

Algorithm for Estimating the Orientation of an Object in 3D Space, Through the Optimal Fusion of Gyroscope and Accelerometer Information.

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Abstract— Two suitable coordinate systems overlapping at the origin were established to estimate orientation by computing the spatial relationship between them. An Inertial Measurement Unit sensor (IMU), consisting of a tri-axial gyroscope and tri-axial accelerometer was used to define reference systems. This work describes an algorithm to estimate the orientation of an object in 3D space through the optimal fusion of gyroscope and accelerometer information. A tri-axial gyroscope was used as a main source of information for assessing orientation during movement. On the other hand, a tri-axial accelerometer was used to compensate drifting deviation on gyroscope measurements. Orientation estimation was performed by using a Direct Cosine Matrix (DCM) as a combination of three consecutive rotations through one of each main axis to the coordinate systems. Rotation matrices also expressed as approximations rather than identities were used to improving orientation computed through DCM. Three different algorithms were proposed to estimate orientation. For assessing differences between them, two different studies were realized: 1.- analysis of behavior in both static position of the sensor and during moving, 2.- estimation of the orientation of the sensor by representing the orientation of an object through a virtual model. The developed algorithm opens opportunities to be used in the evaluation of human body joints.

Keywords—Accelerometer, Director Cosine Matrix, Degrees Of Freedom, Gyroscope, IMU

I. INTRODUCTION

The orientation of an object in 3D space is defined by 3 degrees of freedom (DOF) or 3 components linearly independent. Hence, it is possible to describe both position and orientation of an object through assigning a new coordinate system and studying the existent spatial relationship between them. This relationship is given by both position and orientation of the system associated to the object respect to a reference system. However, assuming both systems overlapping at the origin, it is said that there is no change of position between them [1].

The orientation of an object can be described by three consecutive rotations, whose order is important due rotations into a 3D space are not commutative [2]. There are several ways to describe translational and rotational relationship by establishing systematically a coordinate system like *Euler angles, yaw pitch, roll and rotation matrix* [3]

An Inertial Measurement Unit (IMU) consists of gyroscopes and accelerometers enabling the tracking of rotational and translational movements. To measure in three dimensions, tri-axial sensors consisting of 3 mutually orthogonal sensitive axes are required. An IMU, by itself, can measure an attitude relative

to the direction of gravity which is sufficient for many applications [4] [5] [6]. On the other hand, a gyroscope measures angular velocity which, if initial conditions are known, information may be integrated over the time to compute the sensor's orientation [7] [8]. However, the integration of gyroscope measurement errors leads to an accumulating error in the calculated orientation. Therefore, gyroscopes by themselves are not able to provide an absolute measurement of orientation. An accelerometer measures earth's gravitational and provides an absolute reference orientation. Nonetheless, they are likely to be subject to high levels of noise where accelerations due to motion affect measured direction of gravity. Hence, to compute a single estimate of orientation through the optimal fusion of gyroscope and accelerometer measurements is needed [9] [10].

A direction cosine matrix (DCM) transforms the coordinates of a vector from one frame to another [11] [12]. Since the coordinates of a vector are dependent on the frame that is represented in, then an arbitrary vector can be represented in the rotation frame. Although DCM elements are the direction cosines of the principal axes of one frame to another. To construct DCM only from direction cosines is not only unattractive, but also inconvenient. The main reason is that the orthogonality property is lost, and with it the length and orientation of the transformed vector. To preserve this property a rotation matrix can be used to construct DCM. That is to find the angle of rotation and the axis around which the moving frame has rotated. Two very closely related methods fall under this category; the combination of rotation matrices and Euler angles.

The aim of this project is to estimate the orientation of an object in 3D space though a DCM matrix, by using an IMU. Consequently, it was determined the methodology to fuse together the separate sensor data into a single optimal estimate orientation.

II. METHODOLOGY

A. Sensors

A 10-DOF (Degrees of Freedom) Magnetic, Angular Rate, and Gravity (MARG) module, GY-87, was used by its Inertial measurement unit sensor (IMU), *Invensense MPU6050*, consisting of a tri-axial gyroscope and a tri-axial accelerometer was used to estimate the orientation of an object in 3D space. Gyroscope angular rate (ω) was used as a main source of information establishing a coordinate reference system attached to an object (C_{Obj}) for assessing orientation during movement. On the other hand, accelerometer output (λ) was used to compensate drifting deviation by establishing a coordinate

reference system located at some point on earth but no attached to it, (C_{Ref}).

B. Data acquisition

A 16-bit Digital Signal Controller Microchip (*Dspic30F6014A*) was used to compute orientation. Communication between *Dspic* and *IMU* was established through Inter-Integrated Circuit (I^2C) serial interface protocol at standard operating frequency (100 Kbit/s). Full-scale range of gyroscope information ($\pm 250^\circ/\text{sec}$) and full-scale range of accelerometer information ($\pm 2g$ -1 kHz). The estimated orientation was visualized in MATLAB R2013a through a developed virtual model. Communication between *Dspic* and PC (*Matlab 2013a*) was performed by using a USB UART interface Integrated Circuit Device (*Ft232RL*), any synchronization problem was not found.

C. Orientation estimation

To transform vectors from one coordinate system to another a *Direct Cosine Matrix* (DCM) was used, in which elements are the direction cosines of the principal axes. This DCM was performed by combining three consecutive rotations through one of each main axis to the evaluated system by using rotation matrices, as in Ec. (1).

$$\begin{bmatrix} C(\theta)C(\phi) & -S(\theta)C(\alpha) + C(\theta)S(\phi)S(\alpha) & S(\theta)S(\alpha) + C(\theta)S(\phi)C(\alpha) \\ S(\theta)C(\phi) & C(\theta)C(\alpha) + S(\theta)S(\phi)S(\alpha) & -C(\theta)S(\alpha) + S(\theta)S(\phi)C(\alpha) \\ -S(\phi) & C(\phi)S(\alpha) & C(\phi)C(\alpha) \end{bmatrix} \quad (1)$$

Where C=cos and S=sin, while α, ϕ and θ represents roll, pitch and yaw rotation angles through x, y and z axes of the coordinate systems.

However, it is worth noting that if the rate of rotation movement of an object (σ) given by α, ϕ and θ are small enough, e.g. $\sin^2(\sigma) \approx 0$, $\cos(\sigma) \approx 1$. DCM elements can be expressed as approximations rather than identities, improving orientation computing as shown in Ec. (2), [13].

$$\begin{aligned} \cos(\sigma) &\approx 1 & \text{Assuming } \begin{cases} \sin(\alpha) \\ \sin(\phi) \\ \sin(\theta) \end{cases} \approx \begin{cases} \alpha \\ \phi \\ \theta \end{cases} \\ \sin(\sigma) * \sin(\sigma) &\approx 0 & \end{aligned} \quad (2)$$

$$DCM = \begin{bmatrix} 1 & -S(\theta) & S(\phi) \\ S(\theta) & 1 & -S(\alpha) \\ -S(\phi) & S(\alpha) & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\theta & \phi \\ \theta & 1 & -\alpha \\ -\phi & \alpha & 1 \end{bmatrix} \quad (2)$$

The numerical errors (ϵ) inherent to the approximations will gradually reduce orthogonality between the rotation axes to the coordinate systems; nonetheless these differences can be estimated through the scalar product between coordinate systems Ec. (3) and corrected by rotating error axis in the opposite direction Ec. (4).

$$DCM = \begin{bmatrix} 1 & -\theta & \phi \\ \theta & 1 & -\alpha \\ -\phi & \alpha & 1 \end{bmatrix} = \begin{bmatrix} r_{xx} & r_{xy} & r_{xz} \\ r_{yx} & r_{yy} & r_{yz} \\ r_{zx} & r_{zy} & r_{zz} \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$[\epsilon_x, \epsilon_y, \epsilon_z] = [r_{xx}, r_{xy}, r_{xz}] \begin{bmatrix} r_{yx} \\ r_{yy} \\ r_{yz} \end{bmatrix} = X * Y^T \quad (3)$$

$$[X', Y', Z'] = \left[X - \frac{[\epsilon_x, \epsilon_y, \epsilon_z]}{2} Y, Y - \frac{[\epsilon_x, \epsilon_y, \epsilon_z]}{2} X, (X') \times (Y') \right] \quad (4)$$

Where X', Y' and Z' are the orthogonal vectors of each axis, while X_{norm}, Y_{norm} and Z_{norm} are the normalized vectors used to conform and improved DCM Ec. (5).

$$DCM = \begin{bmatrix} X_{norm} \\ Y_{norm} \\ Z_{norm} \end{bmatrix} = \begin{bmatrix} x' & y' & z' \\ |x'| & |y'| & |z'| \end{bmatrix}^T = \begin{bmatrix} 1 & -\theta & \phi \\ \theta & 1 & -\alpha \\ -\phi & \alpha & 1 \end{bmatrix} \quad (5)$$

D. Estimating orientation from gyroscope information

Coordinate system C_{obj} is defined by the tri-axial gyroscope and each point along each gyroscope axis (G_x, G_y, G_z). Consequently, gyroscope output ($\omega_x, \omega_y, \omega_z$) of each axis was used to compute a single estimation of gyroscope orientation (σ_G) by relating the time rate of change (Δt) in the orientation of the object to its rotation rate as shown in Ec. (6).

$$\sigma_G = \omega * \Delta t \quad (6)$$

Where $\Delta t = 0.0017 \text{ s}$, based on the response of gyroscope information over the time.

It is noted that σ_G can be used to estimate the rate of rotation movement to an object through DCM integration Ec. (7).

$$DCM = DCM_{t-1} * DCM_t \quad (7)$$

However, integration of gyroscope measurement errors will lead to accumulating offset and gyro drift in DCM elements.

E. Offset and drifting estimation

Offset compensation was corrected before estimate orientation by computing gyroscope offset ω_{off} Ec. (8) and subtracted from ω as in Ec. (9).

$$\omega_{off} = \frac{\sum_{i=0}^n \omega}{n} \quad (8)$$

$$\omega = \omega - \omega_{off} \quad (9)$$

Where $n = 1000$ samples

On the other hand, drifting compensation was realized by relating C_{Ref} to a local level coordinate system λ , being $(\lambda_x, \lambda_y, \lambda_z)$ accelerometer information referred to each axis of C_{obj} . Accordingly, λ was used to assess the drifting deviation assuming that the direction of gravity (\hat{g}) is coincident with z axis $[0, 0, g]$ of the inertial frame C_{ref} . Therefore, by computing the cross product between an inertial direction vector (\vec{v}) as the best estimation of λ Ec. (10) and a gravity vector (\hat{v}) estimated from the relationship between \hat{g} and DCM, Ec. (11), a rotational correction vector ($\vec{\mu}$) was estimated Ec. (12).

$$\vec{v} = \frac{\lambda}{|\lambda|} \quad (10)$$

$$\hat{v} = \hat{g} * DCM_{G_t} \quad (11)$$

$$\vec{\mu} = \vec{v} \times \hat{v} \quad (12)$$

F. Data integration

Conventionally, drifting deviation in applications under low levels of noise is performed by feeding back μ to σ_G to obtain a corrected orientation vector σ_c as in Ec. (13).

$$\sigma_c = \sigma_G + \vec{\mu} \quad (13)$$

However, σ_c could be subject to high levels of noise due the accelerations during movements. Therefore, a *Proportional*

Integral Feedback Controller (PI) was implemented for updating σ_G . In this process μ is multiplied by a weight W_μ and feeded to a PI controller (μ_c), Ec. (14) to be added to σ_G forcing coordinate systems C_{Obj} and C_{Ref} to converge and, in this way, removing drift Ec. (15).

$$\mu_c = K_p f(W_\mu * \mu) + K_i \int_0^t f(W_\mu * \mu) dt \quad (14)$$

$$\sigma = \sigma_G + \mu_c \quad (15)$$

$W_\mu = 0.5$, $K_p = 0.15$ and $K_i = 0.002$ were established as the optimal conditions to estimate orientation from both gyroscope and accelerometer information, based on “the good gain method” [14]. Note that between σ and σ_c there is a straightforward relationship since σ corresponds to the corrected orientation of gyroscope σ_G . Consequently $\sigma_G = \sigma$ to estimate the rate of rotation movement through DCM.

III. RESULTS AND DISCUSSION

A. Proposed methods

Three different algorithms were proposed in order to estimate σ as σ_G considering a tri-axial gyroscope ω as a main source of information: 1.- estimation of σ_G by including ω_{off} in ω ; 2.- estimation of σ_G as σ_c by including both ω_{off} and μ computed from λ , without any reduction of noise and; 3.- estimation of σ_G as σ using a *PI feedback controller* in order to update σ_G removing drift and reducing noise. Fig. 1 shows a block diagram representation of each orientation estimation algorithm.

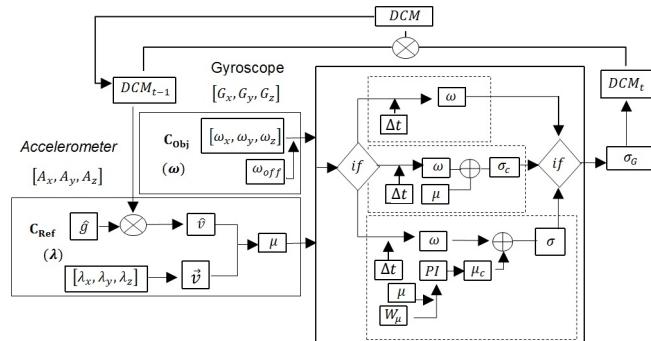


Figure 1. Block diagram representation of each orientation estimation algorithm; ω -orientation from a tri-axial gyroscope compensating offset deviation, σ_c - orientation by merging gyroscope and accelerometer information without any reduction of noise, σ -orientation by merging gyroscope and accelerometer information by using a PI feedback controller.

For assessing differences between the orientation estimation algorithms, two different studies were realized: 1.- analysis of behavior of the sensor in both static position and during moving and; 2.- Estimation of the orientation of the sensor by representing the orientation of an object through a virtual model.

B. Analysis in static position

For the analysis in static position the estimated orientation was represented as *Euler angles* computed from DCM, Ec. (16).

$$\begin{aligned} \alpha &= \text{atan}2(DCM_{23}, DCM_{33}) \\ \phi &= \text{asin}(DCM_{13}) \\ \theta &= \text{atan}2(DCM_{12}, DCM_{11}) \end{aligned} \quad (16)$$

Five minutes of the obtained information were recorded allowing to compare the drift deviation on the static position of the sensor in ω , σ_c and σ for each axis, Fig.2.

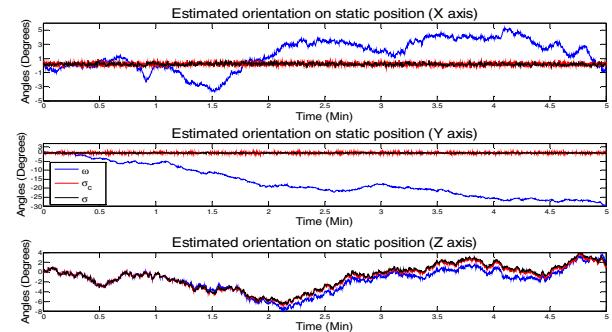


Figure 2. Analysis of behavior in static position for X, Y and Z axes: ω -behavior of a 3-axial gyroscope compensating offset deviation, σ_c - behavior of merging gyroscope and accelerometer information without any reduction of noise, σ -behavior of merging gyroscope and accelerometer information by using a PI feedback controller.

The degree of deviation between proposed algorithms was estimated by computing the coefficient of variation (C_v) of the obtained information (ρ) Ec. (17).

$$C_v = \frac{\text{std}(\rho)}{|\text{mean}(\rho)|} * 100 \quad (17)$$

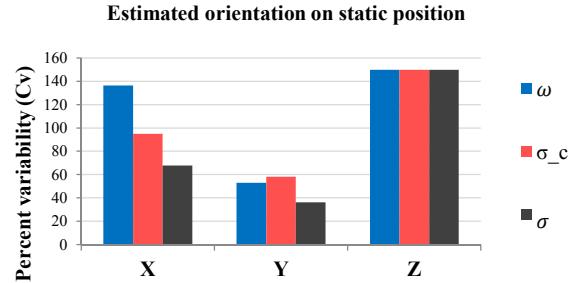


Figure 3. Coefficient of variation in static position.

Consequently, from Fig. 2 and Fig. 3, it can be verified that the integration of gyroscope measurement errors over the time leads to an accumulative drifting deviation even in static position. Besides, both σ_c and σ demonstrate that λ can be used to compensate drift deviation. Because, λ is defined by the acceleration of gravity, drift compensation can be realized only for X and Y axes, so an additional source of information such as magnetometer or GPS is needed to compensate drift deviation in Z axis.

Although, both σ_c and σ could be used to estimate orientation through a DCM, it can be observed that, even without any presence of movement, there is a considerable difference between noise at least on X and Y axes while Z axis keeps the same behavior either in ω , σ_G and σ .

C. Analysis during movement

For the analysis during movement five repetitions from 0 to 90 degrees, for each of the main axis of the module sensor were performed, comparing each method to estimate drifting deviation during movement. In order to assure repeatability, as well as to prevent mechanical errors during the assessment a graduated rotatory device, specifically designed for the assessment was used (the afore mentioned rotatory device can be observed in fig.5).

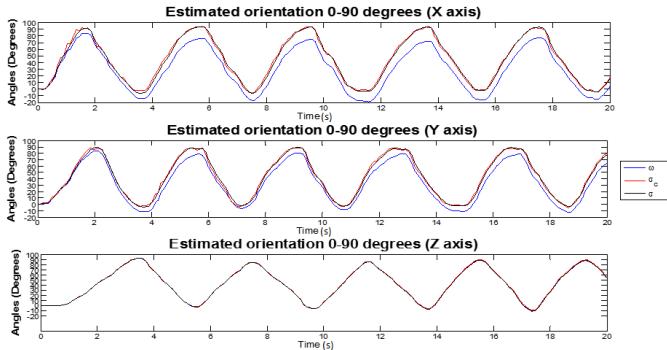


Figure 4. Analysis of behavior during movement for X, Y and Z axes.

From, Fig. 4, it can be observed that drift deviation leads to the loss of orientation during movement in ω , while σ_G and σ for X and Y axes do not, as it was assumed on the analysis of behavior in static position. Despite, an additional source of information is needed to compensate Z axis, neither ω , σ_G or σ demonstrate drift deviation during movement evaluation at least on this axis.

D. Orientation of an object in 3D space.

To transform coordinates between reference systems C_{obj} and C_{Ref} on 3D space, DCM was conformed from σ_G instead of only Euler angles because of its singularity-free orientation representation. Considering that both position and orientation of an object in 3D space are usually represented by a coordinate triplet, the orientation of an object in 3D space can be referred to a V_{xyz} vector. Consequently, transformation between coordinate systems will be determined as in Ec. (18).

$$V_{xyz} = V_{xyz_{t-1}} * DCM \quad (18)$$

Accordingly, V_{xyz} will define the orientation of an object in 3D space based on the sensors orientation represented through a virtual model while performing a trajectory between 0 to 45 degrees. As shown in Fig. 5.

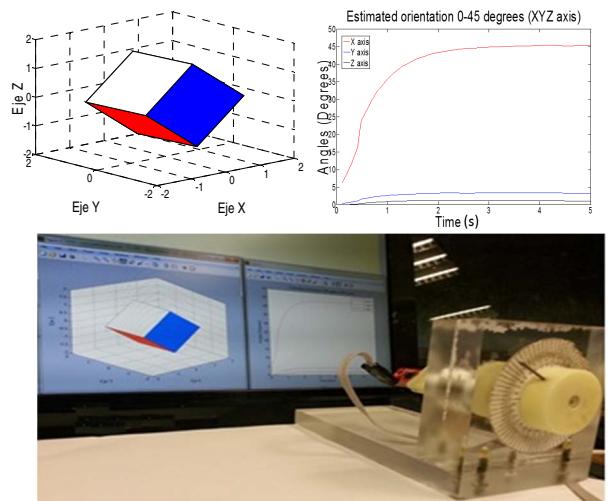


Figure 5. Representation of the orientation of an object in 3D space through a virtual environment.

E. Discussion

Comparing the results obtained with those of [15], [16], the use of DCM is an alternative way to estimate the position and orientation of an object in 3D space through the fusion of gyroscope and accelerometer information. Besides, in the future, the proposed algorithm is attempted to be used for the analysis of the upper limb behavior (shoulder, elbow and wrist).

IV. CONCLUSION

Gyroscopes by themselves cannot be used to provide an absolute measurement of orientation. The main reason is that the integration of measurement errors, in the estimated orientation, leads to both an accumulated offset and gyro drift causing loss of orientation over the time. Experimental results demonstrate that the fusion of gyroscope and accelerometer information, by integrating accelerometer data in gyroscope estimated orientation, can be used to compensate offset and gyro drift. The optimal fusion of this integration was achieved by using a PI controller to feedback accelerometer information into gyroscope estimated orientation removing drift and reducing noise.

The proposed algorithm can be used in applications related with the human motion analysis, such as: the evaluation of the range of movements of the body joints, gait analysis, upper limb motion, etc.

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