# Surface Simplification (Gueziec's Method)

601.455/655



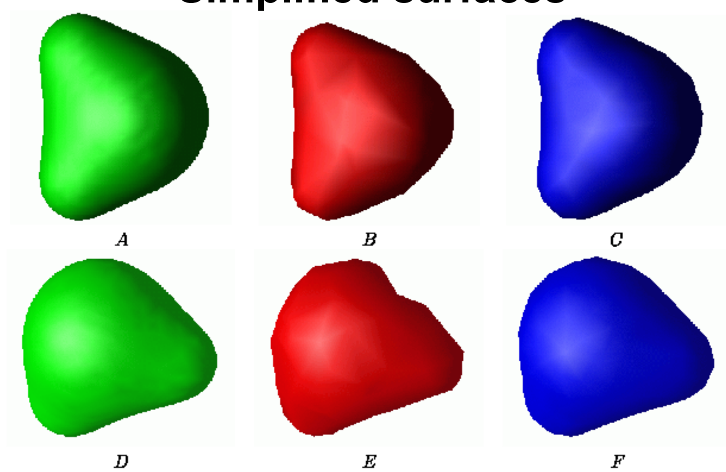




## Gueziec's Method

- Reference
  - A. Gueziec, "Surface Simplification inside a tolerance volume", *IBM Research Report RC20440,* 5/20/97
- Essentially "triangle decimation" done correctly
  - Preserves topology
  - Preserves volume
  - Provable error bound

## Simplified surfaces




Original
(1747 vertices)

Shroeder
(145 vertices)

Gueziec
(145 vertices)




3

## Approximated bunny

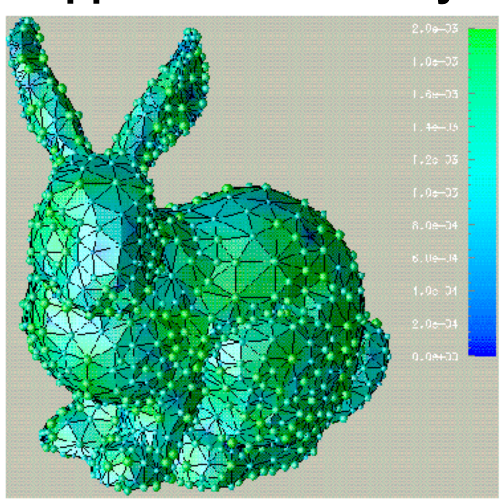
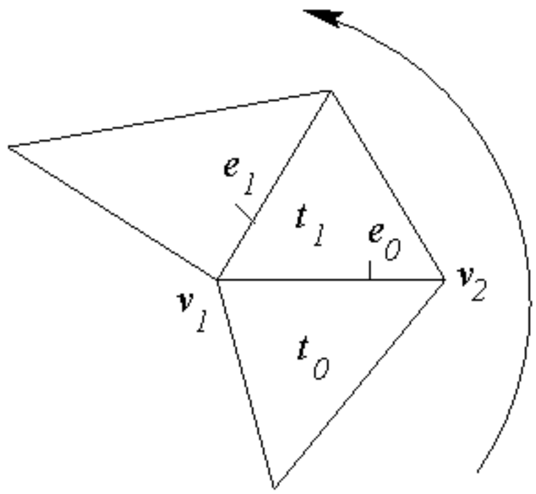


Figure 3: Error volume partially represented using color and spheres centered at surface vertices. The radius of each sphere equals the error value at the vertex, here bounded by .8 % of the bounding box diameter.




4

# Building a vertex star








5

# Notation: “star” v* of a vertex v

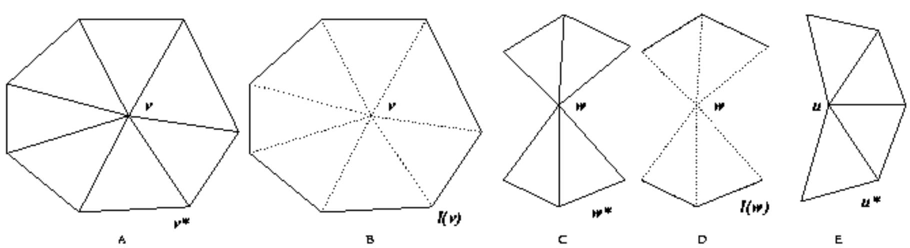


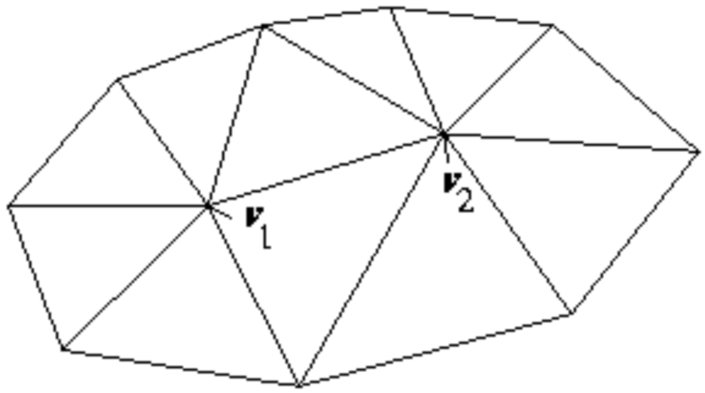

Figure 4: A: the star $\mathbf{v}^*$ of a regular vertex $\mathbf{v}$ of valence seven. B: the link $\ell(\mathbf{v})$ of the regular vertex $\mathbf{v}$, composed of one simple closed polygonal curve. C: the star $\mathbf{w}^*$ of a singular vertex $\mathbf{w}$ of valence four. D: the link $\ell(\mathbf{w})$ of $\mathbf{w}$, composed of two disconnected polygonal curves. E: the star $\mathbf{u}^*$ of a boundary vertex of valence five.





6

# Notation: "star" $(v_1,v_2)^*$ of edge $(v_1,v_2)$

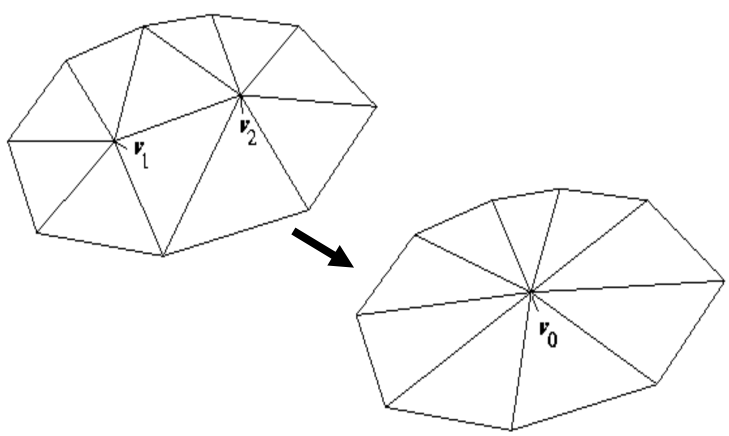







7

# Fundamental operation: Edge Collapse

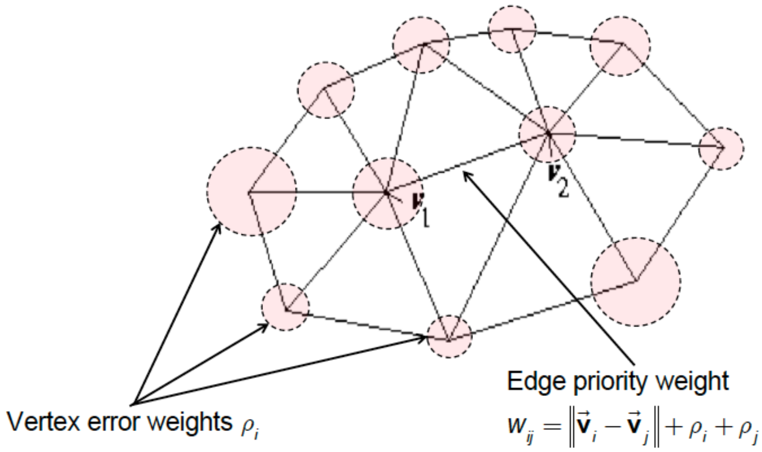







8

# Priority Weighting





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9

# Algorithm outline

Until Queue/ Last Bucket Empty:

- Take edge with low(est) weight
- If edge can be safely collapsed
  1. If valence does not exceed maximum
  2. If simplified vertex is regular
  3. If triangle normal rotation is acceptable
  4. If triangle compactness is acceptable
  5. If error does not exceed tolerance

  □ Change neighboring configuration

  □ Remove all edges of the star from the queue

  □ Reinstate new edges in the queue
- Else
  remove edge from queue

Figure 6: Simplification algorithm.


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10

# Book-keeping to remember hierarchy

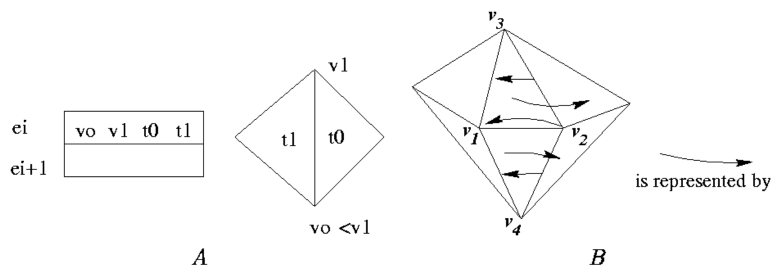
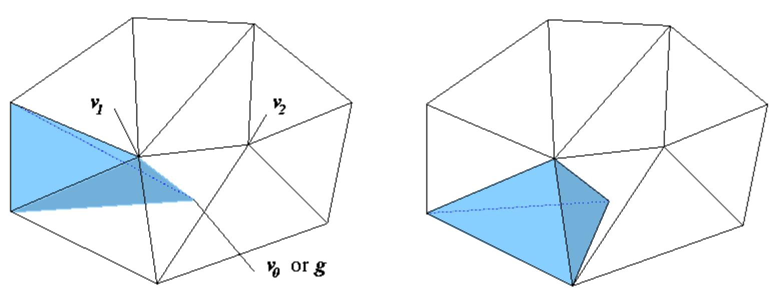



Figure 7: A: an edge refers to four indices. B: defining parents of surface elements during an edge collapse.




11

# Picking new vertex to preserve volume




Volume associated with edge star is sum of tetrahedra

$v_0$ = vertex associated with simplified vertex star

$g_0$ = centroid of edge star




12

## Picking new vertex to preserve volume

Given edge star $(v_1, v_2)^* = \{v_3, \ldots, v_n\}$. Let $T_{12}$ be the set of all triangles $t$ in $v_1^* \cup v_2^*$ and let $vertices(t) = \{v_{t1}, v_{t2}, v_{t3}\}$ be the set of verticies associated with a triangle t. Compute the centroid $g = \sum_{i=3}^{n} \frac{v_i}{n-2}$ of $(v_1, v_2)^*$. Then the volume associated with the $(v_1, v_2)^*$ is

$$V_{1,2} = \sum_{t \in T_{12}} V_{tetra}(g, v_{t1}, v_{t2}, v_{t3})$$

We want to pick $v_0$ such that

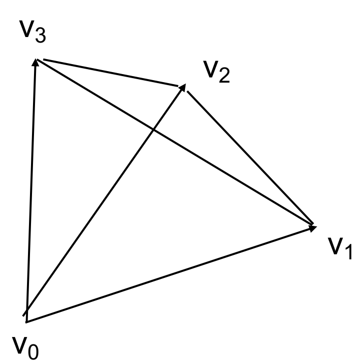
$$\sum_{i=3}^{n-1} V_{tetra}(g, v_0, v_i, v_{i+1}) = \sum_{t \in T_{12}} V_{tetra}(g, v_{t1}, v_{t2}, v_{t3})$$





13

## Volume of a tetrahedron



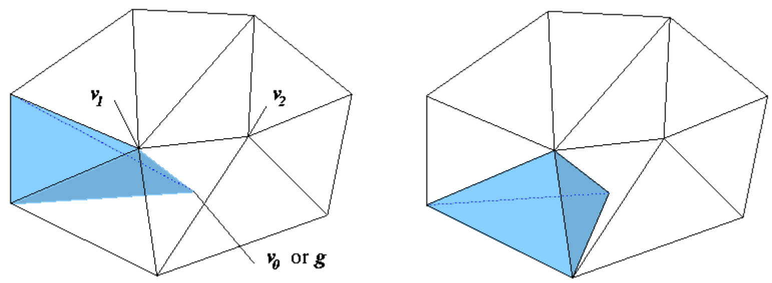

$$V_{tetra}(v_0, v_1, v_2, v_3) = \frac{1}{6}(v_1 - v_0) \bullet (v_2 - v_0) \times (v_3 - v_0)$$





14

## Picking new vertex to preserve volume



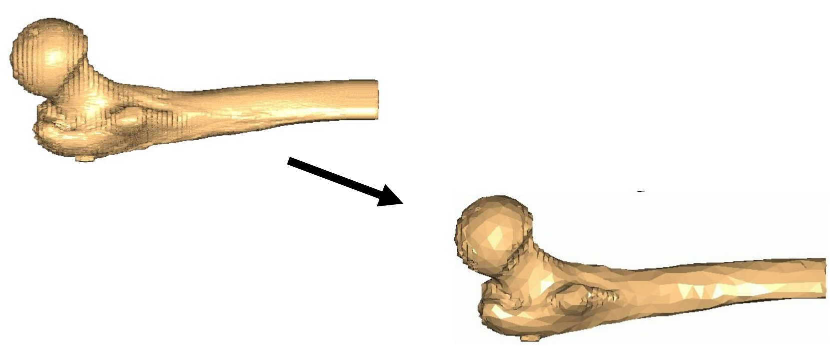

- Volume preservation constraint defines a plane on which $v_0$ must lie.
- Select the point on this plane that minimizes sum-of-squared distance to planes of all triangles being collapsed




15

## Volume preservation results

| model | Femur | Buddha | Deino |
|---|---|---|---|
| original # of triangles | 180,854 | 333,586 | 44,954 |
| simplified # of triangles | 3,124 | 49,106 | 19,490 |
| original volume | 233,462.7455 $mm^3$ | 23,048,568.98 $pixel^3$ | 230,276.599 $mm^3$ |
| vol. after simplification | 233,462.7452 $mm^3$ | 23,048,569.03 $pixel^3$ | 230,276.600 $mm^3$ |






16

# Triangle compactness improvement

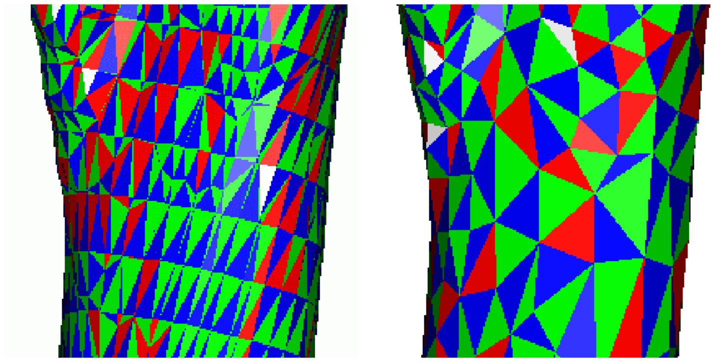


Figure 11: A visual inspection of these triangles extracted from the shaft of the Femur model shows that facets are more regular in the simplified femur model produced by the our algorithm (Right) than they are in the original output of an iso-surface algorithm (Left). In particular, most "sliver" (very narrow) triangles have been removed. Histograms of triangle compactness presented in Fig.10 confirm this observation.






# Triangle compactness improvement

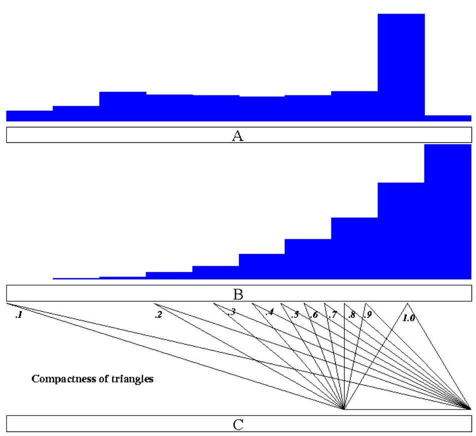
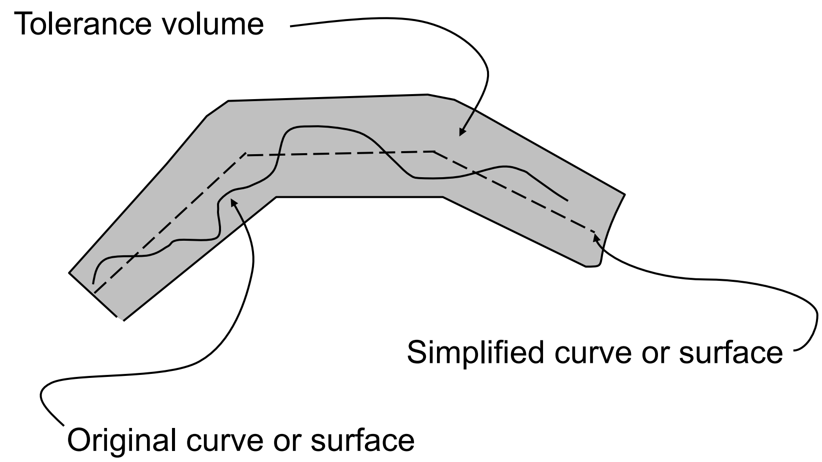



Figure 10: A: histogram of compactness values before simplification for the Femur example. B: histogram of compactness values after simplification. C: example of triangles of compactness values of .1 to 1 in .1 increments, corresponding to boundaries between the histogram bins. A flat triangle has a compactness of zero.

## Basic idea of tolerance volumes



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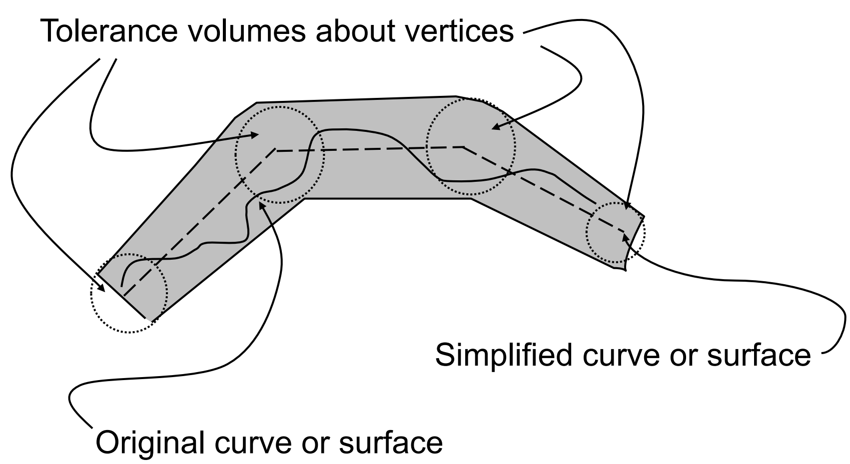
19

## Generate volumes from vertex tolerances



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20

# Error "tube" around an edge

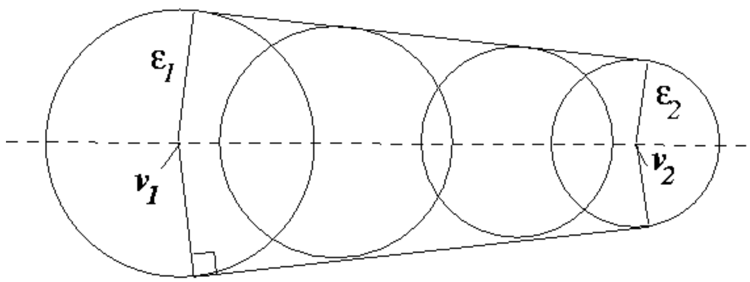



Figure 14: An edge tube







# Triangle tube or "fat" triangle

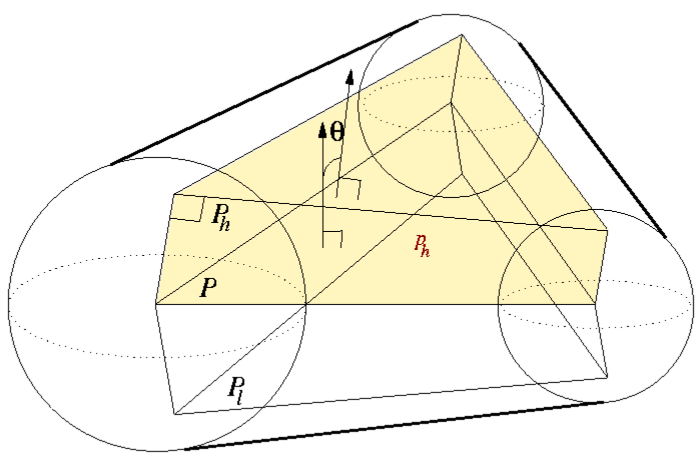



Figure 15: A triangle tube.

# Local enforcement of tight tolerance

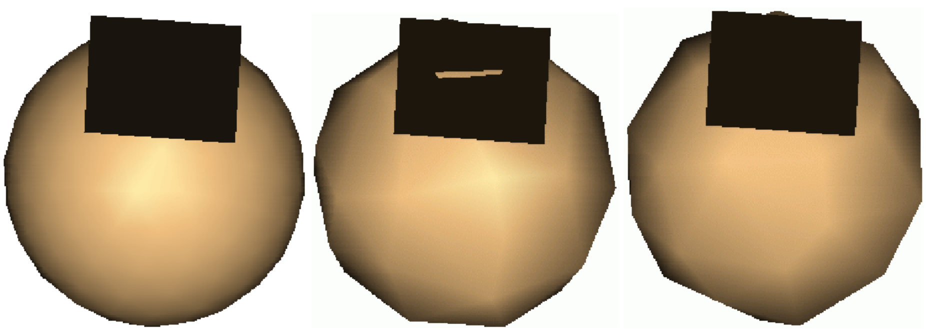


Figure 13: Using the tolerance volume to maintain tangency of two surfaces after simplification. From Left to Right, original sphere model (320 triangles) represented with tangent plane, simplified sphere model (82 triangles) with a tolerance equal to 10 % of the sphere diameter; the tangency condition is violated. Simplified sphere model (88 triangles) with the same global tolerance, but with a tolerance of 0 at the contact point.







# Error Volume & tolerance volume

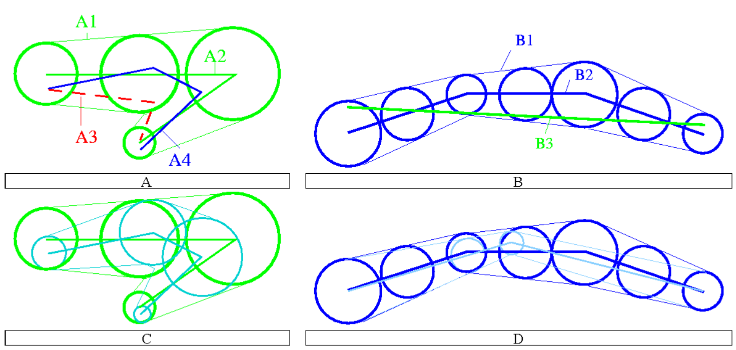
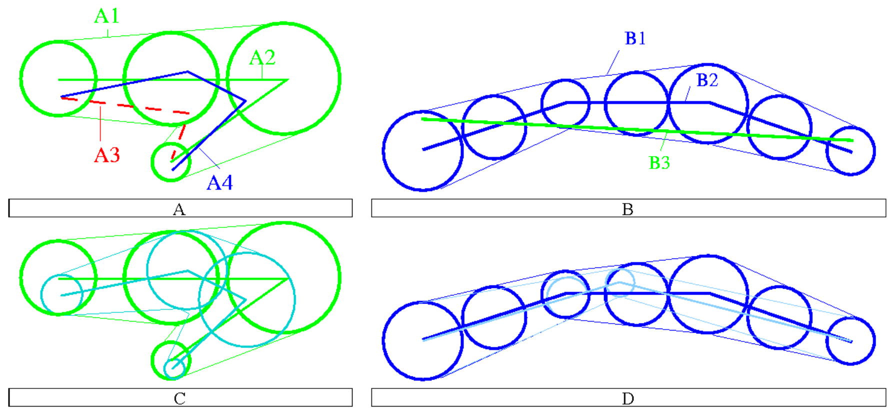



Figure 12: A: Error volume, in green (A1), centered on the simplified surface (A2). The original surface (here, curve) is not only contained in the error volume, in dashed red (A3), but also intersects all the spheres, in blue (A4). B: Tolerance volume, in blue (B1), centered on the original surface (B2). The simplified surface (B3) is contained inside the tolerance volume and intersects all the spheres. C: The final error volume contains all the intermediate error volumes. D: The initial tolerance volume contains all the intermediate tolerance volumes. A simplification operation is rejected if the width of the resulting tolerance volume is negative.

# Error Volume & tolerance volume



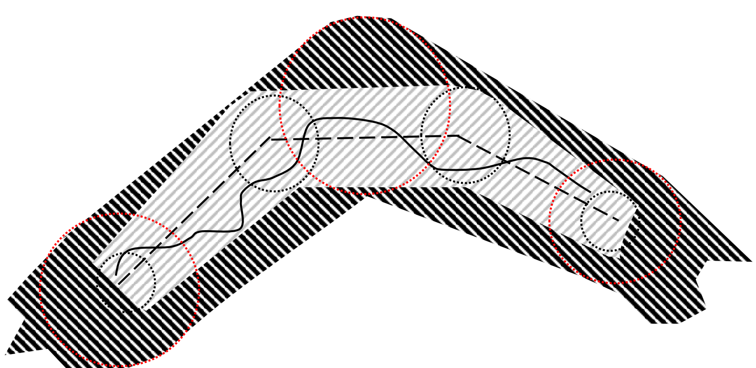





25

# Merging Rule

When merge, assign (enlarge) vertex tolerances so that old surface shell is guaranteed to be completely inside the new surface shell







26

# Simplest method to assign vertex tolerances

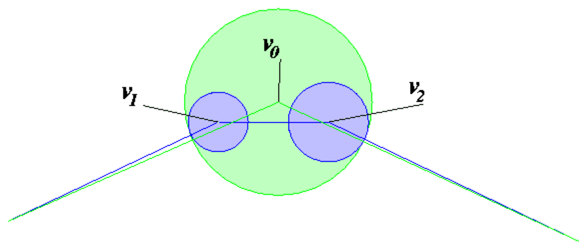



Figure 16: Simplest method (Contraction Method) for computing error values.




27

# Subdivision Method

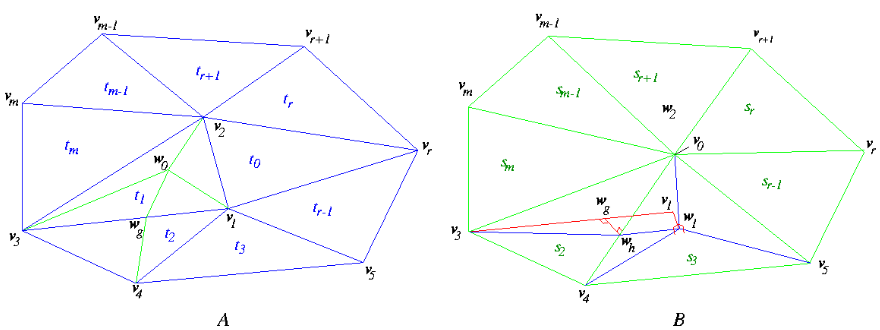



Figure 17: Notations for the Subdivision Method. A: graph $G$. B: graph $H$




28

## Projection Method

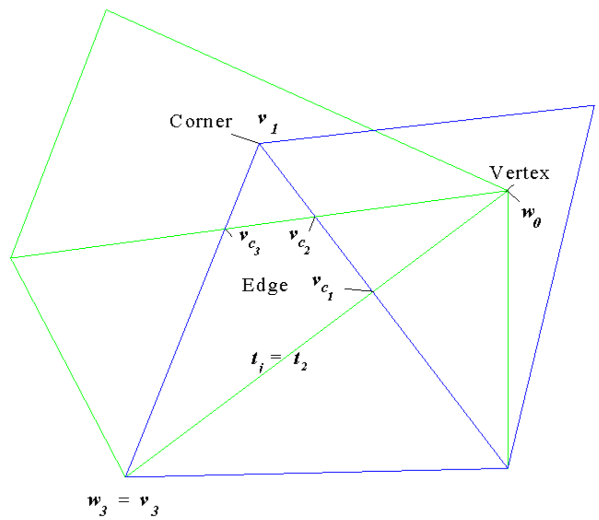



Figure 35: Types of Constraints generated with the Projection Method.







## Boundary Collapse

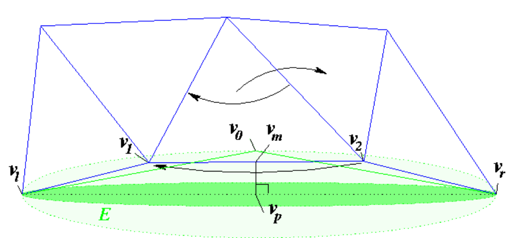



Figure 21: Collapse of a Type I boundary edge. Arrows indicate vertex, triangle and edge representatives after collapse.

# Buddah

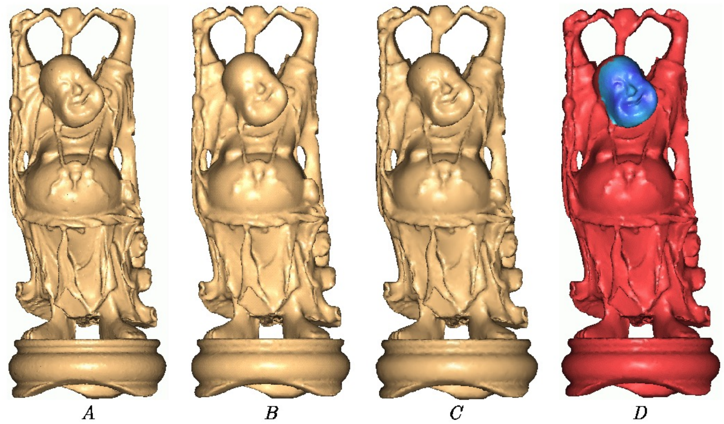



Figure 24: A: Buddha model of 334 K triangles. B: simplification with 46 K triangles using a uniform tolerance. C: simplification with 49 K triangles using a variable tolerance. D: coloring of the tolerance volume for the surface of C, with increasing values from blue to red in a Rainbow colormap.



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31

# Lamp

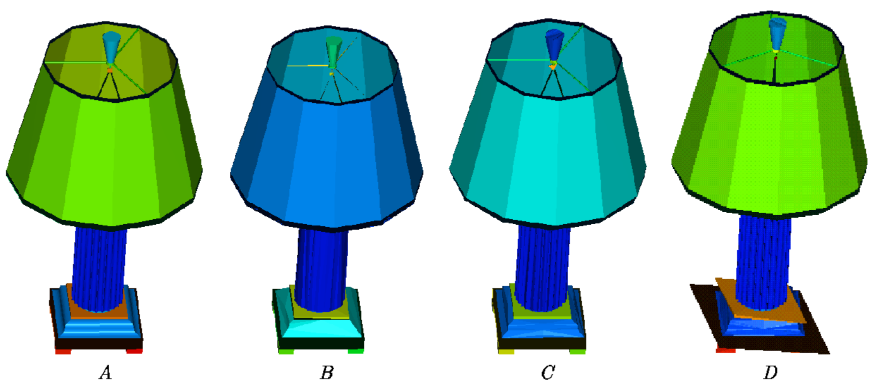


Figure 25: A: Lamp model with 5,052 triangles, flat shaded; Rainbow colors are assigned to surface components by order of vertex count. B: Simplified model (686 triangles) with maximum error of 1% of the bounding box diameter. B: Simplified model (2,035 triangles) with volume preservation. D: Result of a method that does not bound the distance from simplified to original.



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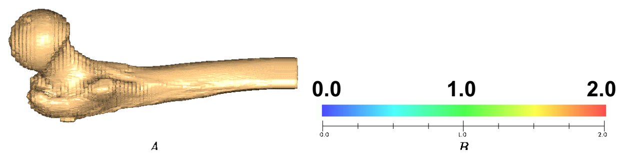
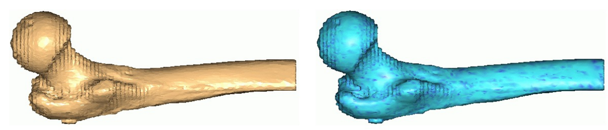
32

# Femur Simplification




Original (181 K triangles)

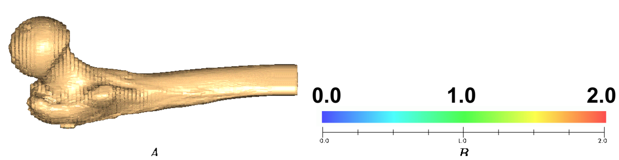


0.5 mm tolerance (26.8 K triangles)

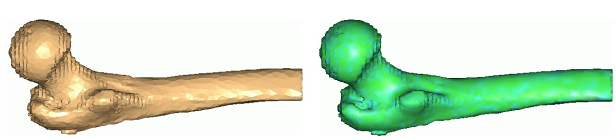





# Femur Simplification

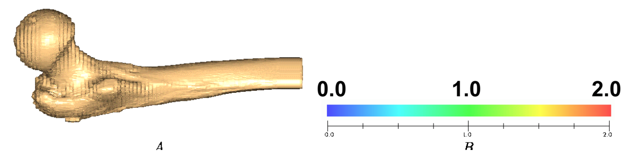



Original (181 K triangles)

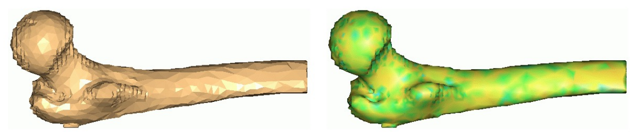


1.0 mm tolerance (9,592 triangles)

# Femur Simplification




Original (181 K triangles)

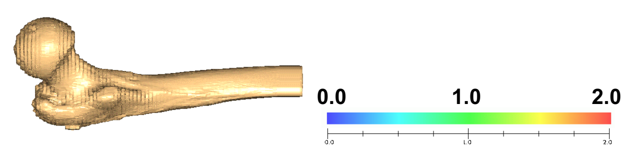


1.6 mm tolerance (4,618 triangles)

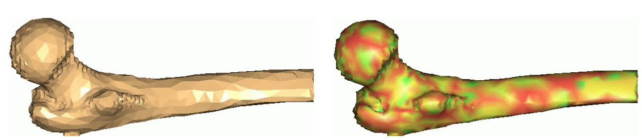



35

# Femur Simplification




Original (181 K triangles)



2.0 mm tolerance (3,124 triangles)




36

## Carotid artery

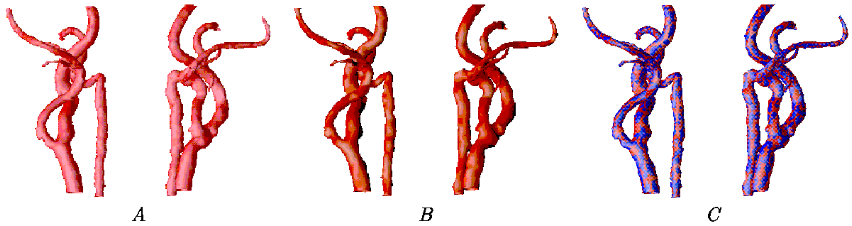


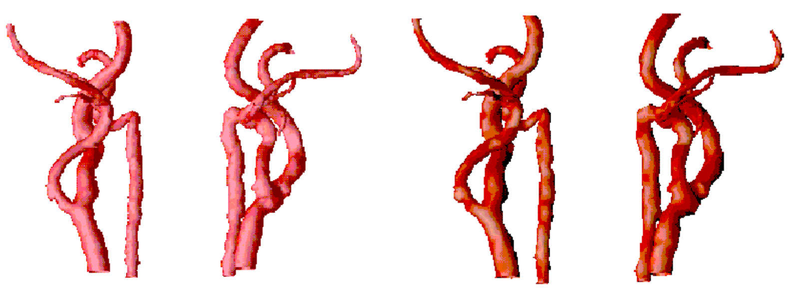
Figure 27: A: Carotid Arteries (57 K triangles). B: Simplification (5.6 K triangles) with a maximum error of 0.8%. C: Superimposition of A and B.





37

## Carotid artery



Original (57 K triangles)

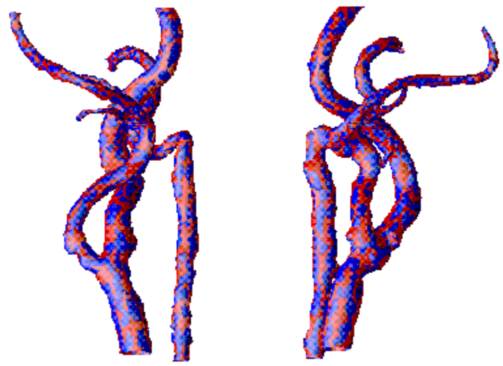
Simplified to 0.8% tolerance (5.6 K triangles)





38

## Carotid artery



Original and simplified superimposed







## Engine noise data

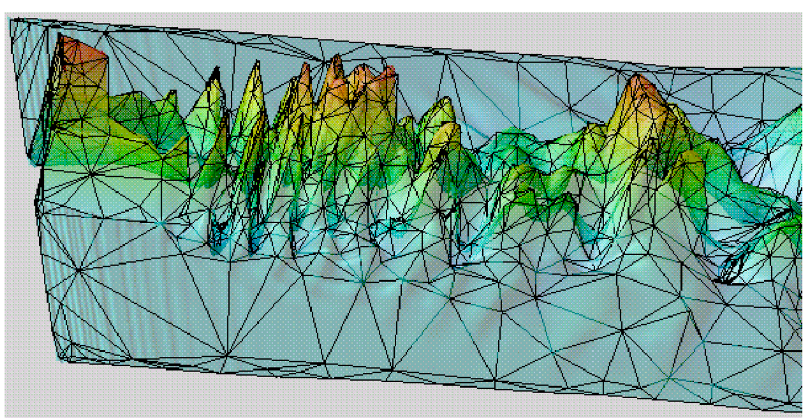


Figure 28: Engine Noise level Data: simplified from 660 K triangles down to 6.7 K, with an error tolerance of 4% of the surface largest dimension.

# Terrain/humidity map (original)

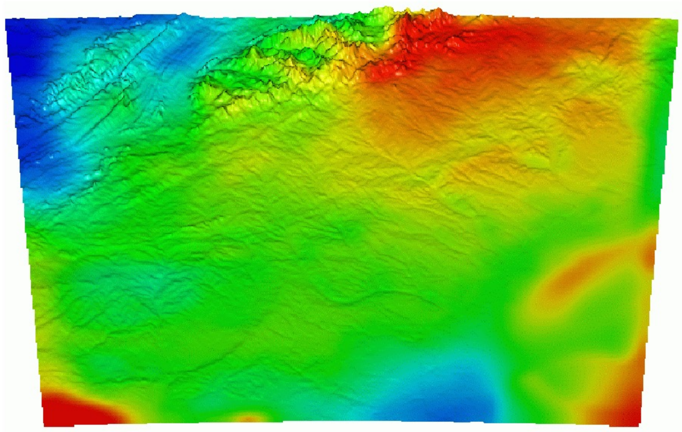


Figure 29: Terrain model of the region of Atlanta with humidity values color coded using a Rainbow colormap (309 K triangles).





41

# Terrain/humidity map (Simplified)

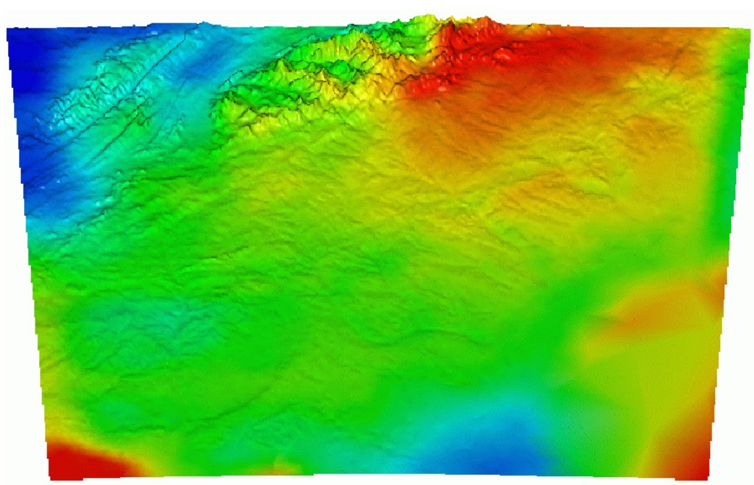


Figure 30: Simplified terrain without color preservation (89 K triangles).





42

# Terrain/humidity map (Simplified) preserving humidity attribute

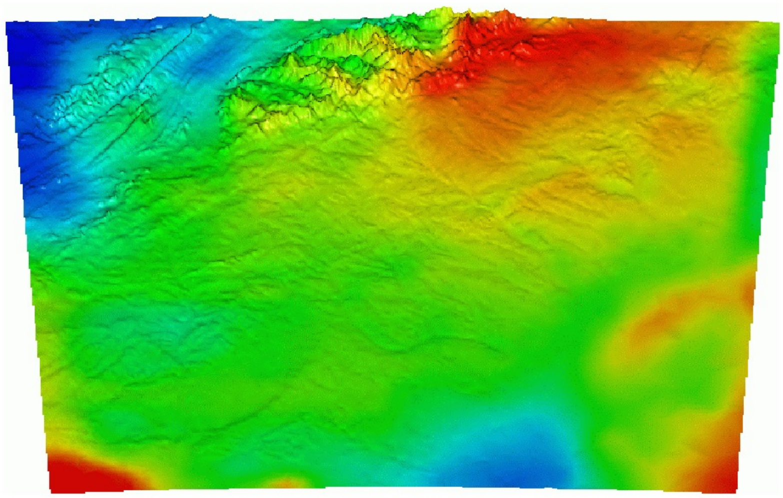


Figure 31: Simplified terrain with color preservation (92 K triangles).





43

# Bunny Simplification

Original

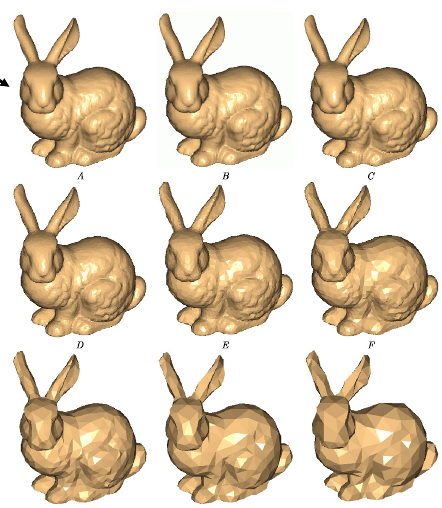



Figure 32: Successive simplifications of the Bunny model, flat shaded. A: original. B: tolerance of 1/32% of bounding box diameter. C: tolerance of 1/16%. D: 1/8%. E: 1/4%. F: 0.5%. G: 1%. H: 1.5%. I: 2%.

52





44

# Phone

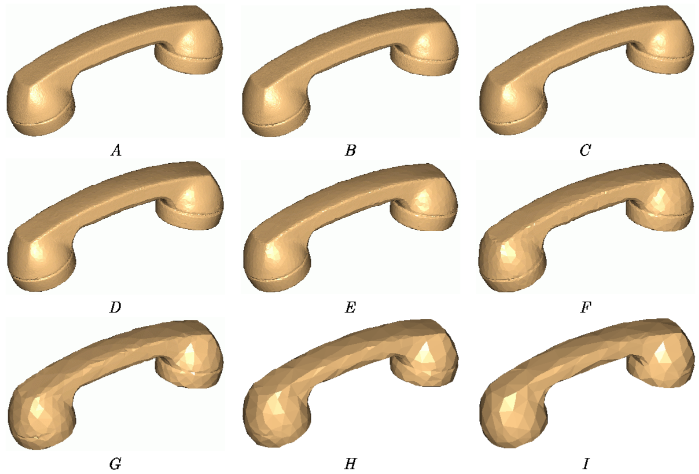



Figure 33: Successive simplifications of the Phone model, flat shaded. A: original. B: tolerance of 1/64% (96 K triangles). C tolerance of 1/32% (53 K triangles). D: 1/16% (24 K triangles). E: 1/8% (10.4 K triangles). F: 1/4% (4,618 triangles). G: 0.5% (1,943 triangles). H: 0.8% (986 triangles). I: 1% (743 triangles).





45

# Phone

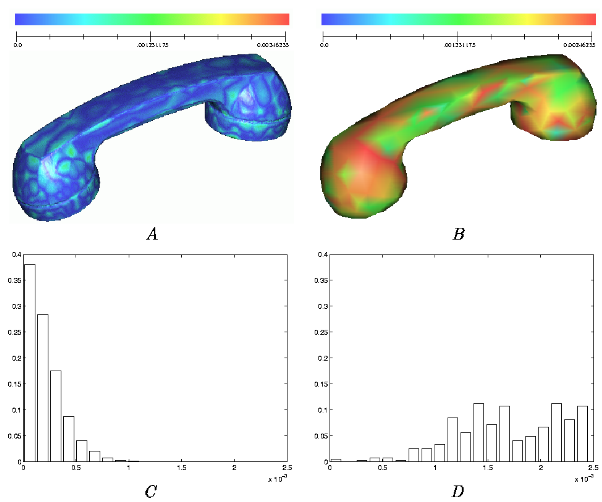



Figure 34: A: visualization of distances from each point of the original Phone to the 1% Model, with the colormap used for A and B. B: visualization of the error volume of the 1% Model. C: histogram of distances measured from points of original Phone to 1% Model. B: histogram of error values on the 1% Model.





46