Design of a Parallel Robot for Needle-Based Interventions on Small Animals

Özkan Bebek, Member, IEEE, Myun Joong Hwang, Member, IEEE, and M. Çenk Çavuşoğlu, Senior Member, IEEE

Abstract—In this paper, a novel 5-degrees-of-freedom robot for performing needle-based interventions on small animal subjects is presented. The robot can realize dexterous alignment of the needle using two parallel mechanisms, and has a syringe mechanism to insert needles to subjects. Operations on small animals require high accuracy positioning during needle insertion. The kinematic calibration procedure of the robot using an optical tracker as an external sensor is presented to enhance accuracy of the system. After the kinematic calibration, the positioning accuracy of the needle tip is measured as 0.4 mm RMS. The robot design is light weight, and has a motion bandwidth of 4 Hz. The robot can track reference trajectories with a closed-loop controller.

Index Terms—Calibration and identification, kinematics, medical robotics, needle insertion, parallel robots, robot-assisted biopsy.

I. INTRODUCTION

THIS paper explains the design of a novel robot to perform needle-based interventions on small animal subjects. Small animals are used in various applications for biomedical research, such as to develop drugs or therapies before they are used to treat human diseases. Manual needle insertion in small animals is time consuming and most of the time target tissues cannot be accurately reached due to their small sizes. Image-guided needle insertion has been preferred for its accuracy and safety [1], for which researchers have used X-ray computed tomography (CT) [2]–[5], magnetic resonance (MR) imaging [6]–[10], micro-postion emission tomographic (PET) [8], [11], and ultrasound imaging [12]–[15]. Even though image guidance is helpful during needle insertion, it is still difficult to manually insert a needle to the small sized moving target. Another area of concern is inaccurate needle positioning, which can destroy tissues and organs, and cause trauma. In cases of insertion failures or missed targets, a new animal is used to repeat the experiment. Robotic-assisted autonomous needle insertion offers a convenient and reliable solution to this accuracy problem. Based on the nature of the problem, the robot is required to have high resolution and accuracy, and should have a light structure and high motion bandwidth, the criteria that are addressed in the design of the parallel robot explained here.

The objective of this paper is to explain the design of the small animal biopsy robot (SABiR) and present its forward and inverse kinematics, dynamics, calibration, and validation of its accuracy after calibration. This compact robot design to insert needles to small animal subjects was first introduced in [16]. SABiR, shown in Fig. 1, is operational and has five degrees of freedom (DOF) with two gimbal joints that carry a needle mechanism. It employs a parallel design to achieve low inertia. The robot has high position resolution and can realize dexterous alignment of the needle before insertion. The design is light weight, and has...
high motion bandwidth, so that biological motions at the target could be canceled while performing needle placement.

The actuated joints of the robot are driven by tendon mechanisms. There are slight differences in the calculated and the actual transmission ratios (or gear ratios). Also, there are offsets in the joint angles affected from initial starting position of the system. The distance and alignment of the parallel mechanisms should be measured precisely for accurate forward kinematics implementation. Therefore, we presented a method to compute transmission ratios, angle offsets of each actuated joint from their actual position, and constant lengths in the robot frame [17]. In this paper, a more reliable version of the kinematic calibration is performed on the robot by measuring actual position and orientation of the robot extremities using an external sensor. Finally, the results are presented regarding accuracy of the robot before and after the kinematic calibration.

In Section II, relevant literature on robotic systems for small animal interventions is presented. Section III describes the design specifics of the developed robotic system. Section IV explains the robot kinematics and calibration process, and gives calibration results. Finally, dynamics of the robot and its dynamic capabilities are presented in Section V.

II. SMALL ANIMAL BIOPSY SYSTEMS

There are several studies on image-guided automated robotic systems for small animal interventions, which focus on the development of robotic systems, their calibration and data registration, and image reconstruction.

Kronreif et al. [4] developed two robotic systems, B-Rob-I and B-Rob-II, for CT- and US-guided biopsies. B-Rob-I has a 4-DOF gross positioning system consisting of XYZ Cartesian linear axes and an additional rotational joint, and a needle positioning unit consisting of three linear DOF. The system is evaluated with image-guided phantom experiments on peas, and the measured targeting accuracy is $1.48 \pm 0.62$ mm. The second version of the system, called B-Rob-II, was developed with the same basic principles of its predecessor but was modular for clinical setups. Image-guided positioning accuracy of this system to a penetrable phantom gel was $0.66 \pm 0.27$ mm.

The AcuBot, developed by Stoianovici et al. [5], incorporates a percutaneous access to the kidney radiolucent needle driver, a remote center of motion (RCM) module, a 3-DOF Cartesian stage for translational positioning, and a passive positioning arm. CT-guided in vitro accuracy of the robot is $0.61^\circ \pm 0.33^\circ$ angular and $1.66 \pm 0.65$ mm linear\(^1\) [18].

An MR-compatible assistance system, called INNOMOTION [6], was developed by Melzer et al. Targeting precision of the pneumatically driven 6-DOF system was determined with a mechanical FARO arm in dry lab experiments. Mechanical target precision of the tool tip point, which is defined as the tip of a virtual instrument at 100-mm distance from the tool center point, is $0.7 \pm 0.5$ mm under dry lab conditions\(^2\) [19].

Huang et al. [8] designed a stereotactic image-guided system that consists of an image fusion software and an autonomous robot arm. In this study, micro-PET images are fused with MR images. For vertical needle insertion, a commercial 4 DOF CAST-PRO II robot arm is used; however, the needle orientation was constrained despite having installed an additional motor. The position accuracy of this robotic system in free space was about $0.05 \pm 0.02$ mm, and the image registration accuracy was $0.37 \pm 0.18$ mm. Overall system integration accuracy was reported as $1.20 \pm 0.39$ mm.

The MrBot robot is designed for image-guided percutaneous needle interventions of the prostate by Stoianovici et al. [7]. Multiimager-compatible MrBot robot was constructed in the form of a platform supported by pneumatic linear actuators in a 5-DOF parallel link structure. With the system, average seed deployment position error to an agar phantom gel was $0.652$ mm [1].

Waspe et al. [12] developed a 6-DOF RCM robot to control needle orientation. In their design, they used a 3-DOF Cartesian translation stage, and two parallel four-bar linkages arranged in a parallelogram structure to produce a 2-DOF RCM stage. For intraoperative image guidance, ultrasound imaging is used. The average overall error for positioning the needle tip in free space was $145 \pm 33 \mu$m for the 2-DOF RCM portion of the robot.

Ayadi et al. [2] installed a pneumatic gripper into a 6-DOF industrial Staubli Rx60 robot. For image guidance, micro-CT was used. The position and orientation of the needle can be controlled freely using this robot; however, it takes up a large space. Generally, commercially available robotic systems are too bulky to perform operations on small animals. In a more recent study, Ayadi et al. [3] have performed experiments with a Mitsubishi RV 1A 6-DOF industrial robot. They were able to guide needles through a test bed structure with 2-mm diameter holes using visual servoing.

Kazanzides et al. [11] developed a 4-DOF robot for image-guided procedures using micro-PET. Their design consists of a $X-Y$ platform and two vertical slides. A force sensor is attached to the robot for force control. The robot can position a measurement probe in free space with a mean error less than $0.4$ mm. However, the needle is attached at a fixed orientation to the 4-DOF robot. Such a constrained orientation can hinder generating various trajectories for needle insertion.

A 5-DOF robot for percutaneous needle insertion was developed by Bassan et al. [20]. The manipulator can perform 2-DOF orientation around its RCM, insertion, and rotation of the needle and linear motion of the stylet. Their experimental setup includes a 3-D ultrasound probe, which was used for needle tip position verification. The average root-mean-square (RMS) targeting error for needle insertion experiments in agar phantoms was found to be $1.45 \pm 0.62$ mm.

More recently, Neubach and Shoham [15] have presented an ultrasound-guided system for flexible needle steering. A 6-DOF revolute, spherical, prismatic, revolute parallel plate robot was used to drive the needle. Their US imaging accuracy was $1.23 \pm 0.87$ mm. Targeting accuracy of the system during needle insertion to a two-layer dermasol phantom under closed-loop image guidance was about $1$ mm.

Needle insertion to small animals cannot be completed using a simple vertical insertion since organs or bones can be in

\(^1\)Calculated from the plots given in [18].

\(^2\)Calculated from the histogram plot given in [19].
between the entry point and the target point. Therefore, the robotic system should be capable of inserting needles in different orientations for convenient needle positioning. Needle positioning error, including the robot error and the calibration error, should be below 1 mm to precisely reach the target points such as tumors.

Although there are robotic systems that can achieve positioning accuracies below 1 mm, none of the systems discussed previously report dynamic motion bandwidth capabilities sufficient for canceling biological motion at the target while performing percutaneous needle operations. The earlier systems explained previously are either bulky, off-the-shelf industrial robots with needle mechanism attachments, or custom built robots that are incapable of canceling biological motion.

Any error in the kinematics of these robotic systems may cause inaccuracies in the motion. In the literature, kinematic calibrations are performed on robots by measuring actual robot position with external sensors or placing the robots to fixtures. There are several studies in the literature for kinematic calibration of manipulators, e.g., [21]–[25]. In medical applications, even the slightest position errors might cause harm to the subjects. Therefore, accurate kinematic parameters should be found to improve accuracy of the manipulators. Beasley et al. presented a kinematic error correction method based on the error for the Jacobian matrix of a minimally invasive surgical robot [26]. Chung et al. presented a calibration method using optical tracker in macro–micro surgical robotic system [27]. Chung’s work is based on the calibration of Denavit–Hartenberg (D-H) parameters.

The robotic system, SABiR, is a novel needle robot with parallel mechanisms. Kinematic calibration of parallel mechanism robots are not trivial, and is an open problem [28]–[30]. In this paper, we propose a novel method to calibrate all calibration parameters of the robot with an external sensor, i.e., an optical tracker. Only the needle mounting calibration has to be repeated when a new needle is attached.

III. ROBOT DESIGN

Design details of the SABiR are presented in this section. First, we overlay the necessary requirements to build a system for small animal interventions. Then, we explain the robot design and its components.

A. Design Requirements

We designed a manipulator that can insert and retract a needle at a given position and orientation. The system can follow a desired insertion and extraction trajectory to perform operations on small animals. This section describes the design requirements to build such a manipulator.

Force measurement experiments: We did experiments on a small animal (mouse) to determine the force needed to insert a needle to the animal tissue. 26GA\(^3\) \(\times 3\frac{1}{2}\) in needles [31] were inserted manually, about 1.5–2.0 cm deep, to various locations such as muscle, internal organs, and tumor growths. The forces at the base of the needle were measured with an ATI Nano17, 6-axis F/T sensor with force resolution of 1/160 N and torque resolution of 1/32 N·mm [32] (see Fig. 2). The magnitude of the resultant force was calculated from the recorded force/torque data.

We have collected three datasets. Each data recording was 150 s long and each set included five needle insertions to different locations. Fig. 3 shows an insertion and retraction into a muscle tissue. During these experiments, the maximum measured force was 0.8 N that was observed when the needle was inserted to the muscle tissue. Retraction forces were relatively small, around 0.1 N, compared to insertion forces. These measurements constitute one of the basic design requirements of the robot.

Manipulator workspace: Another design requirement was the workspace needed for inserting the needle into the small animal that is placed within the workspace of the manipulator. Once the robot is placed next to the animal subject, the robot should be able to position the needle in 3-D space. Then, it should align the needle before insertion along the insertion path. In order to achieve this needle movement, a manipulator with a minimum 5 DOF is needed. With a 5-DOF manipulator, the needle can be oriented in such a way that straining at the penetration site is prevented during insertion, while the needle tip and the penetration point are tracked separately.

Accuracy: The robotic system should realize various needle position configurations with high accuracy. With image guidance, the needle tip positioning error should be below 1 mm to realize the planned needle operations on the small animals. In order to achieve submillimeter accuracy, the positioning resolution of the system should be below 100 μm.

\(^3\)26GA needle has 0.46 mm external and 0.26 mm internal diameters.
Fig. 4. 5-DOF small animal drug delivery/biopsy robot. Some of the base frame beams are not shown to reveal the robot mechanisms.

**Safety:** The maximum kinetic energy that the manipulator can exert should be kept at minimum for safety reasons, and actuators should be selected accordingly.

**Speed/bandwidth:** In small animals, the frequency of breathing decreases as the animal size increases [33]. Small animals’ respiration rate is about 80–160 per min. Therefore, the manipulator should have a compact size and be light weight, such that the robot would have enough dynamic motion bandwidth and speed to track a periodic motion at the rate of 2–3 Hz, such as breathing, and cancel it while performing operations.

### B. Mechanical Design

In Fig. 4, the schematics of the 5-DOF robot design is shown. The robot consists of three main parts: front stage, back stage, and syringe mechanism. By size, it is one of the smallest that were developed in the literature for small animal operations. The developed robotic system prototype is shown in Fig. 1.

1) **Front Stage:** The front stage is a 2-DOF parallel manipulator, which is a five-bar linkage mechanism that is actuated with two electric motors fixed to the base link. Five-bar linkage mechanisms consist of four moving links and a fixed base link. This kind of parallel mechanisms are used in medical robot designs [34], [35]. One of the advantages of the five-bar linkage mechanisms is that bulky parts, such as motors, can be positioned at the base. This provides lighter links.

All four links of the mechanism are 10 cm long, and the upper links are connected to the base from the same axis. Therefore, the mechanism’s geometry is always a rhombus, making kinematics of the robot simpler. The parallel mechanism provides stable guiding of the needle. The links attached to the base are actuated with a tendon-driven mechanism, where a capstan pulls the tensioned cable that rotates the disk around its axis. This transmission provides low-friction motion without slipping or binding. Maxon RE-25 Motors with 500 counts per turn (CPT) encoders [36] are used to drive the front- and back-stage five-bar linkage mechanisms.

A **2-DOF gimbal joint** is attached to the end effector of the five-bar linkage mechanism. The needle can go through the center of the gimbal through a replaceable guide (see Fig. 5). The gimbal is attached to one of the links with a 45° angle offset, in order to provide an almost symmetric workspace. Fig. 6 shows the needle tip workspace of the robot when a 6-in-long needle is used. The gimbal joints do not exert any bending moments on the needle as a result of its low-friction bearings.

2) **Back Stage:** The back stage of the robot has the same five-bar linkage mechanism as the front stage. In addition, this stage is able to rotate around the base link’s pitch axis. This motion provides the push force needed for inserting the needle into the
tissue. The additional axis of the back stage is driven by a Maxon RE-30 Motor with 1000 CPT encoder [36]. Depending on the orientation of the five-bar mechanisms, a position resolution of 5–50 μm is achievable with these position encoders. A 2-DOF gimbal joint attached to the end effector of the five-bar linkage mechanism carries the needle and syringe mechanism.

3) Syringe Mechanism: This robot can be used in a wide range of biomedical applications such as biopsy, drug delivery, or cell seeding by changing the needle mechanism. For initial experimentation, a needle and syringe mechanism is designed to deliver drugs to the target tissue. The mechanism is attached to the back-stage end effector with a gimbal joint to allow free rotation. The syringe mechanism has a Maxon RE-10 motor with gear head and encoder to operate the syringe plug. The motor’s rotational motion is transformed into linear motion that is needed to insert or retract the plunger of the syringe.

The prototype syringe mechanism with its gimbal weighs about 118 g. A similar shaped, hand-held steerable needle device designed by Okazawa et al. [37] weighs approximately 250 g. The total weight of the moving parts attached to the front and back stages of the robot, i.e., the two five-bar linkage mechanisms with gimbals, and the syringe mechanism, is 292 g. A light-weight system is desired for safety reasons, and also to enable a high-bandwidth robot by exerting minimal load to the actuators.

IV. ROBOT KINEMATICS

Forward and inverse kinematics of the robot are derived in the following sections. Following are brief outlines of the forward and inverse kinematics derivations.

In the forward kinematics, needle tip position is calculated from the joint angles. First, a relation between the front-stage and back-stage coordinate frames is defined. This relation is considered as the mounting offset. Then, end-effector positions of the front and back stages are found. In addition to the previous findings, measured needle and syringe lengths, and needle mounting angle are used to calculate the position of the needle mounting point. Finally, the needle tip position is calculated from the end-effector positions and the needle mounting position.

In the inverse kinematics, joint angles are calculated from the needle tip position and the needle direction. First, the front-stage end-effector position is calculated by solving the line equation, which connects the needle tip and the front-stage end effector. Front-stage joint angles can be solved from its end-effector position. Then, the back-stage end-effector position and the needle mounting position can be derived as functions of back-stage joint angles. Finally, an optimization problem can be set using these two functions to solve the back-stage joint angles.

A. Forward Kinematics

To find the position of the needle tip, we derived the forward kinematics of the robot. Assigned coordinate frames and joint angles, \( \theta_i \) for \( i = 1, \ldots, 5 \), of the robot are shown in Fig. 7. \( l_f \) and \( l_b \) are link lengths of front stage and back stage, respectively. \( l_{fe} \) and \( l_{be} \) are link lengths between five-bar linkage end effector and gimbal joints of front stage and back stage, respectively. The front- and back-stage bases are mounted to the robot frame. These base links’ orientations cannot be exactly aligned with each other. Therefore, a relation between the coordinate frame of the front stage and that of the back stage is defined with \( B^f T \). \( \{F\} \) is the coordinate frame of the front stage, and \( \{B\} \) represents coordinate frame of the back stage. Therefore, \( B^f T \) is a homogeneous transformation, which defines mounting offset between the front and back stages, with respect to back stage’s coordinate frame, \( \{B\} \). End-effector positions of front and back stage are given in (1) and (2), respectively, as follows:

\[
\begin{bmatrix}
  x_f \\
  y_f \\
  z_f \\
  1
\end{bmatrix} = 
B^f T 
\begin{bmatrix}
  F \\
  y_f \\
  z_f \\
  1
\end{bmatrix} = 
B^f T \begin{bmatrix}
  l_f \cos(\theta_1) + l_f \cos(\theta_2) + l_{fe} \cos(\theta_2 - \frac{\pi}{4}) \\
  l_f \sin(\theta_1) + l_f \sin(\theta_2) + l_{fe} \sin(\theta_2 - \frac{\pi}{4}) \\
  d_{fgim} \\
  1
\end{bmatrix}
\]

\[\text{(1)}\]

\[
\begin{bmatrix}
  x_b \\
  y_b \\
  z_b \\
  1
\end{bmatrix} = 
B^f T \begin{bmatrix}
  l_b \cos(\theta_4) + l_b \cos(\theta_5) + l_{be} \cos(\theta_5 - \frac{\pi}{4}) \\
  l_b \sin(\theta_4) + l_b \sin(\theta_5) + l_{be} \sin(\theta_5 - \frac{\pi}{4}) \cos(\theta_3) \\
  -d_{bgim} \sin(\theta_3) \\
  \left( l_b \sin(\theta_4) + l_b \sin(\theta_5) + l_{be} \sin(\theta_5 - \frac{\pi}{4}) \right) \sin(\theta_3) \\
  +d_{bgim} \cos(\theta_3) \\
  1
\end{bmatrix}
\]

\[\text{(2)}\]
Fig. 8. Coordinate points on the syringe mechanism used in the derivation of forward and inverse kinematics of the robot.

Fig. 8 shows the assigned coordinates of the syringe mechanism. \( d_{fgim} \) and \( d_{bgim} \) are the mounting offsets of the front- and back-stage gimbals to the five-bar end effectors. \( p_n = (x_n, y_n, z_n) \) is the position of the needle tip, and \( \vec{r}_n = (r_1, r_1, r_k) \) is the unit vector that is along the direction of the needle. \( p_{be} = (x_{be}, y_{be}, z_{be}) \) is the point of the joint in the back stage which is connected to syringe mechanism. \( p_f = (x_f, y_f, z_f) \) is the center of rotation of the front-stage gimbal. \( p_s = (x_s, y_s, z_s) \) is the needle mounting point to the syringe. \( l_q \) is the sum of syringe mechanism length, \( l_n = |p_n p_s| \), and length of the needle, \( l_n = |p_n p_n| \).

We incorporated a needle mounting offset angle in 3-D space, between the centerline of the needle \( p_n p_s \), and that of syringe \( p_b p_f \), shown as \( \beta \), in Fig. 8. In the figure, centerline of the syringe is not aligned with that of the needle. Although this angular offset is relatively small, it could cause few millimeters kinematic error at the needle tip if not accounted.

From (1) and (2), \( B \beta_f R \) and \( B \beta_b \) can be found. \( B \beta_R \) is the relative transformation of the back-stage end-effector coordinate frame, \{Be\}, with respect to back-stage coordinate frame. \( B \beta_f R \) can be calculated from joint angles, as in (3)

\[
B \beta_f R = \begin{bmatrix}
-\sin(\theta_3 - \frac{\pi}{2}) & -\cos(\theta_3 - \frac{\pi}{2}) & 0 \\
\cos(\theta_2 - \frac{\pi}{2}) \cos(\theta_3) & -\sin(\theta_2 - \frac{\pi}{2}) \cos(\theta_3) & -\sin(\theta_2) \\
\cos(\theta_2 - \frac{\pi}{2}) \cos(\theta_3) & -\sin(\theta_2 - \frac{\pi}{2}) \cos(\theta_3) & \cos(\theta_3)
\end{bmatrix}.
\]

In order to calculate the needle tip position \( p_n \), needle mounting point \( p_s \), and the angles of the back-stage passive joint angles, \( \gamma_x \) and \( \gamma_y \), should be found. \( \gamma_x \) is the joint angle of first passive joint (pitch angle of the syringe mechanism), and \( \gamma_y \) is the joint angle of the second passive joint (yaw angle of the syringe mechanism).

The needle mounting offset can also be defined as a rotation matrix from back-stage gimbal center coordinate frame, \{Bb\}, to the coordinate frame attached to needle hub (proximal end), \{Bs\}. Then, the needle mounting offset can be defined with angles, \( \alpha_x \) and \( \alpha_y \), and these angles can be used to calculate the rotation matrix, \( B_{bR} = R_y(\alpha_x)R_z(\alpha_y) \), where \( R_x \) and \( R_y \) are basic \( x \) and \( y \)-axis rotation matrices. \( B_{bR} \) can be calculated using the angular mounting offsets derived from the kinematic calibration.

Then, front gimbal center with respect to back-stage gimbal center can be calculated as

\[
B_{bR} p_f = B_{bR} \begin{bmatrix} 0 \\ 0 \\ l_n \end{bmatrix} + \begin{bmatrix} B_{bR} x_f \\ B_{bR} y_f \\ B_{bR} z_f \end{bmatrix} = \begin{bmatrix} \frac{B_{bR} x_f}{l_n} \\ \frac{B_{bR} y_f}{l_n} \\ \frac{B_{bR} z_f}{l_n} \end{bmatrix}
\]

and the mounting offset \( \beta \) is

\[
\beta = \pi - \cos^{-1} \left( \frac{\sqrt{l_n^2 + l_n^2 - p_f p_f}}{l_n} \right).
\]

Solving (7) and (8), we can find \( \gamma_x \) and \( \gamma_y \), respectively. Then, \( B_{bR} R = R_x(\gamma_x - \frac{\pi}{2})R_z(\gamma_y) \) and \( B_{bR} R = R_y(\gamma_n) \) can be calculated:

\[
B_{bR} y_f \sin(\gamma_x) + B_{bR} z_f \cos(\gamma_x) = \frac{1}{2} \sin(\gamma_x) \left( B_{bR} x_f^2 + B_{bR} y_f^2 + B_{bR} z_f^2 + l_{be}^2 - p_f p_f \right)
\]

\[
(\frac{1}{B_{be} R})^T (B_{bR} - \gamma_n) = R_y(\gamma_n) (\frac{B_{bR} R}{B_{bR} R}) (B_{bR} y_f)
\]

Finally, we can find \( B_{bR} R = (B_{bR} R) (B_{bR} R) (B_{bR} R) R_y(\gamma_n) \); using \( B_{bR} R \), needle mounting position \( B p_s \), and the needle tip position \( B p_a \), with respect to the back-stage coordinate frame, \{B\}, can be calculated as

\[
B p_s = (B_{bR} R) B p_a + B p_b = (B_{bR} R) \begin{bmatrix} 0 \\ 0 \\ l_n \end{bmatrix} + B p_a
\]

\[
B p_a = (B_{bR} R) B p_s + B p_a = (B_{bR} R) \begin{bmatrix} 0 \\ 0 \\ l_n \end{bmatrix} + B p_a
\]

B. Inverse Kinematics

We also need to derive inverse kinematics in order to find joint angles from the desired needle position \( p_a \), and the direction of the needle tip \( \vec{r}_n \). The known parameters from kinematic calibration are \( B_{bR} R, l_n, \) and \( l_n \).

Also, the crossing point of the needle from the front-stage gimbal joint center \( B p_f \) is on the line that connect \( B p_a \) and
\[ B \mathbf{p}_x \cdot B \mathbf{p}_z \text{ can be found from} \]
\[ B \mathbf{p}_s = -l_{n2} \cdot \hat{\mathbf{c}}_{z} + B \mathbf{p}_n. \]  
(11)

Then, \( p_f \) can be solved from the line equation given as
\[ \frac{x_f - x_s}{x_n - x_s} = \frac{y_f - y_s}{y_n - y_s} = \frac{z_f - z_s}{z_n - z_s} = k. \]  
(12)

After finding \( F p_f = [x_f \ y_f \ z_f]^T \), \( \theta_1 \) and \( \theta_2 \) can be solved from
\[ \theta_1 = \text{atan2}(n_{2f}, n_{1f}) + \theta_2 \]
\[ \theta_2 = \text{atan2}(c_f, -\sqrt{1 - c_f^2}) - \text{atan2}(a_f, b_f) \]  
(13)

where
\[ k_f = \frac{l_{fr}}{\sqrt{2}}, \quad m_f = l_f + k_f, \quad a_f = 2m_f x_f - 2k_f y_f \]
\[ b_f = 2k_f x_f + 2m_f y_f, \quad c_f = \frac{x_f^2 + y_f^2 + 2m_f k_f}{\sqrt{a_f^2 + b_f^2}} \]
\[ n_{1f} = x_f \cos(\theta_2) + y_f \sin(\theta_2) - l_f - k_f \]
\[ n_{2f} = -x_f \sin(\theta_2) + y_f \cos(\theta_2) + k_f. \]  
(14)

\( B \mathbf{p}_s \) and \( B \mathbf{p}_n \) can be derived as functions of \( \theta_3, \theta_4 \), and \( \theta_5 \) using forward kinematics. Then, we can find \( \theta_3, \theta_4, \) and \( \theta_5 \) by minimizing the following cost function:
\[ \arg \min_{\theta_3, \theta_4, \theta_5} \{ ||B \mathbf{p}_s - B \mathbf{p}_s(\theta_3, \theta_4, \theta_5)|| \}
\[ + ||B \mathbf{p}_n - B \mathbf{p}_n(\theta_3, \theta_4, \theta_5)|| \}. \]  
(15)

C. Kinematic Calibration

Kinematic calibration is the process to find errors in kinematic parameters. Kinematic calibration parameters include link lengths, link offsets, joint angle offsets, and so on, which are D-H parameters in most serial manipulators. In our robotic system, we assume that each link in the parallel mechanism has negligible kinematic error since these links are manufactured at a high-tolerance computer numerical control machine. The robot has tendon-driven mechanisms to drive its actuated joints. The transmission ratios at these joints have uncertainties due to difficulty in measuring accurate ratio between the encoder angles and joint angles. Motor encoders used are incremental-type optical encoders; therefore, joint angles have initial offset errors every time the system is started. Therefore, gear ratios and joint angle offsets are selected as calibration parameters, and denoted as \( G_i \) and \( \theta_i, \text{offset} \), respectively, for the \( i \)th motor. Also, the distance between base links of the front stage and the back stage is determined since there can be alignment offsets between the two.

To identify calibration parameters, it is necessary to measure the actual position of the robot using an accurate sensor. In this research, an optical tracker system, NDI Polaris Vicra, is used as the external sensor. However, the control of the robot is performed using only the motor encoders. Polaris Vicra can measure 6-DOF position and orientation using optical marker tools. This sensor’s accuracy is 0.25 mm RMS [38].

The eight steps of the calibration process are represented in Fig. 9. The coordinate frames used in the calibration are shown on the robot mechanisms. \( \{ F \} \) and \( \{ B \} \) refer to the front- and back-stage coordinate frames attached to each stage’s base joint center. \( \{ M_f \}, \{ M_b \}, \) and \( \{ s_m \} \) are the coordinate frames of the optical marker tools’ origin defined by the manufacturer; \( \{ S \} \) is the coordinate frame of the optical sensor.

The front- and back-stage parameters are calibrated separately in order to get a more accurate calibration result. In the following sections, the eight steps of the calibration process are explained.

Offset angles (steps 1–3): The first calibrated parameters are the initial offset angles. Two optical marker tools are used for this step. The first optical marker is attached to the robot base, and the second one is attached to the link. The robot axes are moved to their physical limits individually, and data from optical tracker marker tools are acquired (see steps 1–3 in Fig. 9). The angle difference between the optical marker tools is recorded as the offset angle. We choose to move the robot to its physical limits since this process can be repeated every time the system is started. In fact, moving the robot to predefined stops for start-up calibration is a fairly standard practice in industrial robots. The encoder offsets can be reset at each system start, once the absolute encoder angles are determined at the physical limits. These values are updated as starting encoder offsets, \( \theta_i, \text{offset} \).

Transmission ratios (steps 4–6): After the joint angle offsets are set, the gear ratios \( G_i \) are calibrated. \( G_i \) is the transmission ratio between encoder angle of the \( i \)th motor and the \( i \)th joint angle. In this calibration step, front and back stages of the robot are made to follow a predetermined trajectory (see steps 4–6 in Fig. 9). Simultaneous data were recorded from the motor encoders \( \theta_i, \text{enc} \), and the optical tracker tools attached to the links, \( \theta_i, \text{optIK} \). Transmission ratio is defined as
\[ \theta_i, \text{enc} G_i = \theta_i, \text{optIK}. \]  
(16)

The data collected from the optical tracker and from the encoders of the robot are matched by time stamp synchronization. \( G_i \) is computed using linear least squares as in (17)
\[ G_i = (a_i^T a_i)^{-1} a_i^T b_i \]  
(17)

where \( a_i = \begin{bmatrix} \theta_i, \text{enc}(t_0) \\ \vdots \\ \theta_i, \text{enc}(t_n) \end{bmatrix}, \quad b_i = \begin{bmatrix} \theta_i, \text{optIK}(t_0) \\ \vdots \\ \theta_i, \text{optIK}(t_n) \end{bmatrix}. \)

Front-stage to back-stage transformation (step 7): Optical marker tools are attached to the front-stage base and to the back-stage base facing same direction (see step 7 in Fig. 9). Back-stage pitch joint (\( \theta_5 \)) is moved to its physical limits; hence, the robot base frames are at fixed positions. Optical marker tool data are collected and \( M_{M_f} T \) is computed.

The relation between the base frame of the front stage and that of back stage is then given by
\[ B_F T = (B_{M_b} T)(M_{M_f} T)(M_{M_b} T) B_F T \]  
(18)
where $B_{f1}T$ and $M_fT$ are pure translations calculated from physical dimensions, and $B_{f1}T$ is a rotation around $x$-axis for $\theta_3\text{,offset}$.

Needle mounting angles (step 8): Even though the needle tip position can be computed using forward kinematics equations with acquired joint angles, the needle to syringe mounting offset, described in the forward kinematics, can cause inaccuracies in the needle tip position. In order to find the actual needle mounting angular offsets, an optical tracker marker tool is attached to the syringe mechanism (see step 8 in Fig. 9), shown with coordinate frame $\{sm\}$.

Another optical marker tool was attached to the front-stage base, as in step 7, to find $S_MfT$. This will provide the relative position of the optical sensor base with respect to the robot, so that $B_{s}T$ can be calculated as

$$B_{s}T = (B_{f1}T)(B_{1}T)(M_{b}T)(S_fT)(S_{m}T)(B_{b}T)$$

and then $B_{ps}$ can be found as

$$\begin{bmatrix} x_{s} \\ y_{s} \\ z_{s} \\ 1 \end{bmatrix} = (B_{s}T) \begin{bmatrix} 0 \\ 0 \\ l_{n1} \\ 1 \end{bmatrix}.$$
Also, \( p_f \) can be calculated from (1) of forward kinematics. Using these points, \( \ddot{r}_z \) can be found. Finally, \( \alpha_x \) and \( \alpha_y \) are solved from

\[
\ddot{r}_z = \begin{pmatrix} B \cdot R_y(\alpha_y) \end{pmatrix} \begin{pmatrix} R_x(\alpha_z) \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.
\quad (21)
\]

**D. Kinematic Calibration Verification**

Validation of the calibration is done by comparing the needle tip (end effector) position calculated from motor encoder data using forward kinematics and the position measurement from the external sensor (optical tracker). In order to collect accurate data, an optical marker tool, which weighs about 25 g, is attached to the end effector (shown with coordinate frame \{n\} in step 8 in Fig. 9). The needles used in biopsies are extremely thin to carry such a load without deformation. Hence, we replaced the needle with a solid steel shaft of the same length with 2.85 mm in diameter, in order to demonstrate system’s capabilities. Position data are recorded while the robot is following a needle insertion trajectory in free space. Kinematic calibration is evaluated by comparing the needle tip positions calculated from the encoders and the optical sensor. Using this procedure, four different target points were tested. Before the calibration, the accuracy of the system was found to be around 5 mm. After calibration, the mean of the RMS position error for the four trials was \(0.419 \pm 0.166\) mm.

**E. Calibration Remarks**

In this section, we presented the calibration method used on the 5-DOF parallel robot designed for inserting needles to small animal subjects. We performed the calibration in multiple steps to achieve higher accuracies. A total of 14 parameters were calibrated. Only the needle mounting calibration (see step 8 in Fig. 9) has to be repeated when a new needle is attached, which takes little time to do. Before the calibration, the accuracy of the system was found to be around 5 mm. Using the proposed calibration method, we showed that the system can achieve an end-effector position accuracy of 0.4 mm. The error is likely to be due to the initial assumptions made in Section IV-C. Parallel manipulators with similar position accuracies are reported in the literature [35]. The accuracy of the robot presented here satisfies the millimeter accuracy for injecting needles to target positions. One should note that the accuracy of the system would improve more under real-time intraoperative image guidance.

**V. ROBOT DYNAMICS**

It is necessary to know the dynamic equations of motion of the robot to implement high-performance controllers. The dynamic equations of the front and back stages are in the form

\[
M_f(\theta_f)\ddot{\theta}_f + C_f(\dot{\theta}_f, \theta_f)\dot{\theta}_f + N_f(\theta) = \tau_f
\quad (22)
\]

\[
M_b(\theta_b)\ddot{\theta}_b + C_b(\dot{\theta}_b, \theta_b)\dot{\theta}_b + N_b(\theta) = \tau_b
\quad (23)
\]

where \( \theta_f = [\theta_1 \theta_2]^T \in \mathbb{R}^2 \) and \( \theta_b = [\theta_3 \theta_4 \theta_5]^T \in \mathbb{R}^3 \). \( M_f \) and \( M_b \) are the inertia matrices, and \( C_f \) and \( C_b \) are the Coriolis matrices of the front and back stages, respectively. \( N_f \) and \( N_b \) include the gravitational and other forces—such as friction—that act on the joints, and \( \tau_f \) and \( \tau_b \) are the vectors of the actuator torques.

The dynamic equations of motion were derived using the Lagrangian method [39]. The actual equations are not included here, but available in [40]. Using the derived dynamics, we were able to implement a gravity compensation control scheme. After eliminating the effects of gravity, we were able to position the needle at any direction, and the robot was able to hold the needle as it is positioned.

**A. Experimental Transfer Function Models**

Experimental transfer function models of the robot axes were determined. The control algorithms were executed on a PC equipped with a 2.33-GHz Dual-Core Intel Xeon 5140 processor running MATLAB xPC Target v4.0 real-time kernel with a sampling time of 1 ms. Linear current amplifiers were used to drive the dc motors.

In the frequency-response measurements, the input was the force along the axis and the output was the measured position. During the experiments, the motion of the manipulator was constrained with clamps other than the measured axis. Also, the robot is oriented in such a way that the motion of the modeled axis is parallel to the ground to avoid gravitational forces, because gravitational forces would add a bias to the model and corrupt the captured dynamics.

Experimental transfer function models of the robot axes, at the zero configuration of the robot (shown in Fig. 4), are determined from the frequency-response data. As can be seen in Fig. 10, these linear models approximate the low-frequency behavior of the system, up to 100 Hz. In all five axes, the frequency responses at low frequencies are in the form of \(1/s^2\), which is the form for pure inertial behavior. From the magnitude plot of the frequency response, we can see that robot axes 1 and 2 almost have the same behavior, and similarly, axes 4 and 5. This is due to the symmetry of the five-bar mechanisms. When compared to front-stage axes (1 and 2), back-stage axes (3, 4, and 5) maintain their inertial behavior at lower magnitudes and hold up to a lower frequency. This is because the load that back stage carries, syringe mechanism, is heavier than front stage’s load.

**B. System Bandwidth**

We have performed experimental studies to find the dynamic capabilities of the robot. The first one is to find the system bandwidth in closed-loop control. Proportional-derivative (PD) controllers were implemented to control the axes of the robot. Off-line path and motion planning was performed in the end-effector space. Joint angles were solved from inverse kinematics using the desired needle tip position and needle direction. These motor-angle references were used to follow the desired needle tip trajectory.

In order to determine the accurate working bandwidth of the robot, dynamic testing under harmonic motion was conducted. Harmonic motion trajectories were tracked with the needle tip.
at constant frequencies. A series of sine waves with frequencies from 0.25 to 5 Hz and amplitude of 5 mm were tracked by the needle tip in the XYZ Cartesian coordinates. The sinusoidal motions were placed around the zero configuration of the robot, as shown in Fig. 4. Similarly, the needle tip was pivoted around x-axis (pitch axis) and y-axis (yaw axis) to follow a series of sine waves with frequencies from 0.25 to 5.00 Hz and amplitude of 2.5°. The position data were collected from the robot encoders, and the needle tip reference trajectories were compared to the position of the robot calculated from the forward kinematics.

Closed-loop control frequency response of the robot is shown in Fig. 11. Frequency response is flat up to a resonance observed around 4.5 Hz. Phase difference is about 1°–3° in the 0.25–4.0 Hz range.

C. Tracking Biological Motion

The second experimental study was performed to test the motion-tracking capabilities of the robot. Motion compensation is required to improve the targeting accuracy while operating on live animals. In previous work, we have developed active relative motion compensation algorithms to compensate for biological motion [41]. The idea is to cancel the relative motion between the target and the needle tip, so that the needle operations can be done as if the target is stationary.

The frequency of breathing decreases as animal size increases [33]. Small animals’ respiration rate is about 80–160 per min. Our robot’s bandwidth is enough to track a periodic motion at this rate (i.e., 2–3 Hz).

In order to demonstrate the robot’s ability to track high-frequency biological motion, two types of controllers were implemented to control the axes of the robot: PD control, and receding horizon model-predictive control (RH MPC). We implemented the same RH MPC used in [41]. RH MPC employs a sophisticated feedforward algorithm to control the robot motors, which in return has relatively better tracking performance compared to traditional controllers.

Breathing motion is a simple harmonic motion with only few dominant frequencies. In most cases, two or three frequencies can be combined to mimic the respiratory motion. In order to test the motion compensation capabilities of the robot, three sinusoids in XYZ Cartesian coordinates were tracked by the
needle tip. The resulting motion of the sinusoids is an ellipsoid in 3-D coordinates placed at 45° to the XZ plane. Tracking experiments were performed at constant frequencies from 0.25 to 3 Hz for 120-s-long intervals, and repeated 16 times for each frequency. The deviation between the trials were very small. RMS tracking errors between the needle tip position and the reference position for each frequency are shown in Fig. 12. The robot can track the reference trajectory with an error about 0.38 mm RMS up to 2 Hz. This corresponds to a 6% tracking error up to a bandwidth of about 2 Hz, for a reference motion of magnitude 6.11 mm RMS. This tracking error is due to the dynamics of the robot, which can be reduced by improving the controller.

VI. CONCLUSION

In this paper, we presented the design, forward and inverse kinematics, dynamics, and kinematic calibration of a novel SABIR. Kinematic calibration to improve the accuracy of the robot is performed by measuring actual position and orientation of the robot extremities using an optical position sensor. Before calibration, the accuracy of the system was tested to be around 5 mm. After calibration, the mean RMS position error of the needle tip was reduced to 0.4 mm. This result is similar to the accuracies reported in the literature. However, the proposed system is light weight and has a high bandwidth. Dynamic experiments show that the system’s bandwidth under closed-loop control is about 4 Hz, and the robot can track reference trajectories with 6% tracking error up to a bandwidth of 2 Hz.

ACKNOWLEDGMENT

The authors would like to thank Dr. B. Fei for his help during force measurement experiments and determining design requirements, and F. Liang for his help during system identification.

REFERENCES


This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.


**Özkan Bebek** (S’06–M’09) received the B.S. degree in mechanical engineering from the Middle East Technical University, Ankara, Turkey, in 2001, the M.S. degree in mechatronics from Sabanci University, Istanbul, Turkey, in 2003, and the Ph.D. degree in systems and control engineering from Case Western Reserve University, Cleveland, OH, in 2008. He was a Senior Research Associate in the Department of Electrical Engineering and Computer Science, Case Western Reserve University, from 2008 to 2011. He is currently an Assistant Professor of mechanical engineering at Ozyegin University, Istanbul, Turkey. His research interests include medical robotics, mechatronics, and control systems.

**Myun Joong Hwang** (S’06–M’08) received the B.S., M.S., and Ph.D. degrees in mechanical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in 2001, 2003, and 2007, respectively. He was a Research Associate at the Mechanical Engineering Research Institute, KAIST, in 2007. He was a Visiting Scholar at the University of California, Irvine, in 2005. He was a Research Associate in the Department of Electrical Engineering and Computer Science, Case Western Reserve University, Cleveland, OH, during 2008–2009. Since 2010, he has been a Senior Engineer at the Manufacturing Technology Center, Samsung Electronics Company, Ltd., Suwon, Korea. His research interests include motion planning and control of industrial robots, medical robotics, and cooperation of multirobots.

**M. Cenk Çavuşoğlu** (S’93–M’01–SM’06) received the B.S. degree in electrical and electronic engineering from the Middle East Technical University, Ankara, Turkey, in 1995, and the M.S. and Ph.D. degrees in electrical engineering and computer sciences from the University of California (UC), Berkeley, in 1997 and 2000, respectively.

He is currently an Associate Professor in the Department of Electrical Engineering and Computer Science, Case Western Reserve University, Cleveland, OH. He was a Postdoctoral Researcher and Lecturer in the Department of Electrical Engineering and Computer Sciences, UC Berkeley, from 2000 to 2002. He was a Visiting Associate Professor in the Electrical and Electronic Engineering Department, Bilkent University, Ankara, Turkey, from 2009 to 2010. His research involves applications of robotics and control engineering to biomedical and biologically inspired engineered systems. Specifically, his research interests include robotics, systems and control theory, and human–machine interfaces, with emphasis on medical robotics, haptics, virtual environments, surgical simulation, and biosystem modeling and simulation.

Dr. Çavuşoğlu served as an Associate Editor of the *IEEE Transactions on Robotics* during 2005–2009.