

## Introduction

- Accurately reconstruct a tissue/surface from finite number of force sensor palpation readings
  - Functional Geometry remodeling
    - MSE ~1.17
  - Functional Stiffness remodeling
  - Combined geometry/stiffness remodeling
  - Optimal Palpation trajectory on unknown surface
- Potential application: guiding exploratory surgery
  - Accurate real time localization of tumors

## The Problem

- How can we accurately reconstruct a surface without any assumptions on the underlying structure?
- How do we select the fewest number of points to perform this reconstruction?

## The Solution

- Create Gaussian Process (GP) algorithm to independently model both geometry and stiffness.
  - Use force sensor palpations to measure tissue height and stiffness
  - After each palpation, the two independent GP's will be updated in light of the new data
  - A GP modeling is achieved by sampling from a multivariate Gaussian distribution such that:

$$k(x, x') = \sigma_f^2 \exp\left[-\frac{(x - x')^2}{2l^2}\right] + \sigma_n^2 \delta(x, x')$$

$$\begin{bmatrix} y \\ y_* \end{bmatrix} \sim \mathcal{N}\left[0, \begin{bmatrix} K & K_*^T \\ K_* & K_{**} \end{bmatrix}\right]$$

$$y_* | y \sim \mathcal{N}(K_* K^{-1} y, K_{**} - K_* K^{-1} K_*^T)$$

$$\bar{y}_* = K_* K^{-1} y$$

$$\text{var}(y_*) = K_{**} - K_* K^{-1} K_*^T$$

- GP modeling is advantageous
  - predicted point has an associated confidence interval

Symbol	Meaning
y	Values from training points
y*	Values at test set inputs
K	Training set covariances
K*	Training-test set covariances
K**	Test set covariances
k(x, x')	Covariance element
ξ	Reference Point
ξ̃	Query Candidate

- Approaches taken to selecting the next point:
  - Randomly select nearby points to choose from based on these criteria:

a)  $\max(a * \text{predicted mean} + b * \text{predicted variance})$

b)  $\max(\text{predicted change in variance})$

a) Dynamic Sampling Area

$$\text{Interest Factor} = \frac{|\text{Boring} - \text{Previous}|}{\text{Max Difference} - \text{Minimum L}} + \text{Correction Factor}$$

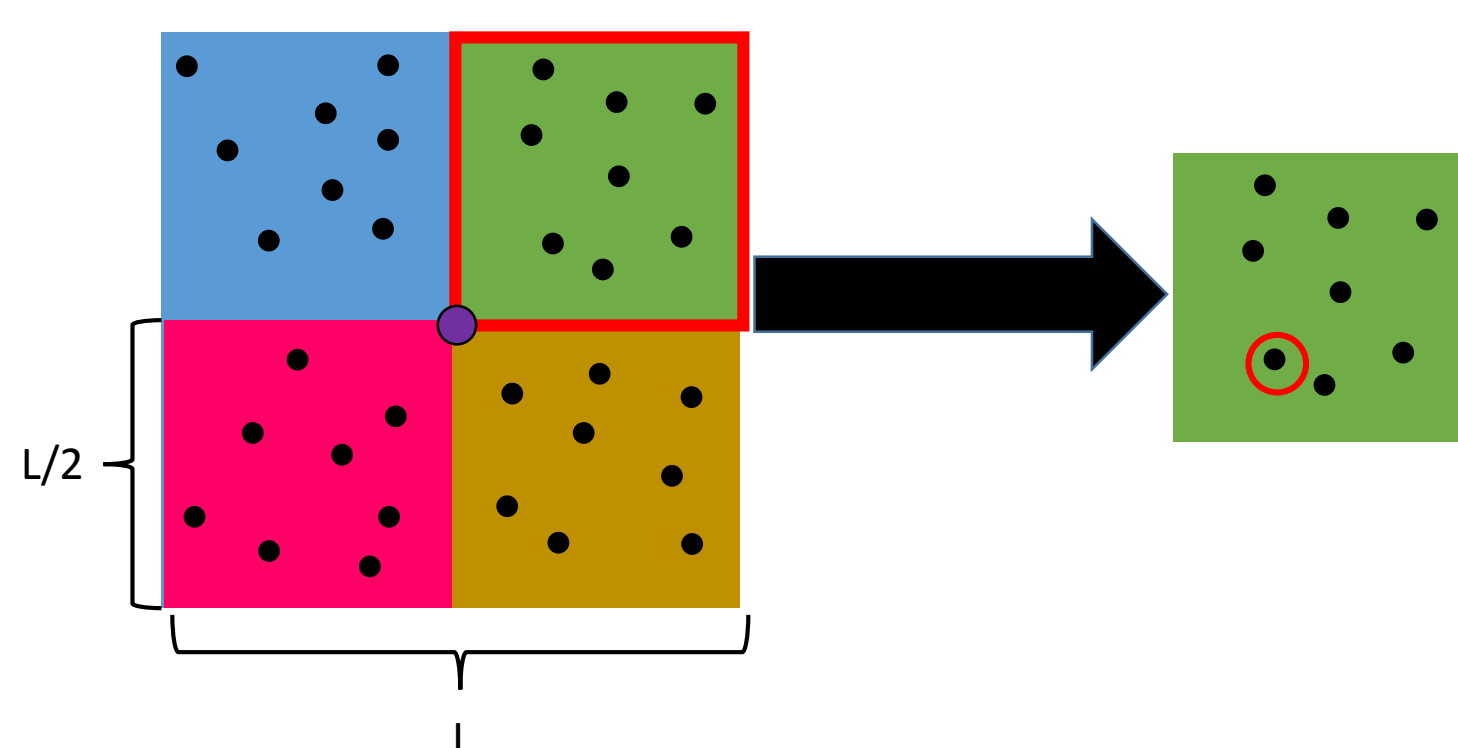
$$L = \frac{\text{Interest Factor}}{\text{Interest Factor}}$$

b) Predicted Change in Variance:

$$\Delta\sigma_{y(\xi)}^2(\tilde{x}) = \frac{(K_N K^{-1} K_* - \text{Cov}(\tilde{x}, \xi))^2}{(\text{Cov}(\tilde{x}, \tilde{x}) - K_*^T K^{-1} K_*)}$$

$$K_N = [C(x_1, \xi), \dots, C(x_N, \xi)]$$

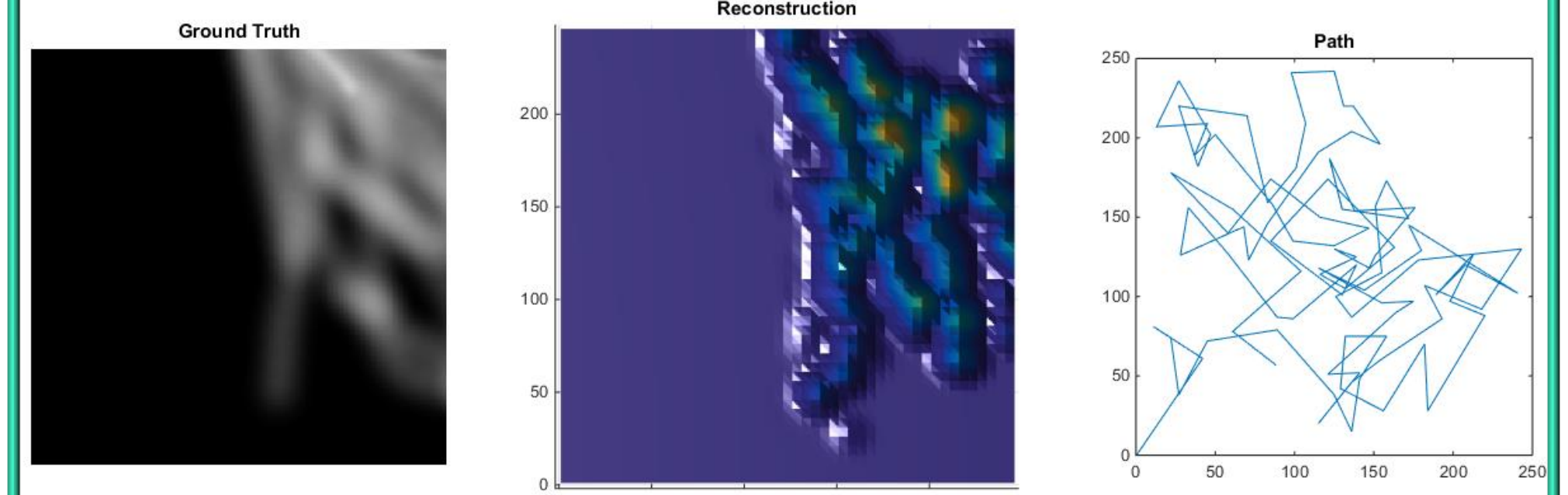
a) Quadseek



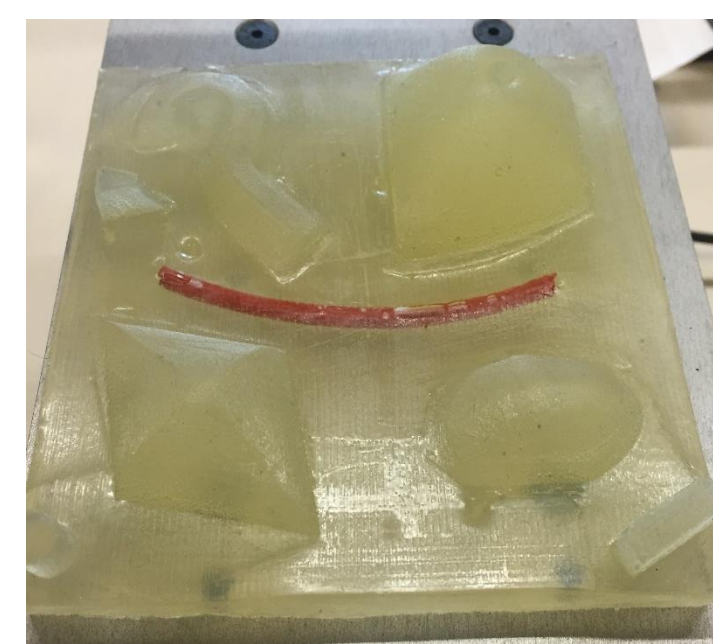
c)  $\max\left(a * \frac{\text{Predicted Mean}}{\text{Previous Predicted Mean}} + b * \frac{\text{Predicted Variance}}{\text{Previous Predicted Mean}}\right)$

## Outcomes and Results

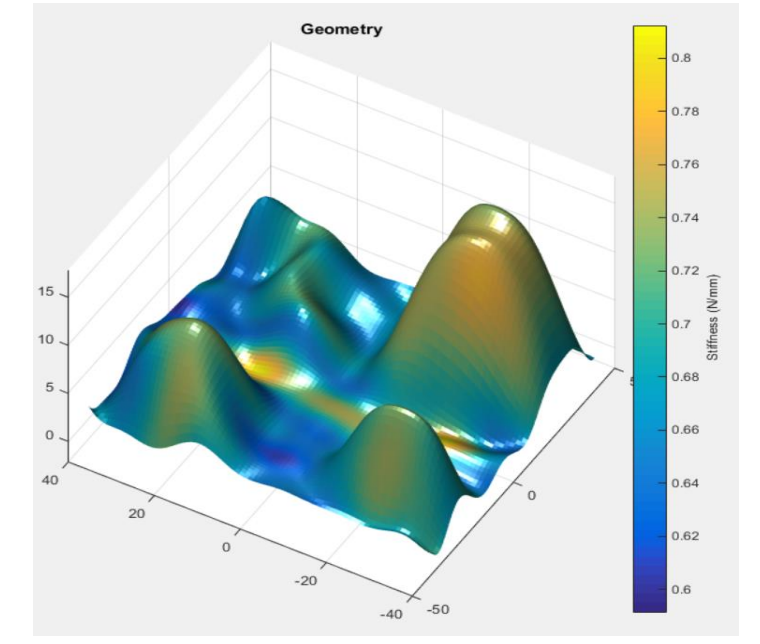
### Max(predicted change in variance)



### Phantom



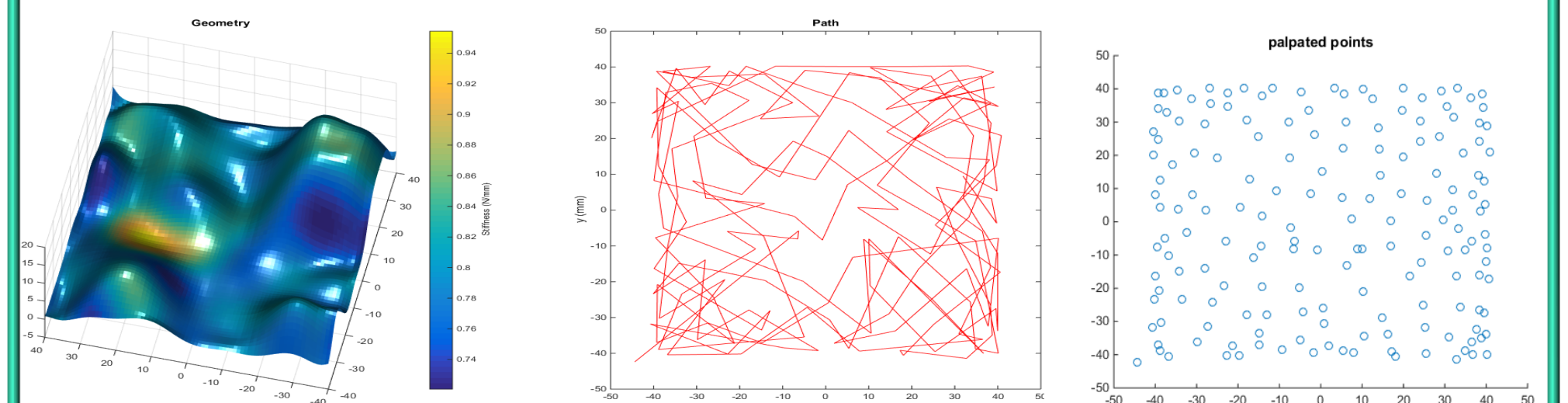
### Uniform (grid) Sampling



Num. points: 324 | Geo. MSE: .755

### Adaptive Maximum Search: Height

$$\max\left(\frac{\text{Predicted Mean}}{\text{Previous Predicted Mean}} + \frac{\text{Predicted Variance}}{\text{Previous Predicted Mean}}\right)$$

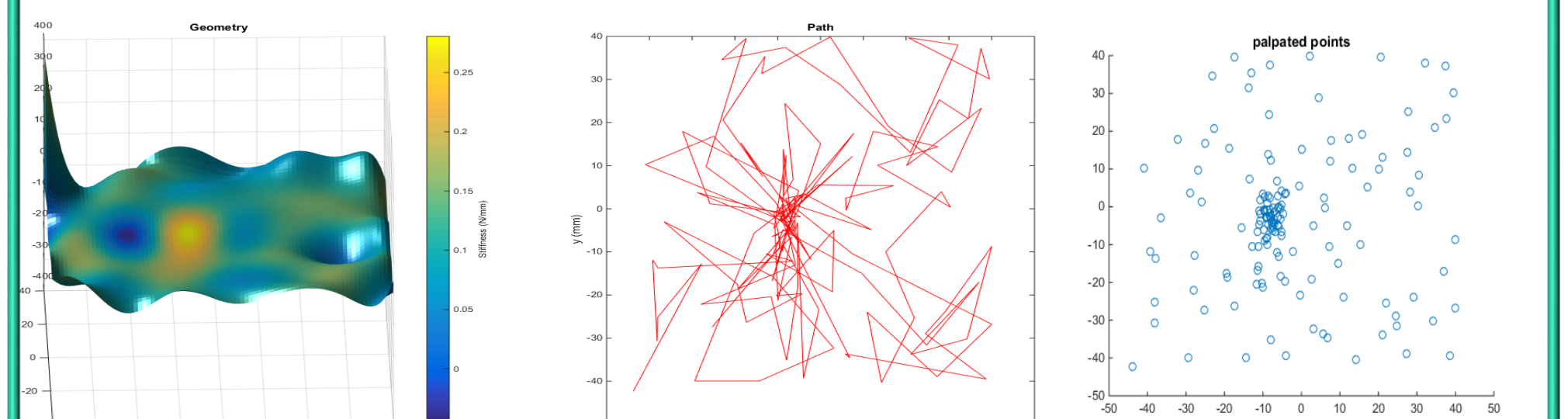


Geo. MSE: 1.17

Num. points: 150

### Adaptive Maximum Search: Stiffness

$$\max\left(\frac{\text{Predicted Mean}}{\text{Previous Predicted Mean}} + \frac{\text{Predicted Variance}}{\text{Previous Predicted Mean}}\right)$$

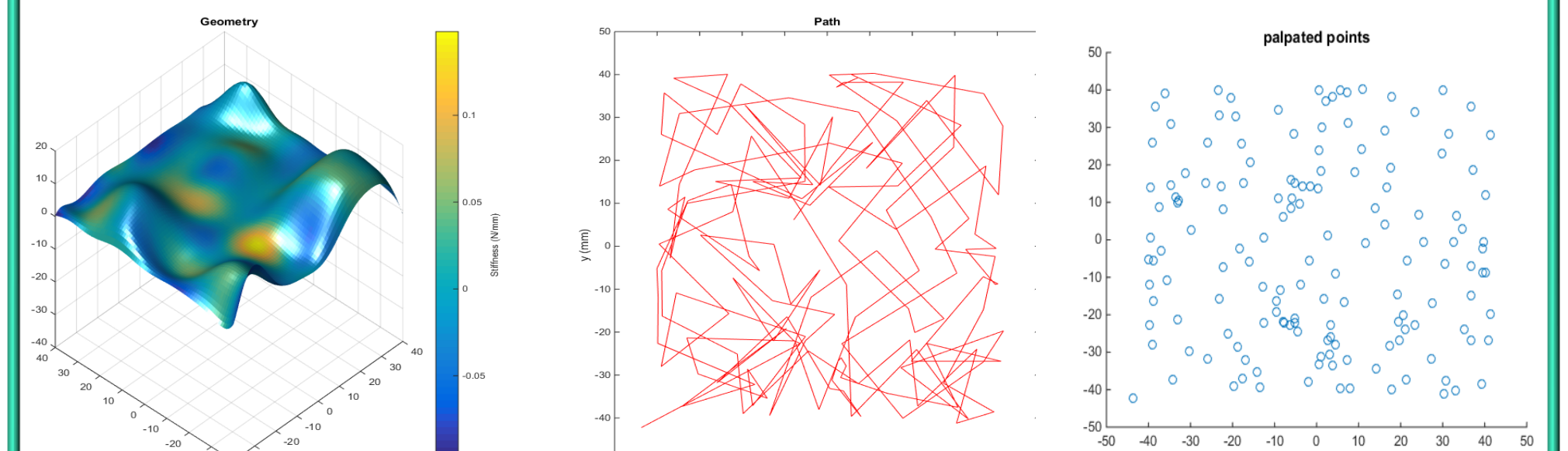


Geo. MSE: 17.84

Num. points: 150

### Adaptive Maximum Search: Height + Stiffness

$$\max\left(\frac{\text{Predicted Mean}}{\text{Previous Predicted Mean}} + \frac{\text{Predicted Variance}}{\text{Previous Predicted Mean}}\right)$$



Geo. MSE: 2.54

Num. points: 150

## Future Work

- Grid Initialization and Adaptive Grid Search
- Test on other stiffness distributions.
- Assume stiffness to be a non-linear model.
- GP with co-dependent outputs

## Support by and Acknowledgements

- CISST/ERC
- Thank you to Preetham Chalasani, Dr. Kobilarov, Dr. Taylor, and our coloborators at CMU and Vanderbilt

## Lessons Learned

- While GPs are versatile, they do have their limits
- Adaptive searching is no small feat
- Algorithms that work well on simulated data may not perform perfectly in practice
- Reports are good for thinking.
- Procrastination is not good ☺