MUSiiC Research Laboratory — Medical UltraSound Imaging and Intervention Collaboration

RPK Robot and Protein Kinematics Laboratory

Laboratory for Computational Sensing + Robotics — The Johns Hopkins University

“An Algebraic Solution to the Multilateration Problem”

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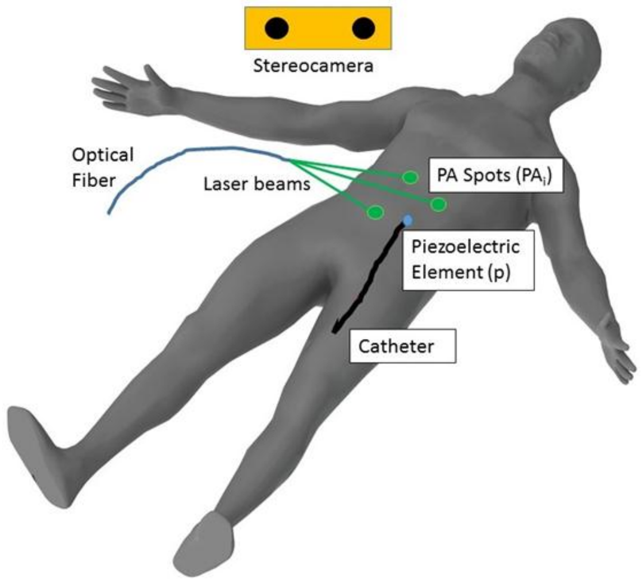
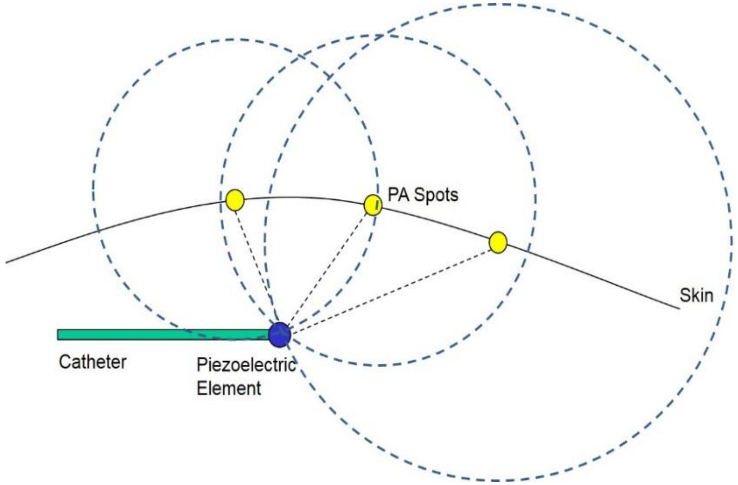
Seminar Presentation

Group 8: iPASS: Photoacoustic Catheter Tracking

iPASS Overview

- Goal: To track a catheter using a stereo camera by applying laser spots on the surface
- Laser spots can be seen by the stereo camera and generate a photoacoustic signal observed by the piezoelectric element





Paper Selection

Norrdine, A. “An Algebraic Solution to the Multilateration Problem,” *In Proceedings of the 15th International Conference on Indoor Positioning and Indoor Navigation*, 2012.

- Desired mathematic formulation of multilateration problem
- Possible to implement in system

Multilateration

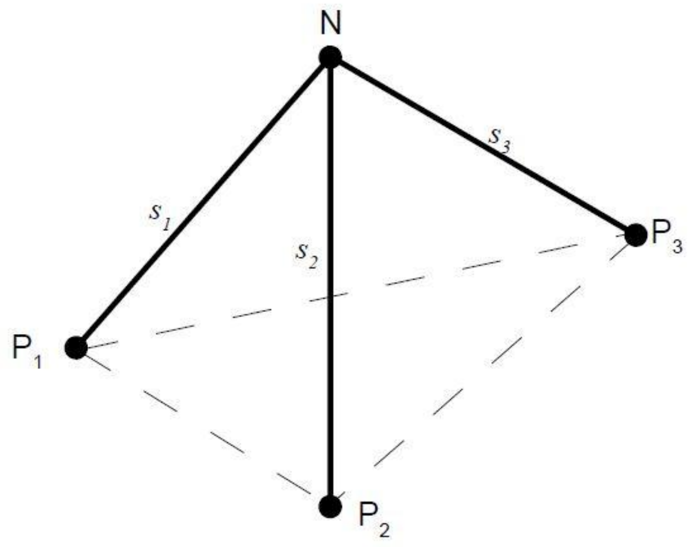
In general:

- Mathematical technique is usually used for calculating the position of a receiver from signals received from several transmitters

In our case:

- Method to be applied for optimizing the number of PA spots when the number of spots is greater than three

Trilateration



- Given three reference points $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, $P_3(x_3, y_3, z_3)$ and the range measurements s_1, s_2, s_3
- Find N(x, y, z)

Step:

1)
$$(x-x_1)^2+(y-y_1)^2+(z-z_1)^2=s_1^2$$
$$(x-x_2)^2+(y-y_2)^2+(z-z_2)^2=s_2^2$$
$$(x-x_3)^2+(y-y_3)^2+(z-z_3)^2=s_3^2$$

2)
$$\begin{bmatrix}1 & -2x_1 & -2y_1 & -2z_1\\ 1 & -2x_2 & -2y_2 & -2z_2\\ 1 & -2x_3 & -2y_3 & -2z_3\end{bmatrix}\begin{bmatrix}x^2+y^2+z^2\\ x\\ y\\ z\end{bmatrix}=\begin{bmatrix}s_1^2-x_1^2-y_1^2-z_1^2\\ s_2^2-x_2^2-y_2^2-z_2^2\\ s_3^2-x_3^2-y_3^2-z_3^2\end{bmatrix}$$

3)
$$A\boldsymbol{x}=\boldsymbol{b}$$

with the constraint: $\boldsymbol{x}\in E$

where $E=\left\{(x_0,x_1,x_2,x_3)^T\in\mathbb{R}^4 \mid x_0=x_1^2+x_2^2+x_3^2\right\}$

Trilateration (cont'd)

Step (cont'd):

4) General solution of $A\boldsymbol{x} = \boldsymbol{b}$ is $\boldsymbol{x} = \boldsymbol{x}_p + t \cdot \boldsymbol{x}_h$

Compute $\boldsymbol{x}_p$ and $\boldsymbol{x}_h$ using the Gaussian elimination method or pseudo inverse of matrix A

5) The solutions are $\boldsymbol{x}_1 = \boldsymbol{x}_p + t_1 \cdot \boldsymbol{x}_h$

$$\boldsymbol{x}_2 = \boldsymbol{x}_p + t_2 \cdot \boldsymbol{x}_h$$

Where $\boldsymbol{x} = (x_0, x_1, x_2, x_3)^T$ $\boldsymbol{x}_p = (x_{p0}, x_{p1}, x_{p2}, x_{p3})^T$ $\boldsymbol{x}_h = (x_{h0}, x_{h1}, x_{h2}, x_{h3})^T$

Multilateration

$$\begin{bmatrix} 1 & -2x_1 & -2y_1 & -2z_1 \\ 1 & -2x_2 & -2y_2 & -2z_2 \\ 1 & -2x_3 & -2y_3 & -2z_3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & -2x_n & -2y_n & -2z_n \end{bmatrix} \begin{bmatrix} x^2+y^2+z^2 \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} s_1^2 - x_1^2 - y_1^2 - z_1^2 \\ s_2^2 - x_2^2 - y_2^2 - z_2^2 \\ s_3^2 - x_3^2 - y_3^2 - z_3^2 \\ \vdots \\ s_n^2 - x_n^2 - y_n^2 - z_n^2 \end{bmatrix}$$

$$A\boldsymbol{x} = \boldsymbol{b}$$

with the constraint: $\boldsymbol{x} \in E$

Additional reference points and distances

Step:

1) The solution in the sense of least squares method

$$\hat{\boldsymbol{x}} = \left(A^T A\right)^{-1} A^T \boldsymbol{b}$$

Multilateration (cont'd)

2)

2.1) First candidate: from step 1)

2.2) Further candidates by using the Recursive Least Squares:

2.2.1) Select one of two solutions from Trilateration problem, which is closer to the first candidate, as a starting point

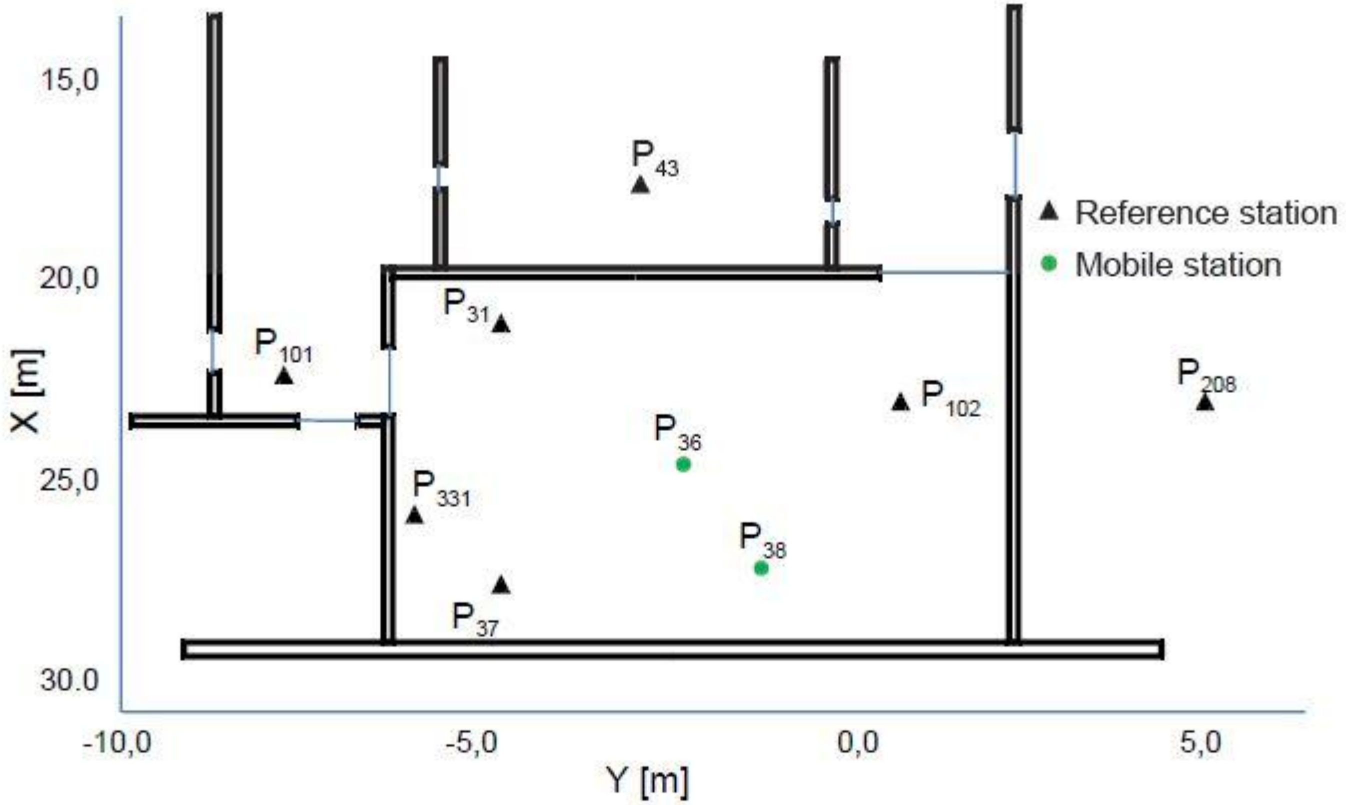
2.2.2) Let x_0 be the initial solution, then x_0 is updated to x_1 by every coming distance

2.3) The solution candidate, which minimizes the error square sum, is chosen

$$\min_N \left\{ \left(\| N - P_1 \|^2 - s_1 \right)^2 + \ldots + \left(\| N - P_n \|^2 - s_n \right)^2 \right\}$$

Experiment

- Distance measurement between the stations with positioning system
- Demonstration of the location of the mobile station located on points P_{36} and P_{38}
- Compare the results from the numerical method with the true coordinates



Experimental Results

Solution based on three reference points (Trilateration)

- The true coordinate of the unknown point P_{36} are (24.34, -2.51, 1.13)
- Three reference points are P_{37}, P_{331}, and P_{102}
- The solutions of the trilateration problem are
  - $N_1 = (24.35, -2.48, 1.67)$ and
  - $N_2 = (24.31, -2.52, 1.54)$

Experimental Results (cont'd)

Solution based on six reference points (Multilateration)

- The true coordinate of the unknown point P_{38} are (26.76, -1.34, 1.13)
- Six reference points are P_{37}, P_{31}, P_{102}, P_{43}, P_{208}, and P_{101}
- The solution of the multilateration problem is
  - N = (26.77, −1.34, 1.46)

Assessment

Pros:

- Based on linear algebraic method
- Versatile
- Possible to implement in current system

Cons:

- Unclear in some parts
- Lack of experimentation on moving objects

Questions?