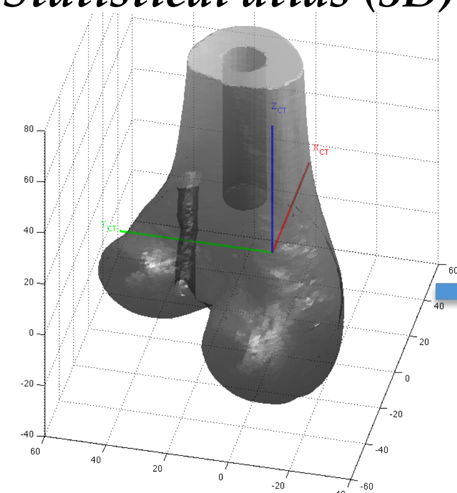
3D-2D Deformable Registration A Survey

X. Kang (Ben)

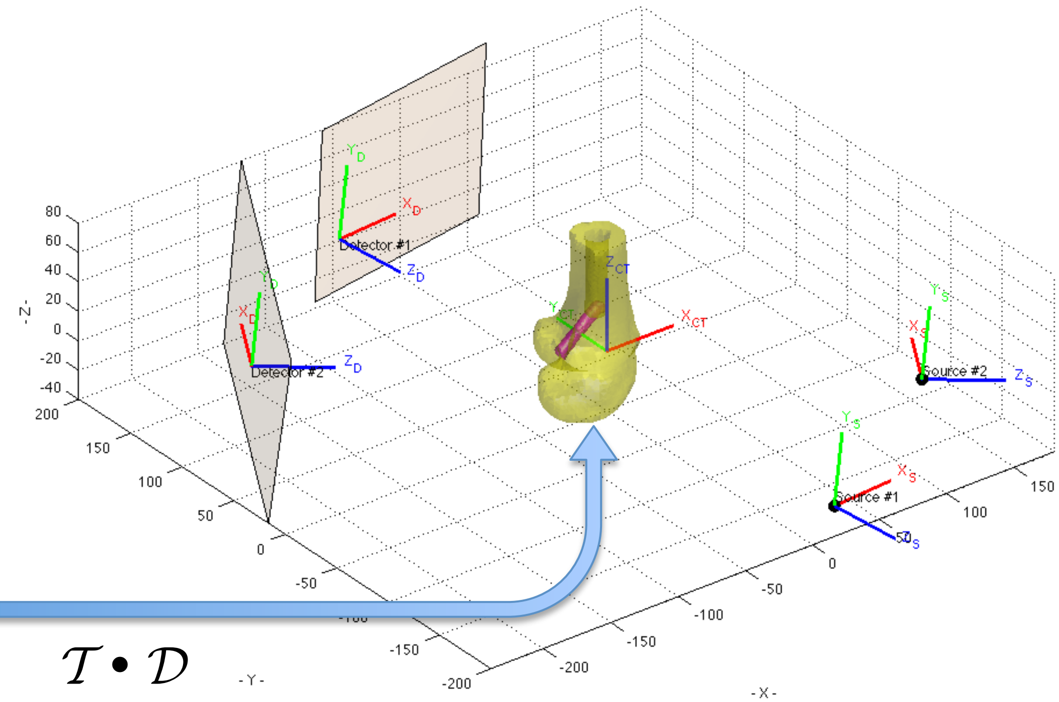
Definition

Plain Image(s) (2D, calibrated)

Statistical atlas (3D)



Imaging geometry



$\mathcal{T} \bullet \mathcal{D}$

$\mathcal{T} \in SE(3)$: ***Transformation***

$\mathcal{D}$ $\mathcal{R}3 \rightarrow \mathcal{R}3$: ***Deformation***

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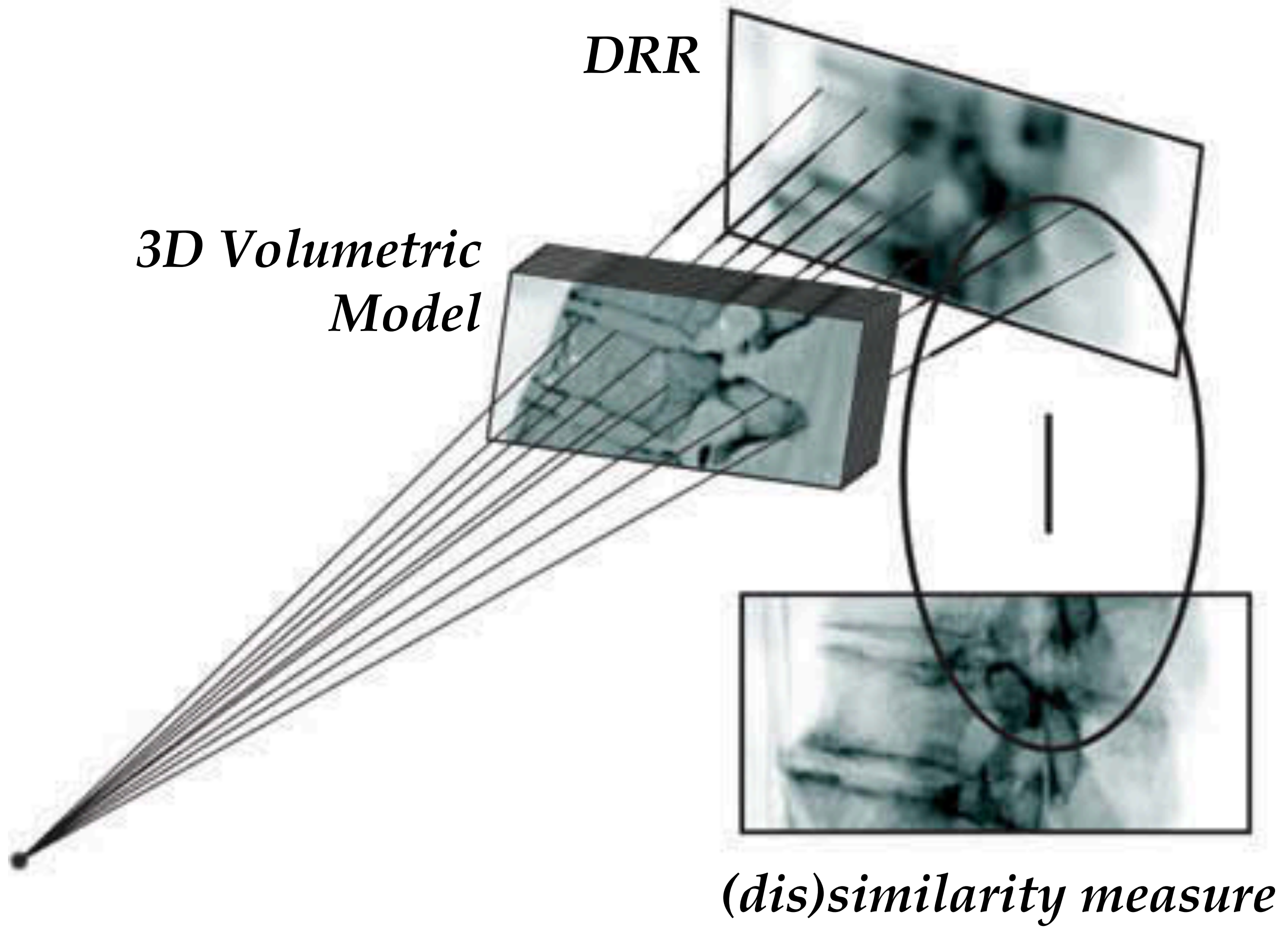
3D-2D Deformable Registration

- *Intensity-based*
  - *Mutual Information (most successful)*
  - *Segmentation Driven*
- *Feature-based*
  - *Point-based (landmark)*
  - *Contour / Silhouette*
- *Hybrid*

Intensity-based



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Intensity-based

J. Yao & R. Taylor **(2003:ICCV, 2003:IJPRAI)**

NMI + Downhill Simplex / Powell's method?

Multiple-layer flexible mesh & mesh deformation

O. Sadowsky et al. **(2006:ISBI, 2007:MICCAI, 2009:MI, 2010:TMI)**

NMI + Downhill Simplex

T.S.Y. Tang & R.E. Ellis **(2005:MICAI)**

MI/NMI + Downhill Simplex

etc.

J. Yao & O. Sawdosky

- ***Normalized Mutual Information***

$$NMI_k = (H(I_k) + H(DRR_k))/H(I_k, DRR_k)$$

H: entropy of pixel intensity distribution

$DRR_k = DRR(t_k, R_k, s, \{w_i\})$

- ***Downhill Simplex***
  - *alternate subsets of parameters (translation (t), rotation (R), global scale (s), mode weights $\{w_i\}$)*
  - *search for the optimal value on each subset*
  - *fix the result when searching the next subset*
  - *multi-resolution, multi-step-size*

Segmentation Driven

T. Brox et al. ***(2005:DAGM)*** *[Saarland University]*

Weighted summation, level set

R. Sandhu et al. ***(2009:CVPR)*** *[Anthony Yezi]*

Active contour w/o edge, gradient flow

T. Brox *et al.* (2005)

Image segmentation coupled w/ pose estimation

$$E(\Phi, \theta\xi) = -\int_{\Omega} \left(H(\Phi)\log p_1 + (1 - H(\Phi))\log p_2\right) dx + \nu \int_{\Omega} |\nabla H(\Phi)|\, dx + \underbrace{\lambda \int_{\Omega} (\Phi - \Phi_0(\theta\xi))^2\, dx}_{\text{Shape}}.$$

$$\exp(\theta\hat{\xi}) = \sum_{k=0}^{\infty} \frac{(\theta\hat{\xi})^k}{k!} \approx \boldsymbol{I} + \theta\hat{\xi}$$

$$\partial_t \Phi = H'(\Phi)\left(\log\frac{p_1}{p_2} + \nu\, \mathrm{div}\left(\frac{\nabla\Phi}{|\nabla\Phi|}\right)\right) + 2\lambda\left(\Phi_0(\theta\xi) - \Phi\right)$$

H: regularized Heaviside function

It can only improve the ***tracking*** *of the object,* ***once a good pose initialization has been found****. How to find such an initialization automatically is a topic on its own.*

R. Sandhu *et al.* (2008)

Active contour w/o edge

$$E = \int_{\mathrm{R}} r_O(I(\mathbf{x}), \hat{c})d\Omega + \int_{\mathrm{R}^c} r_B(I(\mathbf{x}), \hat{c})d\Omega,$$

Gradient descent

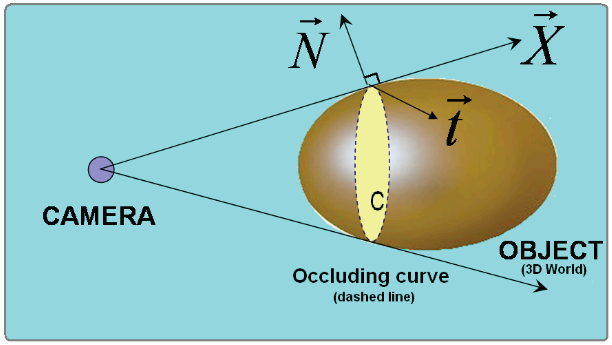
$$\frac{dE}{d\lambda_i} = \int_{\hat{c}} \Big(r_O(I(\mathbf{x})) - r_B(I(\mathbf{x})) \Big) \left\langle \frac{\partial \hat{c}}{\partial \lambda_i}, \hat{\mathbf{n}} \right\rangle d\hat{s} + \int_{R} \left\langle \frac{\partial r_O}{\partial \hat{c}}, \frac{\partial \hat{c}}{\partial \lambda_i} \right\rangle d\Omega + \int_{R^c} \left\langle \frac{\partial r_B}{\partial \hat{c}}, \frac{\partial \hat{c}}{\partial \lambda_i} \right\rangle d\Omega$$

Lifting up to Occluding Contour

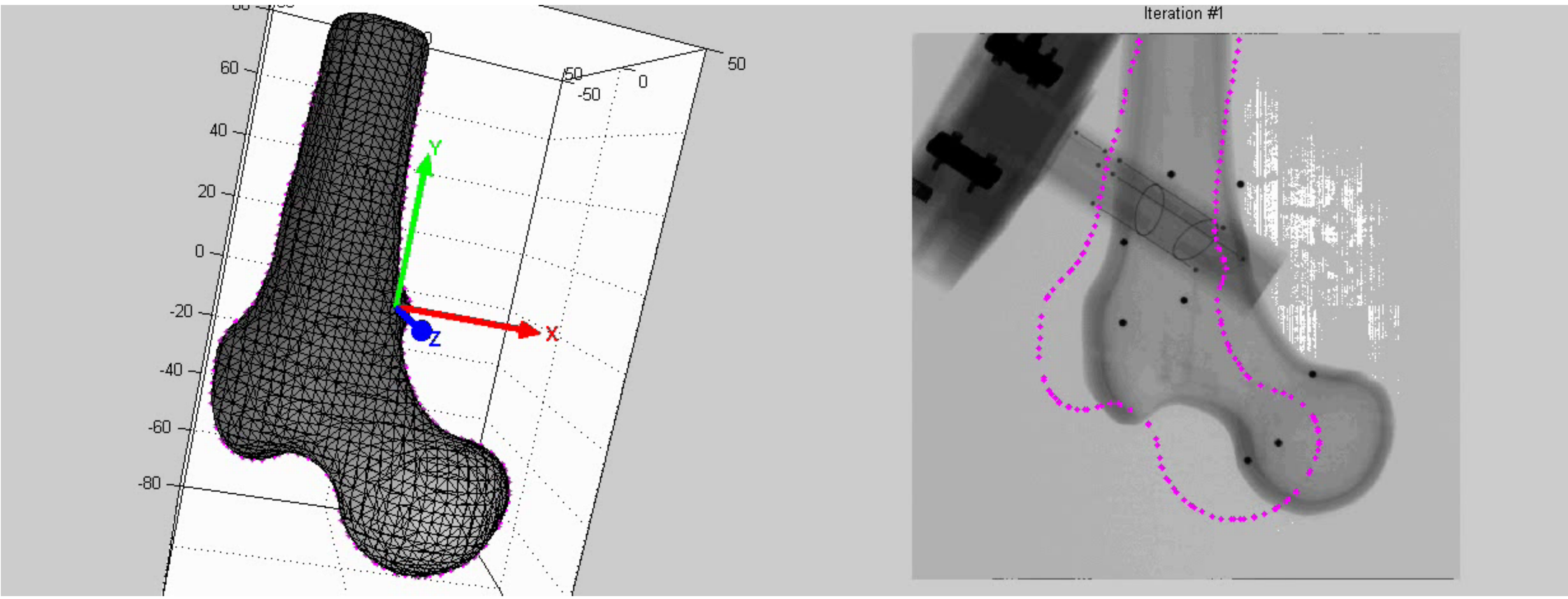
$$\frac{dE}{d\lambda_i} = \int_C \Big(r_O(I(\pi(\mathbf{X}))) - r_B(I(\pi(\mathbf{X}))) \Big) \cdot \frac{\|\mathbf{X}\|}{Z^3} \sqrt{\frac{\kappa_X \kappa_t}{K}} \left\langle \frac{\partial \mathbf{X}}{\partial \lambda_i}, \mathbf{N} \right\rangle ds$$

Deformation

$$\hat{\varphi}(\mathbf{X}_0, w) = \overline{\varphi}(\mathbf{X}_0) + \textstyle\sum_0^k w_i \psi_i(\mathbf{X}_0)$$
$$\text{s.t.} \quad \hat{\varphi}(X_0(w), w) = 0.$$



Demo



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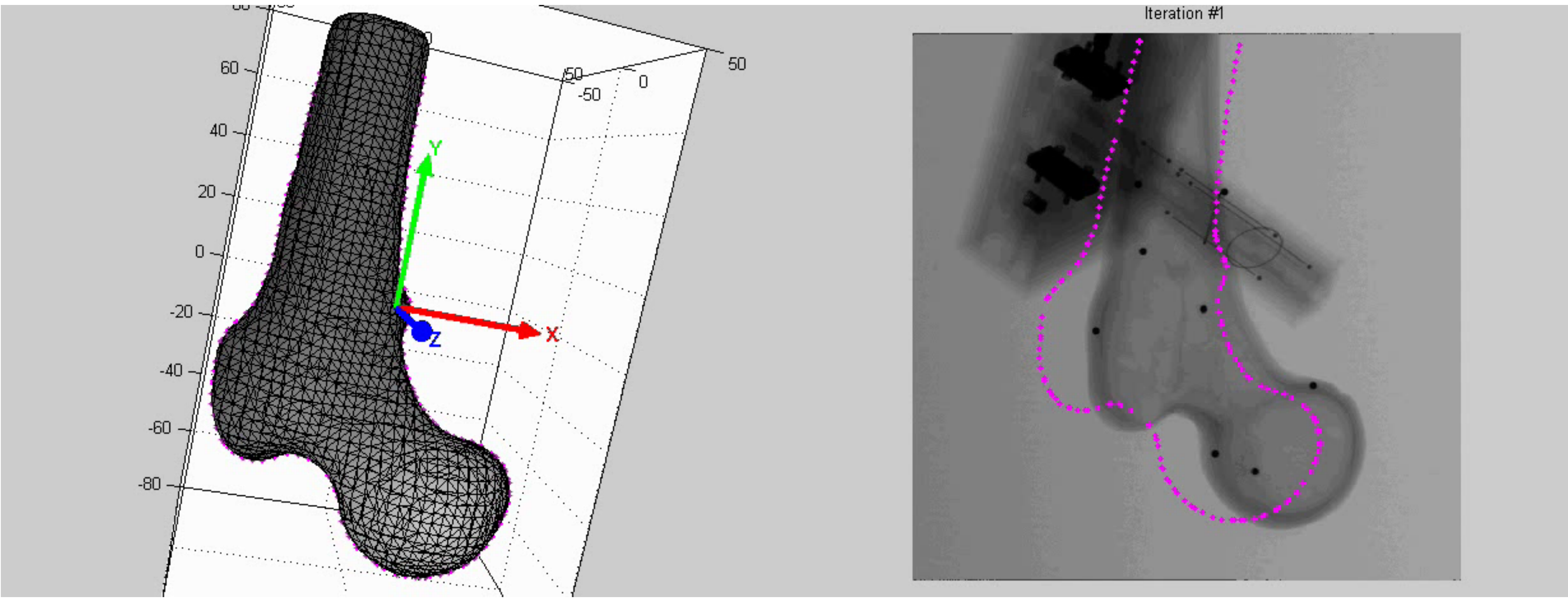
Note this is my implementation.

Demo



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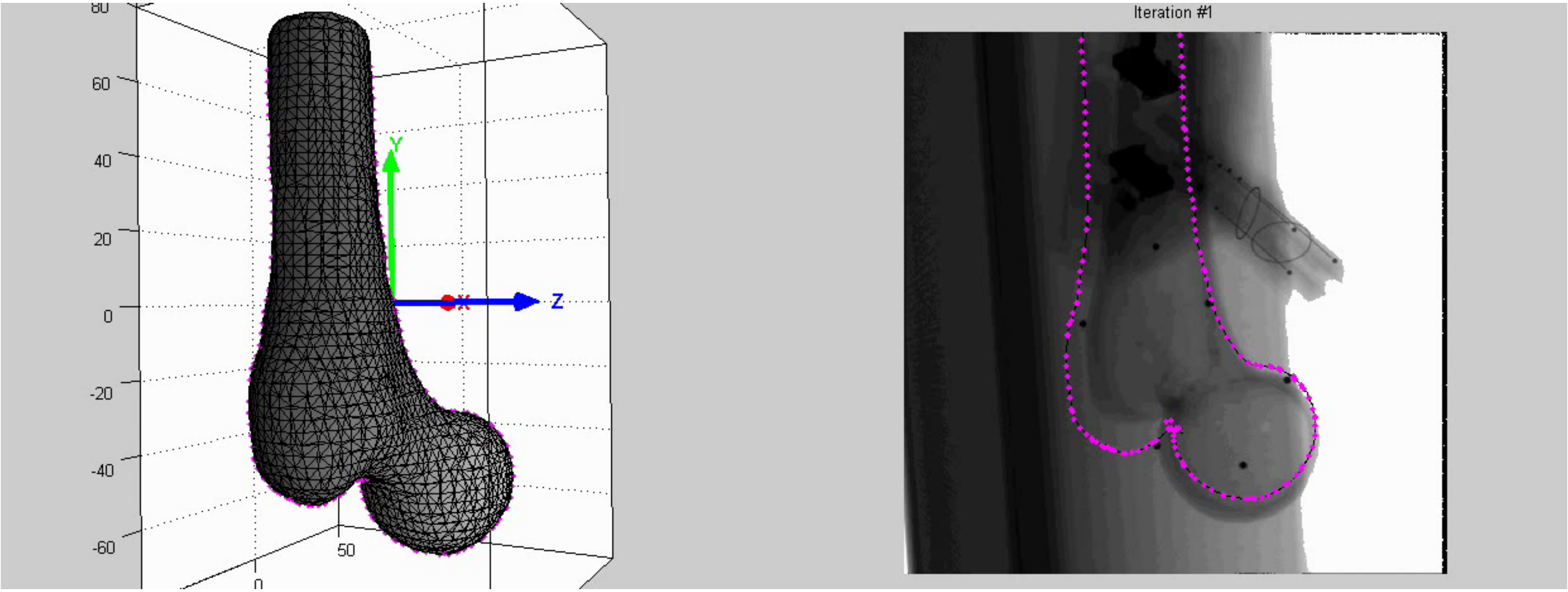
Demo



Note this is my implementation.

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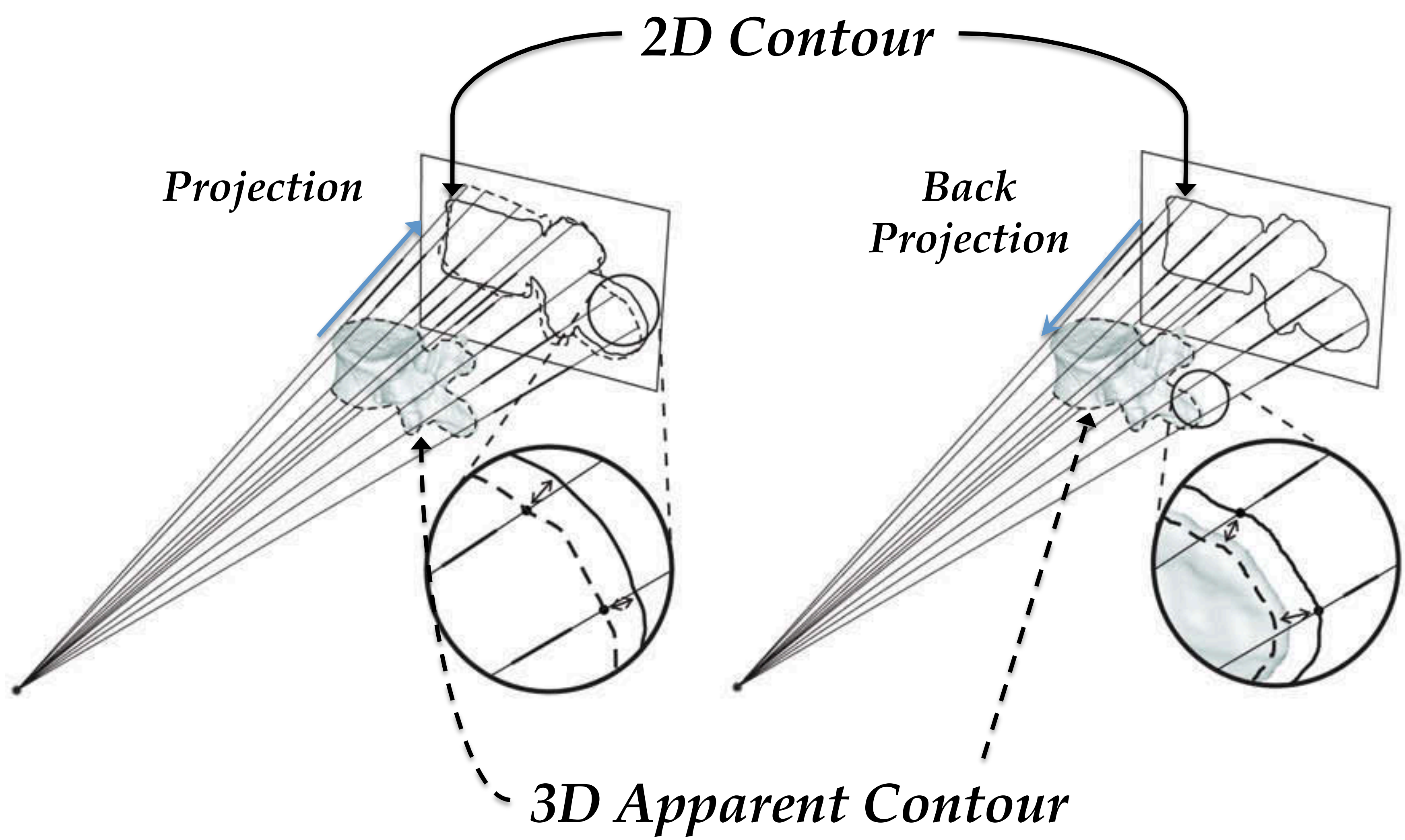
Feature-based



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Silhouette[1]-based

- *Objective function*

$$E(\mathrm{R},\mathrm{T},w_1...w_t)=\sum_{j=1}^{P}\min_{1\le k\le G}\|\mathbf{p}_j-(\mathrm{R}\mathbf{g}_k(w_1...w_t)+\mathrm{T})\|^2$$

projection ray *contour generator points (deformed)*

- *Strategy*
  - *Rigid 3D/2D registration*
  - *Global deformation*
  - *Local deformation*

1. Silhouette is defined as the outer contour of an object.

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Silhouette-based

A. Gueziec & R. Taylor ***(1998:TMI, 1998:SPIE-MI)***

M. Fleute & S. Lavallee ***(1999:MCCAI)***

S. Benameur ***(2003:CVPR, 2003:CMIG, 2005:TBME)***

Gradient descent

H. Lamecker (2006:ICPR)

Gradient descent

R. Kurazume ***(2007)***

Distance map

G. Zheng et al. ***(2007:MICCAI, 2006:MICCAI)*** *[L.P. Nolte]*

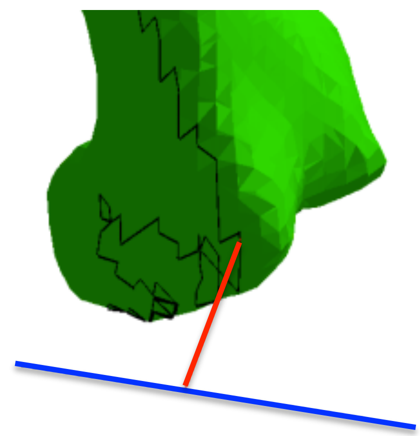
Point-line (vertex-ray) ICP + AX=B (LU)

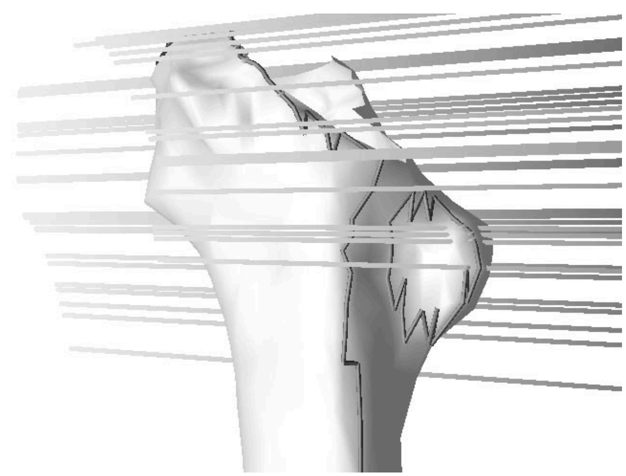
M. Groher et al. ***(2007:MICCAI)***

EBM (EM?)

Variants of ICP

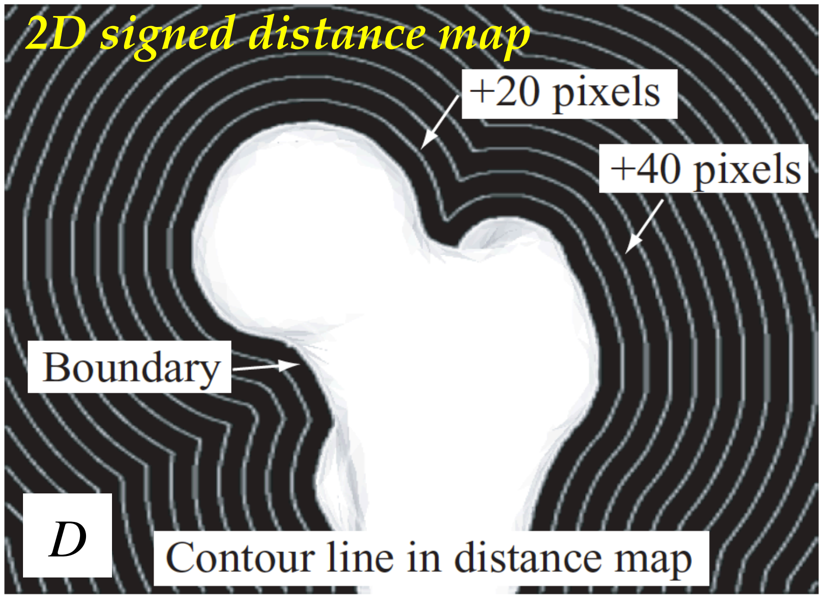
- *Gueziec*
  - *– 3D point-line distance*
- *Fleute*
  - *– 3D line-line distance*
- *Zheng*
  - *– 3D point-line distance*

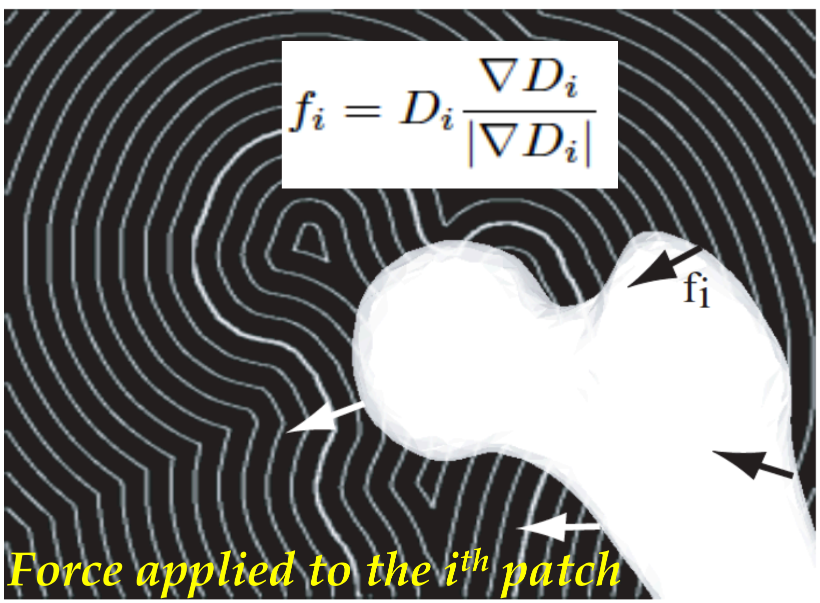




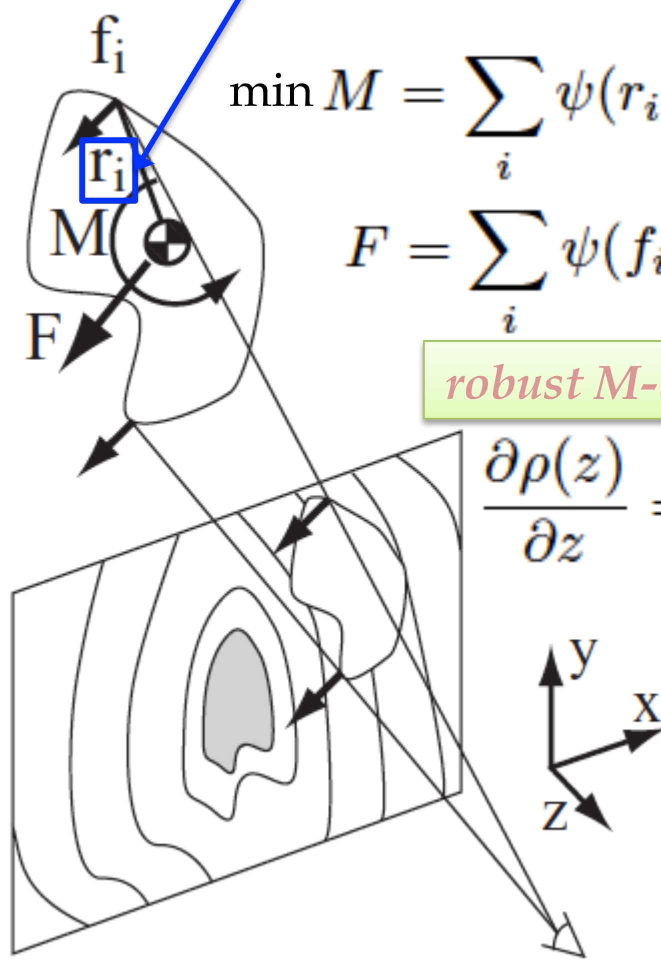
R. Kurazume (2007)

Yoshinobu Sato @ Osaka





a vector from the COG to the i^{th} patch



$$\min M = \sum_i \psi(r_i \times f_i)$$

$$F = \sum_i \psi(f_i)$$

robust M-estimator

$$\frac{\partial \rho(z)}{\partial z} = \psi(z)$$

Total force (F) and moment (M) around the center of gravity (COG).

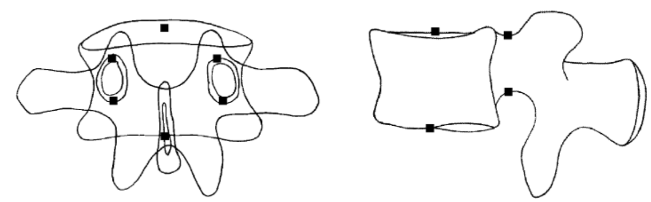
M. Fleute (1999)

$$E(\mathbf{R}, \mathbf{T}, w_1 ... w_t) = \sum_{j=1}^{P} \min_{1 \leq k \leq G} \|\mathbf{p}_j - (\mathbf{R}\mathbf{g}_k(w_1 ... w_t) + \mathbf{T})\|^2$$

- ***Perform rigid 3D-2D registration***
- ***Fix R and T***
- ***Perform Downhill Simplex for*** $\{w_i\}$

S. Benameur (2003, 2005)

Rigid Registration (LS)



- *manually selected landmarks*
- *known correspondences on mean shape*

Global Deformation

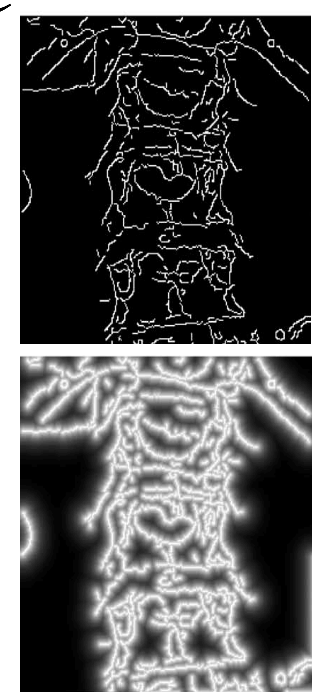
$$-\frac{1}{n_{\mathrm{PA}}}\sum_{\Gamma_{\mathrm{PA}}}\psi_{\mathrm{PA}}(x,y)-\frac{1}{n_{\mathrm{LAT}}}\sum_{\Gamma_{\mathrm{LAT}}}\psi_{\mathrm{LAT}}(x,y)+\frac{1}{2}\sum_{i=1}^{t}\frac{b_i^2}{\lambda_i}$$

projection of the apparent contour *regularization*

$\psi(x,y) =$ *Canny Edge* $\otimes$ *Gaussian*

Gradient descent

\+ *multiple initialization*



$\psi(x,y)$

S. Benameur (2003, 2005)

Local Deformation

$$-\frac{1}{n_{\mathrm{PA}}}\sum_{\Gamma_{\mathrm{PA}}}\psi_{\mathrm{PA}}(x,y)-\frac{1}{n_{\mathrm{LAT}}}\sum_{\Gamma_{\mathrm{LAT}}}\psi_{\mathrm{LAT}}(x,y)+\frac{1}{2}\sum_{i=1}^{t}\frac{b_i^2}{\lambda_i}$$
$$+\frac{1}{2}\sum_{i=1}^{n}\left(\frac{1}{\mu_i^2}\sum_{j\in\mathcal{N}(i)}\|\delta_i-\delta_j\|^2+\frac{1}{\nu_i^2}\|\delta_i\|^2\right)$$

4-neighborhood *Gaussian-Markov process*

Stochastic optimization (Exploration/Selection algorithm[1])

[1] O. François, **Global optimization with exploration/selection algorithms and simulated annealing**, *Ann. Appl. Probability*, vol. 12, pp. 248–271, 2002.

G. Zheng (2007, 2009, 2010)

Rigid Registration (ICP-like, manual initialization)

Global Deformation (Instantiation)

free parameter

$$
\begin{cases}
E_\alpha(\bar{\mathbf{x}}', \mathbf{v}', \mathbf{x}) = (\rho + \log(3n)) \cdot E(\bar{\mathbf{x}}', \mathbf{v}', \mathbf{x}) + E(\mathbf{x}); \\
\mathbf{x} = \bar{\mathbf{x}} + \sum_{k=0}^{m-2} \alpha_k \cdot \sigma_k \cdot \mathbf{p}_k \\
E(\bar{\mathbf{x}}', \mathbf{v}', \mathbf{x}) = (1/n) \cdot \sum_{i=0}^{n-1} \| v_i' - ((\bar{\mathbf{x}}_j)_i + \sum_{k=0}^{m-2} \alpha_k \cdot \sigma_k \cdot \mathbf{p}_k(j)) \|^2 \\
E(\mathbf{x}) = (1/2) \cdot \sum_{k=0}^{m-2} (\alpha_k^2)
\end{cases}
$$

image points (2D) *model point (3D)* *optimize the coeff.*

regularization

Minimize the (average) projection error while regularizing the modes.

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G. Zheng (2007, 2009, 2010)

Local Deformation

deformed atlas

$$E(\mathbf{t}) = (1/l) \cdot \sum_{i=0}^{l-1} ||v_i' - \boxed{\mathbf{t}(v_i)}||^2 + \boxed{\tau} \cdot \boxed{\frac{\log(m)}{\log(3l)}} \cdot \underline{L(\mathbf{t})}$$

free parameter ? *thin-plate spline*

$$\begin{cases} L(\mathbf{t}) = \iiint_{\Re^3} (B(\mathbf{t}))dxdydz; \quad \text{and} \\ B(\cdot) = \left(\frac{\partial^2}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2}{\partial y^2}\right)^2 \\ \qquad + 2\left(\frac{\partial^2}{\partial y \partial z}\right)^2 + \left(\frac{\partial^2}{\partial z^2}\right)^2 + 2\left(\frac{\partial^2}{\partial z \partial x}\right)^2 \end{cases}$$

M. Groher *et al.* (2007)

Segmentation-Driven *2D-3D Registration for Abdominal Catheter Interventions*

3D point cloud —***projection***→ ***2D points***

vessel centerlines & bifurcation locations

vessel centerlines

derivative-mean filter [Can99]

Error measurement (in 2D)

$$\varepsilon(\mathbf{x}) = d(\mathbf{x}, C(\mathbf{x}, \{\mathbf{P}_\Theta \mathbf{X_j}\}))^2$$

distance of closest point

Density function (in 2D)

isotropic Gaussian

$$P(\Theta^{(t-1)} | \ell_\mathbf{x} = 1, I, \mathcal{M}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\varepsilon(\mathbf{x})/\sigma^2}$$

generating seeds for region growing

3D-2D registration

Downhill Simplex

$$\min \sum_\mathbf{x} \varepsilon(\mathbf{x})$$

Initialization

exhaustive search

Hybrid

A. Hurvitz & L. Joskowicz *(2008:IJCARS)*

2D-2D B-spline registration + *2D-3D ICP*

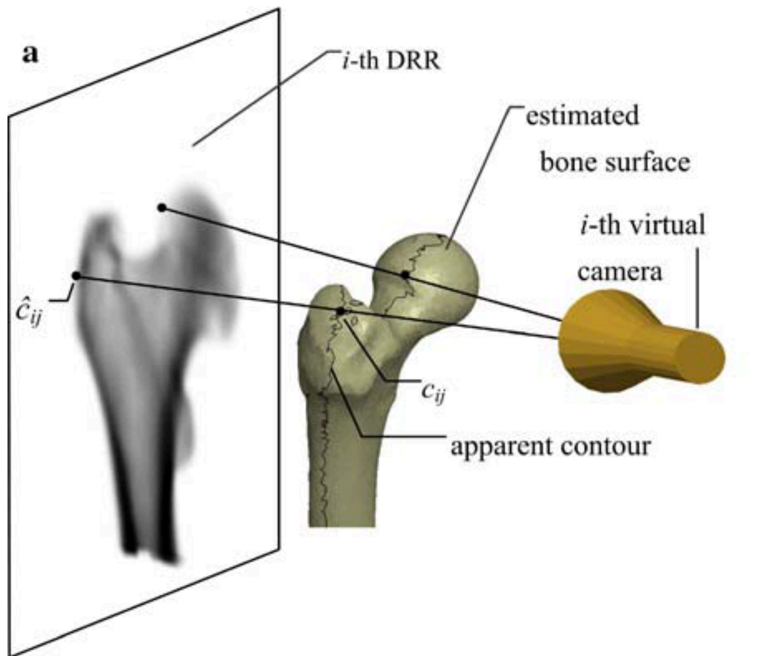
J. Yao & R. Taylor *(2003:IJPRAI)*

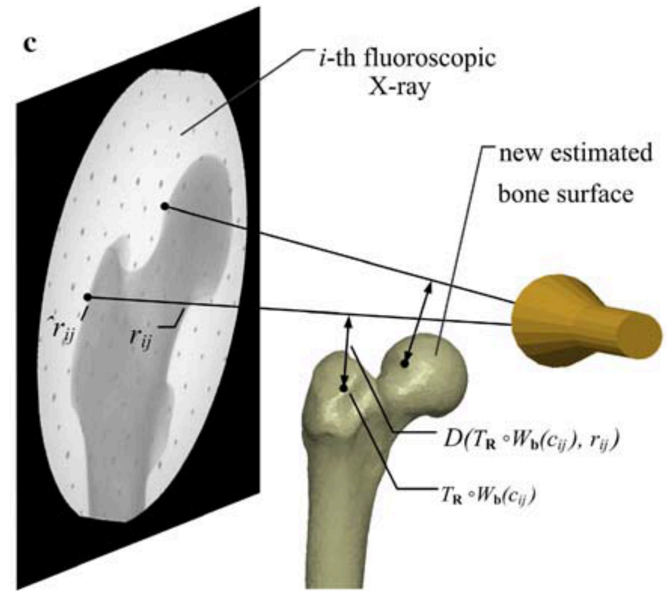
Weighted summation (NMI? + *attribute vector*)

Powell's method

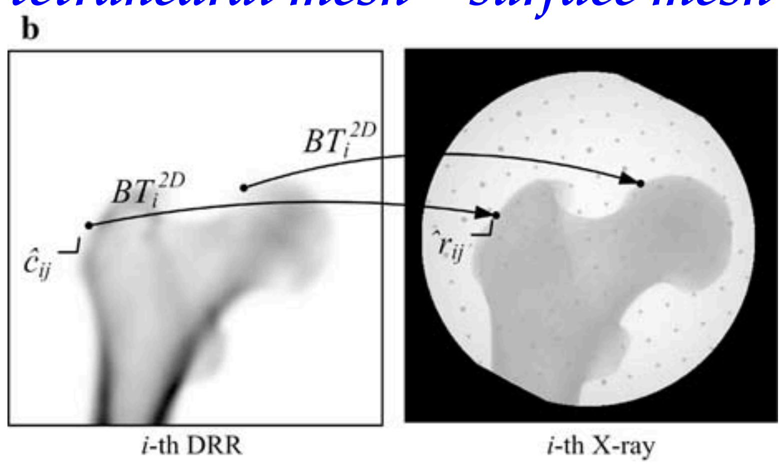
Multiple-layer flexible mesh & mesh deformation

A. Hurvitz & L. Joskowicz (2008)





tetrahedral mesh + surface mesh



a. Project apparent contours

b.1. 2D-2D intensity-based deformable registration using B-spline

b.2. Apply B-spline transformation to the projections of apparent contours

c. Minimize the sum of point-to-ray distances (ICP-like, feature-based) using Pattern Search algorithm

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J. Yao & R. Taylor (2003)

shape difference *density difference*

$$E(mdl, img) = w_s E^{(s)}(mdl, img) + w_d E^{(d)}(mdl, img)$$

$N(v)$ ← all vertices on the model ?

$$E^{(s)}(mdl, img) = \sum_{i=1}^{N(v)} (\mathbf{g}^{(mdl)}(v_i) \cdot \boldsymbol{g}^{(img)}(v_i))$$

surface normal *intensity gradient*

voxel density *image density*

$$E^{(d)}(mdl, img) = \sum_{i=1}^{N(t)} \left(\oint_{\mu} \left(\left(\frac{d^{(mdl)}(t_i, \mu) - d^{(img)}(t_i, \mu)}{d^{(mdl)}(t_i, \mu)} \right)^2 \right) \right)$$

Powell's method

J. Yao & R. Taylor (2003)

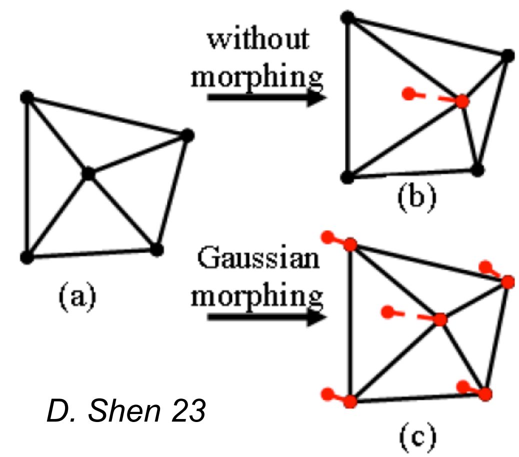
Constrained Local Deformation

- ***Attribute vector***

  *(d ; **g**) = density + gradient*

- ***Gaussian morphing***

  $$\Delta v_0 = v'_0 - v_0 \quad \Delta v_l = \Delta v_0 \cdot e^{-\frac{l^2}{2\sigma^2}}$$



D. Shen 23

- ***Adaptive deformation focus***

  The vertex with highest matched image attribute vector will deform first and drag its neighbors to morph. Then, move the focus to the next highest matched vertex.

- ***Maximum deformation range***

Thank you!