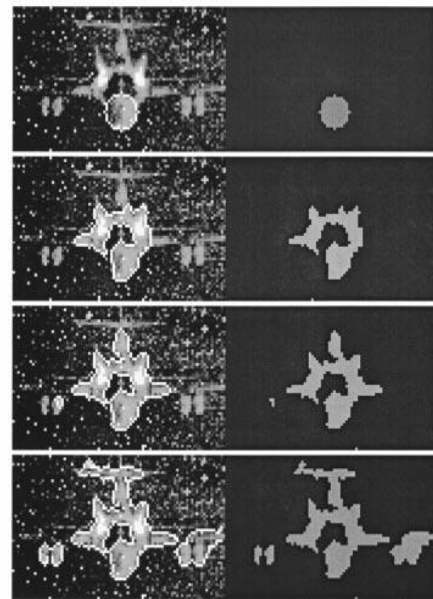Some more experimental results:

Figure 3

Figure 4

6. Assessment of the paper

I think it's a great and very basic paper for object contour detection.

It's relevant to my project as the green frame detection part. Right now, I'm using the simpler edge detectors and the color information to get the frame contour, it works well now when there's not much noise on the image and there are few other objects in the image adjacent to the green frame. But when it is the intraoperative image, where there would be much more noise, jitters and other structures around the fiducial frame, simple edge detector and color information might not be enough to accurately get the contour of the frame. Besides, after getting the contour and trying to locate the three fiducials, since they are all spheres, this contour detection model could be used to help estimate the center of the fiucials when part of the fiducial is hided.

I would implement this model, or some adapted methods of it, if the algorithm I'm using now is not sufficient enough to accurately locate the frame in the intraoperative surroundings, or to detect the positions of the three fiducial which is the goal of the project.