Homework Assignment 1

600.455/655 Fall 2022

Instructions and Score Sheet (hand in with answers)

Name	Name
Email	Email
Other contact information (optional)	Other contact information (optional)
Signature (required) I/We have followed the rules in completing this assignment ____________________________	Signature (required) I/We have followed the rules in completing this assignment ____________________________

Remember that this is a graded homework assignment. It is the functional equivalent of a take-home exam.

1. You are to work **alone** or **in teams of two** and are **not to discuss the problems with anyone** other than the TAs or the instructor.
2. It is otherwise open book, notes, and web. But you should cite any references you consult.
3. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
4. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
5. Sign and hand in the score sheet as the first sheet of your assignment.

NOTE: This assignment has a total of 110 points. However, at most 100 points will count toward your course grade.

Scenario

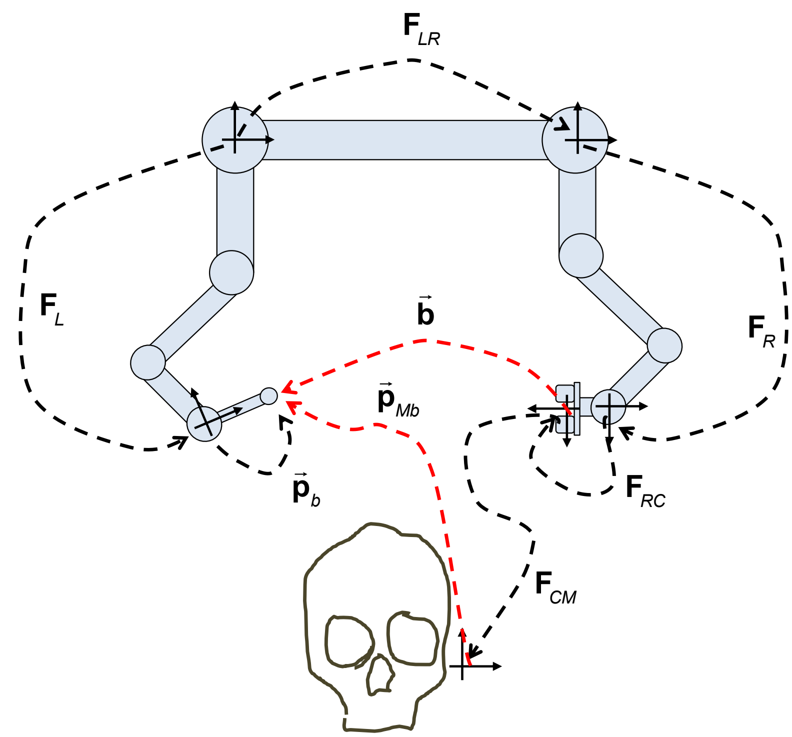

Figure 1: Two-armed robot with camera system and ball probe tool

Consider the two-armed robot shown in Figure 1. The left arm holds a cutting tool with a ball cutter at the end. The right arm holds a stereo video camera. The displacement $\mathbf{F}_{LR}$ of the base of the right arm is fixed and known only approximately, so that the true value is $\mathbf{F}_{LR}^{*} = \mathbf{F}_{LR}\Delta\mathbf{F}_{LR}$. The two robot arms move in space, but they may have some error, so that at time t, the true value of the position of the arms relative to its base are $\mathbf{F}_{L}^{*}[t] = \Delta\mathbf{F}_{L}[t]\mathbf{F}_{L}[t]$ and $\mathbf{F}_{R}^{*}[t] = \Delta\mathbf{F}_{R}[t]\mathbf{F}_{R}[t]$, respectively. ere the context is obvious, we may omit the [*t*]. The displacement $\vec{\mathbf{p}}_{Lb}$ of the ball cutter relative to the tooling attachment plate of the left arm is fixed. Like other quantities, it may be subject to some errors, so that the true position is $\vec{\mathbf{p}}_{LB}^{*} = \vec{\mathbf{p}}_{Lb} + \Delta\vec{\mathbf{p}}_{LB} = \vec{\mathbf{p}}_{Lb} + \vec{\varepsilon}_{LB}$ Similarly, the pose $\mathbf{F}_{RC}$ of the camera system relative to the tooling attachment plate of the right arm is fixed, with the true position given by $\mathbf{F}_{RC}^{*} = \mathbf{F}_{RC}\Delta\mathbf{F}_{RC}$. For each of these $\mathbf{F}_{xy}$ and $\Delta\mathbf{F}_{xy}$ quantities, we will adopt the usual convention that $\mathbf{F}_{xy} = [\mathbf{R}_{xy}, \vec{\mathbf{p}}_{xy}]$ and $\Delta\mathbf{F}_{xy} = [\Delta\mathbf{R}_{xy}, \Delta\vec{\mathbf{p}}_{xy}]$.

We can assume that the various errors are sufficiently small so that we can use linearized approximations. I.e.,

$$
\begin{aligned}
\Delta\mathbf{F}_L[t] &\approx [\mathbf{I}+sk(\vec{\alpha}_L[t]),\vec{\varepsilon}_L[t]] \\
\Delta\mathbf{F}_R[t] &\approx [\mathbf{I}+sk(\vec{\alpha}_R[t]),\vec{\varepsilon}_R[t]] \\
\Delta\mathbf{F}_{LR} &\approx [\mathbf{I}+sk(\vec{\alpha}_{LR}),\vec{\varepsilon}_{LR}] \\
\Delta\mathbf{F}_{RC} &\approx [\mathbf{I}+sk(\vec{\alpha}_{RC}),\vec{\varepsilon}_{RC}]
\end{aligned}
$$

where the $\vec{\alpha}_x$ and $\vec{\varepsilon}_x$ are small values. Many of the questions below ask you to produce or use linearized error estimates for various quantities. You will be expected to show your work, and the final answers should be expressed in a normalized matrix-vector format:

$$\vec{\eta}_{xxx} = \sum_k \mathbf{M}_k \vec{\eta}_k$$

where the $\mathbf{M}_k$ are matrices which will typically depend on various $\mathbf{R}$ and $\vec{\mathbf{p}}$ variables and the $\vec{\eta}_k{}^T = [\vec{\alpha}_k^T, \vec{\varepsilon}_k^T]$. When only an $\vec{\alpha}$ or $\vec{\varepsilon}$ is involved, you can also have terms that look like $\mathbf{M}_k\vec{\alpha}_k$ or $\mathbf{M}_k\vec{\varepsilon}_k$ and (of course) you can leave off the $\mathbf{M}_k$ if it is an identity matrix. However, we do <u>not</u> want to see final answers with terms involving things like $sk(\vec{\alpha}_k)$, although you may have these in intermediate steps showing how you got to the answers. Also, it is fine to have things like $sk(\vec{\mathbf{p}})$, where $\vec{\mathbf{p}}$ is a vector expression not involving any of the small linearized error variables.

Questions

Question 1: (5 Points) Give an expression for computing the position and orientation $\mathbf{F}_{CL}$ of the left-hand-side tool holder relative to the camera. Here, I am looking for an expression using the $\mathbf{F}_L$, $\mathbf{F}_{LR}$, and the other "**F**" variables in the scenario. Here you may assume that all the $\Delta\mathbf{F}$'s are the identity and that any errors are negligible. Do <u>not</u> use $\mathbf{F}_{CM}$, even though it is shown in Figure 1.

Question 2: (5 Points) Expand your answer to provide expressions for $\mathbf{R}_{CL}$ and $\vec{\mathbf{p}}_{CL}$, where $\mathbf{F}_{CL} = [\mathbf{R}_{CL}, \vec{\mathbf{p}}_{CL}]$.

Question 3: (5 Points) Give an expression for computing the position $\vec{\mathbf{b}}$ of the cutter ball relative to the camera system in Figure 1 in terms of $\mathbf{R}_{CL}$, $\vec{\mathbf{p}}_{CL}$, and other quantities described in the scenario, assuming that all errors are negligible.

Question 4: (10 Points) Suppose now that the error term $\Delta\mathbf{F}_{LR} = [\Delta\mathbf{R}_{LR}, \Delta\vec{\mathbf{p}}_{LR}]$ is not negligible. This will introduce some error into $\mathbf{F}_{CL}$, so that the actual value is $\mathbf{F}_{CL}^{*} = \Delta\mathbf{F}_{CL}\mathbf{F}_{CL}$. Give expressions for computing $\Delta\mathbf{R}_{CL}$ and $\Delta\vec{\mathbf{p}}_{CL}$. **NOTE:** In principle, the errors $\Delta\mathbf{F}_L$ and $\Delta\mathbf{F}_R$ may also be non-negligible, but I am saving you some work by not asking for the full error chain.

Question 5: (10 Points) Now give expressions for $\vec{\alpha}_{CL}$ and $\vec{\varepsilon}_{CL}$, where $\Delta\mathbf{F}_{CL} \approx [\mathbf{I} + sk(\vec{\alpha}_{CL}), \vec{\varepsilon}_{CL}]$. Express these in normalized linear format.

Question 6: (10 Points) Suppose, now that $\Delta\mathbf{F}_{RC}$ is not negligable. Produce updated linearized expressions for $\vec{\alpha}_{CL}$ and $\vec{\varepsilon}_{CL}$.

Question 7: (10 Points) Assume now that the values for $\Delta\vec{\mathbf{p}}_{Lb} = \vec{\varepsilon}_{LB}$ is not negligible. The actual position $\vec{\mathbf{b}}^{*}$ of the ball will differ from the computed position by some error, so that $\vec{\mathbf{b}}^{*} = \vec{\mathbf{b}} + \Delta\vec{\mathbf{b}}$. Give an expression for $\Delta\vec{\mathbf{b}}$. Please express your answer in terms of $\mathbf{R}_{CL}$, $\mathbf{p}_{CL}$, $\Delta\mathbf{R}_{CL}$, $\Delta\vec{\mathbf{p}}_{CL}$ and the other quantities in the problem.

Question 8: (10 Points) Give a linearized expression for $\Delta\vec{\mathbf{b}} = \vec{\varepsilon}_b$ under the assumptions above, in terms of the other variables in the scenario. Here, you should use $\vec{\alpha}_{CL}$ and $\vec{\varepsilon}_{CL}$ in your answer.

Question 9: (10 Points) Assume now that all the errors in the scenario are negligible. Suppose that the computer has available to it a CT scan of a patient's skull and that the computer has available software that can locate anatomic landmarks in the CT image that are also visible to the surgeon. Assume, also that the surgeon is able to use hand-over-hand guidance of the left arm (similar to what is done in Robodoc) to move the left arm in space. Describe a procedure for determining a registration transformation $\mathbf{F}_{CM}$ such that the position of the ball relative to the camera frame is given by $\vec{\mathbf{p}}_{Mb} = \mathbf{F}_{CM}^{-1}\vec{\mathbf{b}}$. This will necessarily involve moving at least the left arm to multiple positions $\mathbf{F}_L[t]$. Please provide a step-by-step workflow, specify what data will be collected or computed, and how the calculation will be performed. Here, you should use only mathematical methods covered in the lectures so far. In this problem, you can ignore the diameter of the cutter ball (i.e., assume that it has a very small diameter). **NOTE:** Your method may not be as accurate as methods covered in later registration lectures. But that is fine.

Question 10: (10 Points) Assume now that there is some error in $\mathbf{F}_{CM}$, so that the actual value is $\mathbf{F}^*_{CM} = \mathbf{F}_{CM}\Delta\mathbf{F}_{CM}$. Assume that there is some error in the arms, so that $\vec{\mathbf{b}}^* = \vec{\mathbf{b}} + \Delta\vec{\mathbf{b}} = \vec{\mathbf{b}} + \vec{\varepsilon}_b$. Assume that the other quantities in the scenario have negligible error. Produce a formula for computing the error $\Delta\vec{\mathbf{p}}_{Mb}$ in $\vec{\mathbf{p}}_{Mb}$ such that the actual position is $\vec{\mathbf{p}}^*_{Mb} = \vec{\mathbf{p}}_{Mb} + \Delta\vec{\mathbf{p}}_{Mb}$ in terms of $\vec{\mathbf{b}}$, $\Delta\vec{\mathbf{b}}$, $\mathbf{F}_{CM}$, and $\Delta\mathbf{F}_{CM}$.

Question 11: (10 Points) Give your answer to Question 10 in terms of the "**R**", "$\vec{\mathbf{p}}$", "$\Delta\mathbf{R}$"," $\Delta\vec{\mathbf{p}}$" variables.

Question 12: (10 Points) Now, produce a linearized error estimate for $\Delta\vec{\mathbf{p}}_{Mb} = \vec{\varepsilon}_{Mb}$, expressed in the normalized form described above, where $\Delta\mathbf{F}_{CM} \approx \left[\mathbf{I} + sk(\vec{\alpha}_{CM}), \vec{\varepsilon}_{CM}\right]$.

Question 13: (5 Points) Suppose that you have another expression for $\vec{\varepsilon}_b = \sum \mathbf{M}_k \vec{\eta}_k$. How would you use this to produce another normalized expression for $\Delta\vec{\mathbf{p}}_{Mb} = \vec{\varepsilon}_{Mb}$ in terms of the $\vec{\eta}_k$, $\vec{\alpha}_{CM}$, $\vec{\varepsilon}_{CM}$ and the other variables in the scenario.