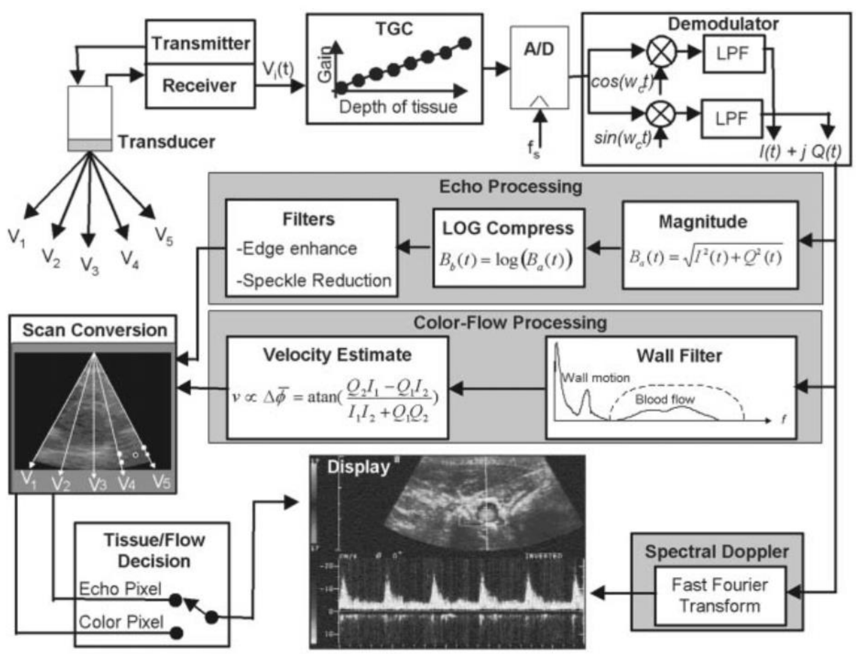
Paper review: Ultrasound Elastography: A Dynamic Programming Approach

A. Background introduction

Ultrasound

Ultrasonography uses a probe containing multiple acoustic transducer to send pulses of sound into a material. Whenever a sound wave encounters a material with a different density (acoustical impedance), part of the sound wave is reflected back to the probe and is detected as an echo. The time it takes for the echo to travel back to the probe is measured and used to calculate the depth of the tissue interface causing the echo. The greater the difference between acoustic impedances, the larger the echo is.

A-mode (amplitude mode) is the simplest type of ultrasound. A single transducer scans a line through the body with the echoes plotted on screen as a function of depth. In B-mode (brightness mode) ultrasound, a linear array of transducers simultaneously scans a plane through the body that can be viewed as a two-dimensional image on screen.



Elastography

Elastography is an emerging medical imaging method with medical applications such as tumor detection, which computes the spatial variation of the elastic modulus of tissue. There are two types of method, which are Quasi-static method and Dynamic method.

In Quasi-static method, constant stress is applied to the tissue. The elasticity can be calculated based on Hooke's law $\delta = E\varepsilon$. This type of method is easy to implement. But if the exact stress distribution inside the tissue is not known, it is not possible to have a precise estimation of the local Young's modulus.

In Dynamic method, a time-varying force is applied to the tissue -- short transient mechanical force or an oscillatory force with a fixed frequency. Based on shear waves propagation theory $E = 3\rho V_s^2$, it can produce quantitative and higher resolution Young's modulus map. However, this will require a complex system which can generate the shear wave and able to image the small displacements induced by the shear wave.

Dynamic Programming

Dynamic Programming is a method for solving a complex problem by breaking it down into a collection of simpler sub-problems, solving each of those sub-problems just once, and storing their solutions. The next time the same sub-problem occurs, instead of re-computing its solution, one simply looks up the previously computed solution, thereby saving computation time at the expense of a modest expenditure in storage space.

B. Paper content

This paper introduces a 2-D strain imaging technique based on minimizing a cost function using dynamic programming (DP). The cost function incorporates similarity of echo amplitudes and displacement continuity. Since tissue deformations are smooth, the incorporation of the smoothness into the cost function results in reduced decorrelation noise. Paper shows that the method is more robust to signal decorrelation in comparison to the standard correlation techniques. The method operates in less than 1 s and is potentially suitable for real time elastography.

One-dimensional Displacement Estimation using DP

For one-dimensional displacement estimation, we need to consider two A-lines acquired before and after compression. For cost function, there are two terms, unary term and binary term. Unary tem is used to quantify the absolute difference between corresponding echo signals. It seems like L1 norm has better robustness against outliers.

$$\Delta(i,d) = |g(i) - g'(i+d)|$$

Where $g(i)$ $g'(i)$ are echo signals sampled from these two A-lines. d is the displacement at the sample i.

Binary term is the smoothness constraint of the displacements

$$S(d_i, d_{i-1}) = (d_i - d_{i-1})^2$$

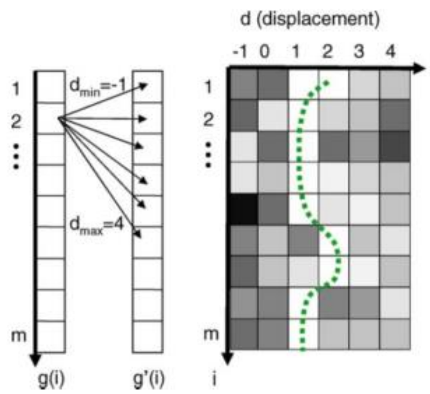


Fig. 1. In the left, values of $g(i)$ and $g'(i+d)$ corresponding to precompression and postcompression RF data are compared. Right shows the cost function C of (4) (white and black represent low and high cost values, respectively).

The cost function C at a point I and associated displacement d_i is defined as a recursive function to allow for DP.

$$C(i, d_i) = \min_{d_{i-1}}\{C(i-1, d_{i-1}) + wS(d_i, d_{i-1})\} + \Delta(i, d_i)$$

where w is a regularization weight which governs smoothness.

In DP, one important thing is to backtrack optimal solution. The function to achieve this here is

$$M(i, d_i) = \underset{d_{i-1}}{\operatorname{argmin}}\left\{C\left(i-1,\ d_{i-1}\right) + wS(d_i, d_{i-1})\right\}$$

Subsampling pre-compressed A-line signal and upsampling post-compressed A-line signal is used to speed up computation and improve displacement estimation accuracy.

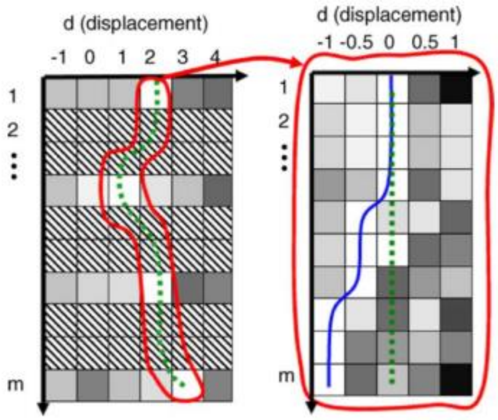


Fig. 2. In the left, the cost function C is shown when DP is performed on $g^*(i)$ ($g(i)$ downsampled by a factor of β) and $g'(i)$ (not downsampled). Hashed squares indicate no cost calculation is performed due to downsampling of $g(i)$, and white and black representing low and high cost values respectively. Displacement is calculated at m/β samples in this stage ($\beta = 3$ in this figure). In right, a new cost function around the optimum path of the first stage's cost function (the dashed line) is created, giving a $1/\gamma = 1/2$ pixel displacement accuracy at m samples.

2-D Displacement Estimation

One assumption for one-dimensional displacement estimation is that there is only pure axial compression inside the tissue. However, lateral displacement in a soft material is inevitable. Therefore, 2-D displacement estimation seems to be a better choice.

The cost function C at a point i, associated axial displacement d_a and lateral displacement d_l is defined as a recursive function

$$C_j(d_a, d_l, i) = \min_{\delta_a,\delta_l}\left\{\frac{C_j(\delta_a,\delta_l,i-1)+C_{j-1}(\delta_a,\delta_l,i)}{2} + wS(d_a, d_l, \delta_a, \delta_l)\right\} + \Delta(d_a, d_l, i)$$

The difference between pre and post-compressed corresponding signals:

$$\Delta(i,j,d_a,d_l) = \left|g_j(i) - g'_{j+d_l}(i+d_a)\right|$$

The smoothness of the displacements for adjacent locations in strain images:

$$S(d_a,d_l,\delta_a,\delta_l) = (d_a-\delta_a)^2 + (d_l-\delta_l)^2$$

Results

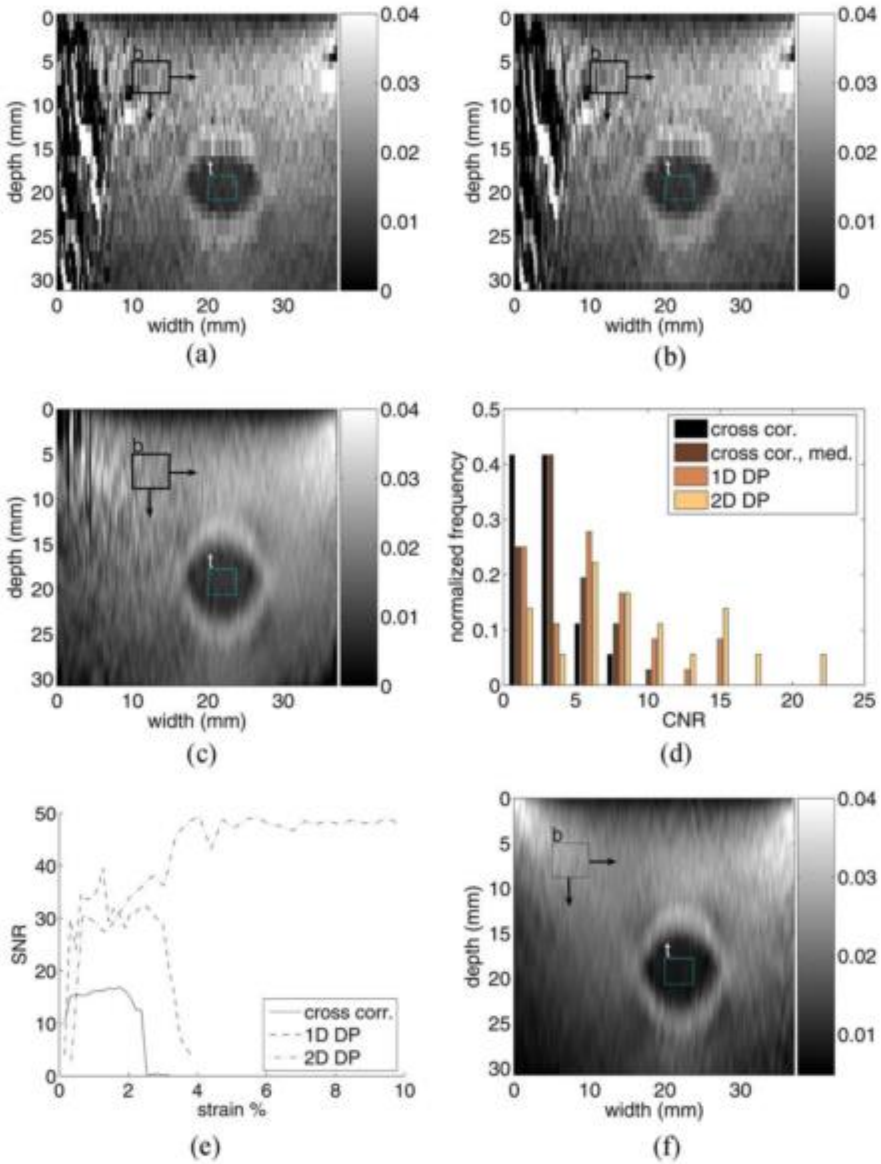


Fig. 3. (a)-(c) strain images obtained from freehand palpation of the phantom using cross correlation, cross correlation with a 3×3 median filter applied on the displacement image and 1-D DP respectively. The target window is fixed on the lesion and the background window is moved to allow multiple CNR calculation. (d) Normalized CNR values of the lesion, obtained by dividing each bin by the total of 36 CNR measurements. (e) SNR values of the cross correlation and 1-D DP techniques. (f) Strain images obtained from freehand palpation of the phantom using 2-D DP.

C. Assessment

Pros:

- More robust to the signal decorrelation than standard cross correlation techniques.

- Able to generate low-noise elastogram using almost any two frames in free-hand palpation, given that they both belong to the same compression or relaxation cycle of the palpation excitation.
- No post-processing step such as median filtering needed.
- 1 second processing time per frame, potential real-time elastography with further optimization.

Cons:

- Need to tune the smoothness parameters to control the degree of smoothness in generated elastogram.
- Lateral search is only to decrease the noise and increase robustness of the axial strain, not suitable for calculating lateral strain.
- Intrinsic difficulty of static elastography – unknown stress distribution inside tissue, which leads to only qualitative results.

D. References

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[2] Gennisson, J. L., Deffieux, T., Fink, M., & Tanter, M. (2013). Ultrasound elastography: Principles and techniques. *Diagnostic and Interventional Imaging*, *94*(5), 487–495. https://doi.org/10.1016/j.diii.2013.01.022

[3] Rivaz, H., Boctor, E., Foroughi, P., Zellars, R., Fichtinger, G., & Hager, G. (2008). Ultrasound elastography: A dynamic programming approach. *IEEE Transactions on Medical Imaging*, *27*(10), 1373–1377. https://doi.org/10.1109/TMI.2008.917243